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論文内容の要旨

Simultaneous estimation of a high-dimensional parameter is and will be a very important subject of research. The recent rapid progress in computational environment has made it easy to collect a complex dataset. The role of a statistical model containing a high-dimensional parameter, which is suited for such a complex dataset, is getting more and more significant in accordance with this progress.

The aim of this dissertation is investigate simultaneous estimation of a high-dimensional parameter from a view point of a Pythagorean relationship both theoretically and practically. Theoretical researches on estimation in such complex models are needed from the viewpoint of application. Also actual estimation procedures with nice properties are in high demand. The James-Stein estimator (James and Stein, 1961) is a breakthrough estimator in this area. A number of works have been devoted to elucidating the reason why the James-Stein estimator or its modifiers perform well. There seems to be the following two approaches to casting light to the reason. One is a Pythagorean relationship holding among the maximum likelihood estimator (MLE), the James-Stein estimator and the true parameter, which was pointed out first by Stein (1981). The other approach is the interpretation of the James-Stein estimator as an empirical Bayes estimator, as proposed in Efron and Morris (1973).

The theoretical aspect of the motivation of this dissertation is as follows. A Pythagorean relationship is one of the most natural and fundamental notions by which we can improve upon something or show a certain inequality. For example, the Pythagorean relationship pointed out by Stein (1981) makes it clear how the James-Stein estimator dominates the MLE. It is expected that such a Pythagorean relationship will lead to better estimation in a unified way. Yanagimoto (1994, 2000) discussed Stein-type estimation from this point of view. We show in Chapter 3 that a Pythagorean relationship holds in the field of estimating functions, although the original Pythagorean relationship by Stein (1981) does in the field of estimators.

Further, in Chapters 4 and 5 which investigate the Bayesian analysis, a Pythagorean relationship plays another important role. The optimality of the Bayes estimator is understood quite easily through the Pythagorean relationship. The three points, the Bayes estimator, an arbitrary estimator and the true parameter value, constitute on the average a modified right triangle. The modified triangle makes it clear how the Bayes estimator is superior to an arbitrary estimator.

Here we state the practical aspect of the motivation of this dissertation. As indicated by the fact that the James-Stein estimator is regarded as an empirical Bayes estimator (Efron and Morris, 1973), the Bayesian approach seems to be promising in the estimation of a high-dimensional parameter. Although Chapter 3 investigates the problem from the frequentists' viewpoint, some of the obtained estimating functions can be interpreted from the Bayesian point of view. The empirical Bayesian method provides us with practical inferential procedures for a vector parameter. A difficulty in constructing an empirical Bayes estimator lies in that there are a restricted number of families of prior densities. This is why some useful families of prior densities are necessary.

Conjugate priors, originally introduced by Raiffa and Schlaifer (1961, p. 43-58), are of great use for their desirable properties and was assumed by Efron and Morris (1973) in deriving the empirical Bayes estimator. To extend the notion of conjugate priors is of great significance in this respect. A recent and extensive review of the conjugate priors is found in Gutierrez-Pena and Smith (1997). Chapters 4 and 6 present two methods for eliciting prior densities.

The organization of this dissertation is as follows. In Chapter 2, some basic concepts and tools are presented, which lead to better understanding of the subsequent chapters.

In Chapter 3, which is based on Ohnishi and Yanagimoto (2003), a unified approach using a Pythagorean relationship reveals the mechanism through which the maximum likelihood estimation can be improved upon. A Stein-type estimation of location vectors is discussed in terms of estimating functions. We assess the superiority of an estimating equation by its mean squared norm. The Coulomb potential function in electrostatics leads to a Pythagorean relationship with respect to this norm. By making full use of the Pythagorean relationship, we improve upon the likelihood estimating function. A further improvement is shown to be feasible under a certain condition. We pursue possible strong relationships between the superiority over the likelihood estimating function and physical quantities appearing in the theory of electrostatics.

In Chapter 4, we enrich the notion of conjugate prior distributions in two directions and investigate the Bayesian analysis assuming the introduced prior densities. A conjugate prior for the exponential family, referred to also as the natural conjugate prior, is represented in terms of the Kullback-Leibler separator. This representation permits us to extend the conjugate prior to that for a general family of sampling distributions. Further, by replacing the Kullback-Leibler separator with its dual form, we define another form of a prior, which is called the mean conjugate prior. Various results on duality between the two conjugate priors are shown. Implications of this approach include richer families of prior distributions induced by a sampling distribution and the empirical Bayes estimation of a high-dimensional mean parameter. A Pythagorean relationship with respect to the Kullback-Leibler separator is used both to show the optimality of the Bayes estimator and to construct an empirical Bayes estimator. This chapter is due to Yanagimoto and Ohnishi (2002, 2003).

