Column density distribution of the Lyman \( \alpha \) forest – evidence for the minihalo model

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Summary. The column density distribution of the Lyman \( \alpha \) forest is predicted for the minihalo model, in which the baryon clouds responsible for the absorption lines are supposed to be in gravitational equilibrium under the potential of cold dark matter (CDM). It is determined primarily by the density profile across a typical cloud. The evolution of minihalos within various environments is discussed in relation to the redshift dependence of the Lyman \( \alpha \) forest.

1 Introduction

Data on the Lyman \( \alpha \) forest have accumulated rapidly since the classic work of Sargent et al. (1980). Statistical studies indicate several regularities (Bechtold 1987):

(i) the number density evolves as \( dn/dz \propto (1 + z)^\gamma \), with \( \gamma = 2.1 \);
(ii) the \( \text{H} \text{I} \) column density distribution is approximately given by \( dn/dN_{\text{HI}} \propto N_{\text{HI}}^{-\beta} \) with \( \beta = 1.6 \sim 1.9 \) (Murdoch et al. 1986, Carswell et al. 1987), and
(iii) there is no correlation in one-dimensional velocity space except possibly for very small velocity differences.

These characteristics suggest that the Lyman \( \alpha \) forest is not directly related to ordinary galaxies and may involve intergalactic clouds.

Several models relating the origin of the Lyman \( \alpha \) forest to galaxy formation have been proposed (e.g. Ikeuchi 1987, 1988). Self-gravitating clouds cannot be in stable gravitational equilibrium unless their dynamical time is as long as the Hubble time or they are rotating (cf. Black 1981). The clouds could be contracting (Hogan 1987) or expanding (Bond, Szalay & Silk 1988). If they are photoionized by the diffuse ultraviolet (UV) flux from quasars, the temperature goes up to \( \sim 3 \times 10^4 \) K and a cloud with \( \text{H} \text{I} \) column density \( N_{\text{HI}} \sim 10^{14} \text{ cm}^{-2} \) is far below the Jeans mass. This means that the clouds must be confined either by the pressure of ambient intergalactic gas (Sargent et al. 1980; Ostriker & Ikeuchi 1983; Ikeuchi & Ostriker 1986) or by the gravity of dark matter (Ikeuchi 1986; Rees 1986). The former (pressure-
confined) clouds would be expanding with time, as the external pressure falls with the expansion of the universe. The latter clouds, 'minihalos' (Rees 1986, 1987), could be close to gravitational equilibrium.

These alternative models both involve unknown parameters, which relate to the details of galaxy formation and evolution. The 'minihalos' required for the gravitationally confined option are natural products of the cold dark matter (CDM) scenario. In this paper we examine the physical properties and gravitational equilibrium of baryon clouds under the potential of CDM and we calculate the H\textsc{i} column density distribution expected in this model, which depends primarily on the equilibrium density profile of gas in a typical minihalo potential well. In a succeeding paper we shall consider the evolution of clouds with redshift, taking into account the time variation of UV flux, the destruction of minihalos by mergers into layer systems, and the mass accretion onto clouds.

2 Gravitational equilibrium of a minihalo

Here, we assume that the clouds are photoionized and heated by the diffuse UV flux and keep the temperature as $T_c = 3 \times 10^4$ K for simplicity (Ikeuchi & Ostriker 1986), ignoring the slow dependence on the ionization parameter (Black 1981) and possible Compton heating at low density. The basic equations for the gravitational equilibrium of a cloud are as follows:

$$\frac{1}{\rho_b} \frac{dP_b}{dr} = - \frac{G}{r^2} (M_{b,r} + M_{d,r}),$$

$$(1)$$

$$\frac{dM_{i,r}}{dr} = 4 \pi r^2 \rho_i \quad (i = b, d),$$

$$(2)$$

$$P_b = \rho_b k T_c / m_b c_b^2,$$

$$(3)$$

where the suffices b and d denote baryons and dark matter, respectively, and $M_b$ and $M_d$ are the masses of the two components within the radius $r$. Other symbols have their usual meanings. We assume that the CDM obeys the isothermal distribution

