

Observers at the moment of observation

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Abstract

In quantum mechanics, measurer(subject) plays important roles influencing the state of an object under experimental observation. That implies the necessity of inclusion of the subject(observers) at the moment of observation in the final theory. What is "the observers at the moment of observation"? To answer this question, we observe and analyse phenomenologically "observers at the moment of observation" and find that there exists the state which is common, at the moment of observation, to all observers. In this paper, we describe what it is by introducing a vacuum state of observer and observation operators by using analogies. Taking the entangled two neutral kaons as example, we show how they can be used. The state satisfies the requirements of the universality and reproducibility which are necessary for any object studied in physics.

Keywords: observer, object-subject, measurement, entanglement

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1 Introduction

J. S. Bell states[1] “the subject-object distinction is indeed at the very root of the unease that many people still feel in connection with quantum mechanics. The theory is fundamentally about the results of ‘measurements’ and therefore presupposes in addition to the ‘system’(or object) a ‘measurer’ (or subject).”

It is not strange that measurements disturb objects in essential ways in the area in which quantum mechanical effects are conspicuous because objects are fundamental particles (electrons, photons, protons, neutrons, etc.) or made of fundamental particles and the subjects(measurers) are also fundamental particles or made of fundamental particles interacting with the fundamental particles in the objects upon measurement.

In quantum mechanics the effects of observers on objects (the systems to be observed) are expressed not by equalities (as in classical mechanics) but by inequalities such as Heisenberg’s uncertainty relations[2]

$$\Delta E \Delta t \geq h; \quad \Delta p_x \Delta x \geq h. \quad (1)$$

and by words such as by Dirac[3] “.....in a measurement, the subject always causes the system to jump into an eigenstate of the dynamical variables that is being measured.....” In a measurement in Dirac’s statement, we think that the subject includes a person (observer) who observes and recognizes directly or indirectly an outcome of measurements, an eigenvalue.

The situations that Dirac described in the above sentences are understood in the most intuitive way by using the correlated (entangled) two-particle systems such

as two photons, two electrons, and/or a $K\bar{K}$ pair or $B\bar{B}$ pair systems (which are produced by decays of particles), etc., which are used to test the EPR paradox[4].

As an explicit example and short review, we use a $K\bar{K}$ pair from a ϕ meson decay[5] which is expressed by the wave function

$$\psi = \frac{1}{\sqrt{2}}(|K_S\rangle_1|K_L\rangle_2 - |K_L\rangle_1|K_S\rangle_2) \quad (2)$$

where K_L and K_S are long- and short- lived neutral K -mesons respectively and 1 and 2 stand for the directions of emission of two kaons from a ϕ (say in the rest system of ϕ).

Suppose an observer 1 in the direction of 1 performs a measurement on this system expressed by equation (2) at time t_1 and finds K_S . It influences instantaneously the outcome of the measurement which will be performed at t_2 (later than t_1) by observer 2 in the direction of 2 regardless of the distance between the two kaons traveled (in opposite directions away from the point of a ϕ meson decay) and regardless of the size of the time difference between the two observations; $\delta t = t_1 - t_2$.

Namely the measurement by observer 1 makes the wave function of the system given by equation(2) jump/shrink to

$$\psi_{12} = \frac{1}{\sqrt{2}}|K_S\rangle_1|K_L\rangle_2 \quad (3)$$

This is what Dirac meant in his statement. So observer 2 will observe K_L .

How could the measurement by observer 1 influence the outcome of measurement by observer 2 who measures at t_2 (δt could be zero as a limit) being located far from observer 1? : K mesons are electronically neutral and therefore the interaction between K_S and K_L is short-range ($\sim 10^{-13}cm$). In the case observer 1 detected

K_L instead, the state(2) jumps to

$$\psi_{21} = \frac{1}{\sqrt{2}}|K_S\rangle_2|K_L\rangle_1 \quad (4)$$

Though this example is usually used to illustrate the non-local nature of quantum mechanics[6], it also shows that measurement (by the observer, namely the subject) instantaneously and unavoidably alters the system (object) to be observed regardless of the distance between the two observers. Bell's inequality[7] enabled physicists to show quantitatively that this seemingly paradoxical phenomenon inherently and actually occurs in nature.

The "vagueness" in quantum mechanics caused by those two non-equation ways of describing the laws, namely inequalities and words, in no way interferes with the miraculous accuracy of its predictions which have been entirely justified by experimental results. The correctness of quantum mechanics has been shown in many fields of physics such as particle, atomic, molecular, and solid state physics in which a wide range of application of quantum mechanics is flourishing. Yet there are some parts in quantum mechanics which give the unease to many physicists.

As noted by Bell[1], " Now must this subject include a person? Or was there already some such subject-object distinction before the appearance of life in the universe ?" The answers to these questions will lead to another new layer of physics [8] research which must describe at once both the system of objects to be observed and the observer who is made of organic material, as opposed to the inorganic systems traditionally studied in physics [9].