In Chapter 5, we introduce specific location-dispersion models and discuss a conjugate analysis by assuming some prior density. The models are called the l-additive location-dispersion models, and we apply one of the prior elicitation procedures in Chapter 4. The l-additive location-dispersion model is generated by the density function whose logarithm satisfies a certain addition identity, an extension of the addition formula for the (hyperbolic) cosine function. The addition identity can be interpreted in the light of statistical mechanics. We show that l-additive location-dispersion models consist of the familiar five models. The assumed prior density is proved to be closed under sampling. We also calculate the Bayes estimator under a Kullback-Leibler loss function. A unified approach proves that the posterior mode, which has an analytical form, is optimal. Empirical Bayes estimators of location vectors are constructed explicitly in the five l-additive location-dispersion models. This chapter is based on Ohnishi and Yanagimoto (2002)

In Chapter 6, we propose a prior elicitation method other than the one in Chapter 5. The key feature is the use of the likelihood of the distribution of the MLE. The derived prior density is proved to be an extension of a conjugate prior density. Three examples including the l-additive location-dispersion model in Chapter 5 are presented in order to clarify our idea. The applicability of our method is discussed with the use of the Barndorff-Nielsen's p^*-formula (1983).

論文の審査結果の要旨

審査委員会は大西君の論文について慎重に検討し、公開論文発表会を開催し審査を行った。同君は、高次元母数を含むモデルにおける推定問題を (1)ピタゴラス関係、(2)Bayes推定及び(3)物理学的解釈という3つの観点から系統的に論じ、理論と関連する統計的手法において幾つかの重要な結果と種々の興味ある知見を得た。これらの貢献により、同君の論文は博士号取得に十分値する内容を備えていると判断した。

<u>論文の概要及び貢献点</u>：

論文は6章から成っており、その主要な貢献は、第3章と第4章で扱われている、以下の二つの重要課題への取り組みとその解決を図ったことにある。

一つ目は、location family における母数ベクトルの推定問題を、推定関数の一般的立場から論じたことである。申請者は母数の次元が高いときに最尤推定の振舞が必ずしも良くないという事実の理論的な側面を、ある種の統一的な視点－ピタゴラス関係－から明らかにした。推定関数の理論では、推定関数の良さを評価する一つの基準として推定関数の平均2乗ノルムを用いるというトレース基準が知られている。特に、尤度推定関数はこの基準の下で不偏な推定関数の中で最適であることもよく知られている。申請者は、この基準がバイアスのある推定関数に対しても有効であることに注意し、バイアスはあるが尤度推定関数を優越する Stein 型推定関数を次のピタゴラス関係の成立を示すことにより明快に導出した。ここでのピタゴラス関係とは、尤度推定関数を斜辺として、Stein 型推定関数をその他の一辺とする平均的な直角三角形の成立を意味する。この関係を導くときに決定的な役割を果たしているのが，ラプラス方程式の基本解、すなわち1点を除いて調和な関数である。申請者はさらに、ある一定の条件が満たされているときに、Stein 型推定関数を優越する推定関数が構成できることを示した。これらの推定関数と物理学の一分野である静電気学における物理量との関連についても示唆に富む言及を行った。

二つ目は、Kullback-Leibler (K-L) 情報量に着目した共役事前分布の拡張である。高次元母数モデルでは Bayes 推定が良い振舞をすることが広く認識されている。申請者は、理論・応用の両面から興味のある、モデルから自動的に事前分布を導出するスキームに取り組んでいる。指数型分布族における共役事前分布を K-L 情報量を用いて表現すると、二つの方向への拡張が可能である。一つは，指数型分布族以外へ共役事前分布の概念を拡張することである。ここで、事前分布と事後分布が同一の分布族に属するという意味での共役性は満たされなくなる。申請者は、この共役性は指数型分布族に内在するピタゴラス関係と深く結びついていることを強調した。別の拡張として、申請者は共役事前分布あるいはその拡張において用いた K-L 情報量と双対な量を用いて事前分布を構成した。特にモデルが指数型分布族の場合、これを平均共役事前分布と呼んで、新しい事前分布の族が定義できることも明らかにしている。

公開論文発表会の後で、質疑が行われ、統計理論に現れる物理学的視点や、今後の発展の可能性について、申請者の知識と意見が質された。応答は的確で、また関連事項について明快な意見が述べられた。なお、本論文に関連して既に５篇の論文が関連の学会誌等に発表されている。

審査委員会の審議結果

論文報告および質疑応答等に基づき、学位請求論文が統計科学の基盤における難解な部分に独自の統一的観点から意義ある貢献を行っていること、また申請者が統計科学および関連分野の専門知識を十分有していることを認めた。以上の理由により、審査委員会は大西君からの学位請求論文を博士論文として十分なものと判断した。