$$\frac{c_d^2}{\rho_d} \frac{d\rho_d}{dr} = - \frac{G}{r^2} (M_{b,r} + M_{d,r}),$$

$$(4)$$

where $c_d$ is the velocity dispersion of CDM. Since we suppose that the cloud is also isothermal, the distributions of baryons and CDM are related as

$$\rho_b(r) = \rho_b(0)(\rho_d(r)/\rho_d(0))^X,$$

$$(5)$$

with $X = c_d^2 / c_b^2$, the ratio of the dark matter's virial temperature to the gas temperature. Then, if $X \gg 1$ we can consider the distribution of CDM as nearly uniform.

The ionization equilibrium of the cloud medium, assumed to have a primordial abundance of $n_{\text{II}}/n_{\text{He}} = 9$, exposed to a diffuse UV flux $J_0$, is written as

$$J_0 G_i n_i = \alpha_{i+1} n_e n_{i+1}$$

$$(6)$$

with $i, i+1 = (\text{H} \text{I}, \text{H} \text{II}), (\text{He} \text{I}, \text{He} \text{II})$ and $(\text{He} \text{II}, \text{He} \text{III})$. The number density of electrons is given by $n_e = n_{\text{H} \text{II}} + n_{\text{He} \text{II}} + 2 n_{\text{He} \text{III}}$. $G_i$ and $\alpha_{i+1}$ are taken from Black (1981). The H\textsc{i} column density for a line-of-sight through a cloud with impact parameter $b$ is
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\[ N_{H_1}(b) = 2 \int_{b}^{R} n_{H_1}(r) r(r^2 - b^2)^{-1/2} \, dr, \]  

(7)

where \( R \) is the outer radius of a cloud.

The free parameters for the present model are

\[ C = \rho_b(0)/\rho_d(0), \quad D = \rho_d(0)/\rho_{\text{crit}(z=4)}, \quad X = c_s^2/c_b^2 \quad \text{and} \quad J_0. \]  

(8)

The central density of CDM in a minihalo obviously depends on the turnaround redshift (being higher for those that collapse earlier); it also depends on the internal structure within the minihalo - in particular, on whether it is centrally condensed with a small core radius. Here it is normalized to the critical density at \( z = 4 \) for convenience. First, we investigate a simple case when dark matter is exactly uniform (Ikeuchi 1986; Ikeuchi & Norman 1987). (This is actually more relevant to dark matter which is hot, either because it is in low-mass neutrinos, or because it is CDM in an already virialized system of larger scale.) From this, we can see the range of masses and densities of clouds with the \( H_1 \) column density of \( N_{H_1} = 10^{14} \sim 10^{16} \) cm\(^{-2}\). After that, we examine the case of finite \( X \) (Case 2). The results are as follows:

CASE 1

The mass and radius of a baryon cloud embedded in a uniform gravitating medium are shown in Fig. 1 with respect to the central baryon density \( C \) for various \( D \). As is seen, the total mass is approximated as

\[ M_{b(r=R)} \propto f(C) D^{-1/2}, \]

(9)

\[ f(C) \begin{cases} \propto C & \text{at } C \leq 1, \\ \text{constant} & \text{at } 1 < C < 10, \\ \propto C^{-0.1} & \text{at } C > 10. \end{cases} \]

(10)

\[ \begin{array}{cccc}
\text{M}_b \quad (\text{M}_\odot) & \text{D} = 0.1 & \text{D} = 1 & \text{D} = 10 \\
\text{R} \quad (\text{kpc}) & \text{---} & \text{---} & \text{---}
\end{array} \]