If this subject include a person (observer), what is the state of observers relevant to

physics? Can we express it in a fusible form for physics? These questions require first observation of the observers at the moment of observation.

In this paper, we report what we found from the experimental observation (not gedanken experiment) in which we phenomenologically observe the relationship between a person (subject) and object (system to be observed) and show one way to prescribe an observer (a person) at that moment of observation in a fusible form to physics.

2 Observation of observers

When we are observing (attentively seeing with full attention) an event in experiment, we realize that we are just observing and recognizing it at that moment of observation and at that moment we neither think what it means nor judge whether it is correct or wrong. We just observe and recognize the object.

When we think of how we are when we are observing an object, we realize that the observer at the moment of observation, observation-recognition, is universal and does not depend on individual person. This is also understandable from the fact that the laws of physics which were based on observation, have been understood and granted by physicists in many different cultures, societies, races, and gender in different languages in long time span.

Our study in this paper is only about the observer (subject) at the moment of observation.

As usual for phenomenological studies in physics, in order to understand an observer at the moment of observation, observation on observer has to be done though the objects in physics are mostly nonorganic matters[9] existing physically independent from the observers themselves. Namely an observer observes oneself; meaning that an observer becomes an object to be observed at the same time. This kind of extension of objects dealt in physics is necessary because we are interested in the answer to the question which Bell[1] had brought up: How does an observer influence the state of an object under observation? To answer the question, we first have to learn the observer at the moment of observation by observing oneself.

When we are attentively looking at a thing, for an instance, a beautiful flower, we find that we become the flower forgetting ourselves. The time interval for an observer in that state is very short. Needless to say, we are we and a flower is a flower regardless what all the time.

Here we are talking about that instant when one looks at and recognizes the flower, in which there is no separation between the two. Just a short time before looking at and recognizing, we find that there exists a ground state which may be also called a vacuum state of an observer, which is common to all observers at the moment of observation. The word ,ground state, means the most stable state and unable to transit down, but can readily move up to other excited states under various situation.

It is stable in the sense that the observer in any excited state always goes back to the ground state which is the lowest energy state. It is found that the ground state is always present though not always noticeable. This is similar to the case that the ground state of a hydrogen, for instance, does exist even when it is transited up

to an excited state.

It may be said that this state is close to the mind of a physicist who is in deep contemplation of a mathematical problem, or in complete absorption in an experiment, or in an effortless concentration on a physics problem in the state, in which one forgets oneself[10]: No ego nor self-consciousness there. It is the state which is free from all the limitations coming from the fact that one is a living system. It is not possible for anyone to stay in the such state for a long time because of the perturbation from the environment which constantly excites the state and from its own transient nature. There are many sources of disturbance which come from outside as well as from inside of oneself, which can be compared with the induced and spontaneous transitions of atoms.

Denoting the ground/vacuum state of an observer by $||\bigcirc \gg$ [11] , and the operator detecting, say, a cypresstree in a courtyard by $O_{cypresstree}$, (or for the case of the example in the introduction, the operator detecting K_L or K_S , $O_{(K_L \text{ or } K_S)}$) then, we can describe the state of the observer who is being asked a question, ” What kind of tree do you see in the courtyard?” (or ”is it K_L or K_S ?”) by:

$$O_{cypresstree}||\bigcirc \gg = (cypresstree)||\bigcirc \gg . \quad (5)$$

where the right-hand side of the equation $(cypresstree)||\bigcirc \gg$ is what the observer observed at that moment. The ground state of the observer, $||\bigcirc \gg$, has the characteristic of vacuum in the field theory in the sense that it is the lowest energy state and invariant under any transformations such as translations, rotations, space and time inversions, internal space operations etc. Operator $O_{cypress}$, looks like acting as a projection operator. However because the state $||\bigcirc \gg$ is like a vacuum

state rather than a superposition of infinitely many states [12] , we think it image-wisely more like an ordinary operator.

It is also

$$O_{cypress\ tree}||\bigcirc \gg = (cypress\ tree)||\bigcirc \gg = ||cypress\ tree \gg . \quad (6)$$

The last expression of the above equation implies that the observer who is looking at a cypress tree attentively becomes, in our terminology, the cypress tree itself. Needless to say that an observer never become physically a tree; a tree is a tree and an observer is an observer all the time. It means that at the moment an observer is observing the cypress tree, the observer and the tree become inseparable. This is similar to the situation for two flavored-elementary-particles interacting each other: Within the short interaction time, the individual identities of the two particles get lost. The time parameter is not explicitly written here but it is at the same time. Seeing is recognizing at the moment of observation. We are not asking questions such as "What is the same time ?" or "Is there a time difference shorter than 10_{-23} sec between these two states ? "