Figure 1. The dependence of baryonic mass distribution on the central baryon density for 'Case 1', when the dark matter density is uniform. Three cases are plotted, for \( D \) (the ratio of the dark matter density to the mean cosmic density at \( z = 4 \)) equal to 0.1, 1 and 10. The solid and dotted lines denote, respectively, the mass and radius.
As in a neutron star, there is a critical density, $C_{\text{crit}}$, for which the cloud mass has a maximum (Ikeuchi 1986). $C_{\text{crit}}$ is about 10, above which the cloud can be considered to be self-gravitating, and is unstable. The cloud radius is nearly constant. A uniform background medium therefore stabilizes gas clouds for $C \leq 10$. Stable clouds show an exponential density fall-off towards their boundary, and the central condensation increases with the increase of $C$. The H\textsc{i} density scales as

$$n_{\text{H}\textsc{i}} \propto J_0^{-1} C^2 D^2 \quad \text{at} \quad C \leq 1.$$  

(11)

Accordingly, the H\textsc{i} column density scales as

$$N_{\text{H}\textsc{i}} \propto J_0^{-1} C^2 D^{3/2},$$  

(12)

and its profile with respect to the impact parameter $b$ is shown in Fig. 2. For the range $N_{\text{H}\textsc{i}} = 10^{13}-10^{17}$ cm$^{-2}$, the H\textsc{i} column density decreases exponentially with $b$. Therefore, the column density distribution is given as

$$dn/dN_{\text{H}\textsc{i}} \propto N_{\text{H}\textsc{i}}^{-1} \log(N_{\text{H}\textsc{i}})$$  

(13)

This is too flat in comparison with observations.

Figure 2. The H\textsc{i} column density is here plotted versus the impact parameter $b$ for a ‘Case 1’ model (see Fig. 1) with $D = 1$. Three examples are given, corresponding to values of $C$ (the ratio of the central baryonic to the dark matter density) of 0.1, 1 and 10. The present ionizing flux is taken as $10^{-23}$ erg Hz$^{-1}$ s$^{-1}$ cm$^{-2}$ and is scaled to $z = 2.5$, assuming $J \propto (1+z)^4$; the neutral column densities scale inversely with $J_{23}$, except that ‘self-shielding’ (not fully included here) is significant for column densities above $10^{17}$ cm$^{-2}$. 

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CASE 2

Since the baryon distribution resembles a self-gravitating isothermal one, the cloud extends infinitely for \( X < 1 \). We define the cloud boundary (somewhat arbitrarily) as \( \rho_b(R) / \rho_b(0) = 10^{-6} \). (In reality, the quasi-static approximation will break down in the outer part of the clouds, and they will merge into the general intergalactic medium.) The total baryon mass with respect to the central density \( C \) is shown in Fig. 3 for \( D = 1 \). For \( X > 1 \), the result is essentially the same as Case 1: that is, the baryons are more centrally-concentrated than the CDM. For \( X = 1 \), the maximum mass is attained for \( C \sim 1 \) in which the baryons and CDM exactly follow the same distribution. For values of \( X \) below unity, the only realistic models would have \( C < 1 \); otherwise the self-gravity of baryons dominates at large radii and the cloud becomes unstable.

For \( X = 1 \), the baryon density follows the isothermal distribution \( n_b(r) \propto r^{-2} \). Since the cloud is highly ionized, the \( \text{H}1 \) density decreases as \( n_{\text{H}1} \propto r^{-4} \) and its column density as \( N_{\text{H}1} \propto b^{-3} \). Then, the \( \text{H}1 \) column density distribution becomes (Rees 1987; Milgrom 1988)

\[
\frac{dn}{dN_{\text{H}1}} \propto N_{\text{H}1}^{-5/3}.
\]

(14)

This dependence also holds in the outer parts of clouds with \( X < 1 \), as is seen in Fig. 4. This means that the column density distribution law (14) holds for a wide range of cloud parameters. Clouds with \( X > 1 \) yield a somewhat steeper power law than (14). The power \( \beta = 5/3 \) is very similar to the observed value. Then, we may conclude that the \( \text{H}1 \) column density distribution reflects the distribution law of baryons within a minibulge, irrespective of the mass function of minibulges. The distribution of column densities from \( 10^{13} \) to \( 10^{17} \, \text{cm}^{-2} \) can naturally arise from lines-of-sight passing through a fairly standard population of clouds, but with different impact parameters. We regard this as an important advantage of gravitationally-confined over pressure-confined models for the Lyman \( \alpha \) clouds – for the latter, the \( \text{H}1 \) density is fixed once the external pressure is given, and it is hard to explain a big range of column densities without an implausibly large spread in cloud sizes. The column densities

**Figure 3.** The same quantities as in Fig. 1 are here plotted for Case 2, where the dark matter has a rms velocity dispersion \( X \) times the sound speed of the gas. The chosen values of \( X \) are 0.1, 0.8, 1.0, 1.2 and 3.0, with \( D = 1 \) for each example. The dash-dotted lines indicate the results of Case 1 (corresponding to \( X \to \infty \)).

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Figure 4. The H\textsc{i} column density is here plotted versus the impact parameter $b$ for various 'Case 2' models. The parameters shown are $(C, X) = (10, 1), (10, 0.1), (1, 1), (0.1, 1), (0.1, 3)$ and $(0.1, 10)$. The column density falls off as a power law (roughly proportional to $b^{-5}$) for models with $X \leq 1$. Minihalos which formed at $z > 4$, or which are very centrally condensed, may have central dark matter densities of up to $D = 100$, rather than $D = 1$ as shown here. The results for any value of $D$ can be read from this figure by scaling $b$ as $D^{-1/2}$ and the column density as $D^3$. The column density of H\textsc{i} also scales as $J^{-1}$ as in Fig. 2, the curves plotted correspond to a redshift $z = 2.5$, assuming a UV background which, adiabatically expanded to the present epoch, yields $J_{23} = 1$.

above $10^{17}$ may come from genuine protogalaxies. Our present discussion does not apply to these, since we have not consistently included self-shielding of clouds from the background ionizing flux. Over the range $10^{13} - 10^{17}$ cm$^{-2}$, this model predicts a negative correlation between the column densities and transverse dimensions of absorbing clouds.

3 Overview

The minihalo model accounts for the spread in the H\textsc{i} column-density distribution of the Lyman $\alpha$ forest; moreover the distribution of minihalos according to the CDM scenario is indeed expected to be more homogeneous than the galaxy distribution. Two of the three observed characteristics mentioned in Section 1 are thus naturally explained in this model. Since we constrain ourselves to equilibrium minihalos, we cannot discuss the number-density evolution at present. However, we suspect it is influenced by the following effects:

(i) Clouds with $C \gg 1$, where the baryonic gas in the inner parts is self-gravitating and unstable, collapse to dwarf galaxies (Ikeuchi & Norman 1987; Rees 1987). Even if they are stable when first formed, radiative cooling, the decrease of UV flux (leading to self-shielding and cooling of the central regions) and the accretion of baryons are three effects that could
make the clouds unstable. We will investigate the consequent evolution of the clouds in a separate paper.

(ii) The minihalos will merge by their mutual collisions. These merging processes directly decrease the space density of minihalos; the merged halos will tend to have a larger internal velocity dispersion $c_4$; gas will therefore tend to become more centrally condensed and vulnerable to gravitational instability (leading to star formation). The merging will occur more frequently in the large-scale overdense regions than in the rarefied ones. This leads to an 'anti-biasing' of minihalo survival probability (Rees 1987). Large-scale regions void of bright galaxies would not be deficient in minihalos. If absorption-line clouds were absent from any large volume this would have to be due to some other environmental influence on the gas.

The escape velocity is about 30 km $s^{-1}$, which is very small in comparison with that of galaxies. Therefore, the winds from quasars could strip the baryons from minihalos (Rees 1987). This mechanism may offer an alternative interpretation of the inverse effect/proximity effect, that the Lyman $\alpha$ forest decreases nearer to quasars (Tytler 1987; Bajtlik et al. 1988).

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References