As an another example, using observation operators for the entangled state of two neutral kaons, ψ in the previous section , observation of a neutral kaon by observer 2 results

$$O^{(2)}_{(K_L\ or\ K_S)}\psi||\bigcirc \gg_2 \rightarrow |K_L\rangle_1 O^{(2)}_{(K_L\ or\ K_S)}|K_S\rangle_2||\bigcirc \gg_2 \rightarrow (|K_L\rangle_1||K_S \gg_2 \quad (7)$$

where the supperscript 2 at observation operator stands for those for observer 2 and

arrows means, upto normalization constants

or

$$O^{(2)}_{(K_L \text{ or } K_S)} \psi ||\bigcirc \gg_2 \rightarrow |K_S\rangle_1 O^{(2)}_{(K_L \text{ or } K_S)} |K_L\rangle_2 ||\bigcirc \gg_2 \rightarrow |K_S\rangle_1 ||K_L \gg_2 \quad (8)$$

Observation by observer 1 (making observation on the state in equation 7) at a later time, takes to

$$O^1_{(K_L \text{ or } K_S)} |K_L\rangle_1 ||\bigcirc \gg_1 ||K_S \gg_2 \rightarrow ||K_L \gg_1 ||K_S \gg_2 \quad (9)$$

where $||\bigcirc \gg_1$ and $||\bigcirc \gg_2$ are the ground states of the two observers, 1 and 2 in a distance apart and $O^{(1)}_{(K_L \text{ or } K_S)}$ and $O^{(2)}_{(K_L \text{ or } K_S)}$ are observation operators to detect a K_L or a K_S by observers 1 and 2 respectively.

In the equation (7), the observer 2 observed K_S which makes the wave function ψ shrink to ψ_{21} and in the latter equation (8), the observer 2 observed K_L which makes the wave function, ψ , shrink to ψ_{12} . We used a " \rightarrow " instead of a " = " in equations (8) and (7)), because, after observer 2, $||\bigcirc \gg_2$ became $||K_L \gg_2$, namely after observer 2 recognizes a K_L , the expectation value of finding K_L wave function ψ_{21} must be calculated. A full calculation and explanation are given in reference ([13]) including normalization. In Fig 1, it shows figuratively observation by observer 2, $||\bigcirc \gg_2$, on a Φ decay in which the operator $O^{(2)}_{(K_L \text{ or } K_S)}$, projects the entangled wave function ψ in unphysical sheet onto the physical sheet upon measurement by observer 2 : On the lefthand side of picture shows the entangled wave function ψ in unphysical sheet. On the righthand side, it shows the situation that an observation by observer 2 changes the wave function, ψ , to ψ_{12} and the out come of which (K_L in this example) appears on the physical plane.

In the physical sheet, our world of recognition, observer 2 becomes $||K_L \gg_2$ and a

K_L is physically recognized,

The last and most important question is how anyone can be sure that there exists such a state of observers, $||\bigcirc \gg$. To convince oneself, one becomes the observer and the object to be observed at the same time. It is difficult for anyone to keep oneself in the ground state under normal conditions where lots of other more exciting things are going on. That is why under usual circumstances, observers in general never notice the existence of such a state which we call a ground/vacuum state of observers. For those who did not get any kind of idea what it is like by reading the description of the ground state in the previous pages, it requires studying and training under a professor in the field for years to learn how to observe this ground state. One just has to assume that there exists such state of observers till one gets convinced with it. However it is possible to imagine such existence when one recalls the fact that all observers have common sensory functions such as seeing and hearing which receive a similar signal from an object and recognize the object. We find that this ground state, $||\bigcirc \gg$, embodies the common characteristics of all objects (observers) at the moment of the observation, namely observer at the moment of observation, at the moment, subject = object.

We think this, $||\bigcirc \gg$, is the state of observers to be counted into physics if ever. Important point of this is that anyone who is willing to observe it should be able to observe it and can be convinced, that is the case in any field of science. For some people, to be convinced of the existence of the state $||\bigcirc \gg$ is as hard as to be convinced that a quark has spin one-half. It requires studying and learning. Until one is becoming sure of it, it is an assumption for that person.

We came to notice that there are many researchers who have made observations of observers at the moment of observation and found the ground state which is exactly like the one we found, namely $||\bigcirc \gg$. We call these researchers who found it independently from us, zenists. In the next section, partly for the sake of proofs of it's universal existence and partly for the sake of understanding it, some of zenists' expressions for the state $||\bigcirc \gg$ are given.

3 Description of the ground state

For physicists, the nature of this ground state $||\bigcirc \gg$ is expressed best by the statement[14]

being is being

because being is not being

This sounds contradictory to the logic which physicists use.

We re-express the sentence in a somewhat more acceptable-looking expression;

A is called A because A is not A.

We recall that an observer is an observer (subject) and an object(to be observed) at the same time at the time of observation of observer (oneself) here. An observer (being) is an observer who is the object to be observed.

The following story[15] about the word, being, is useful for understanding what is meant by in the above statement: At an "university" in China, a professor who has been observing himself wishes to decide upon his successor of his field. He asked

his students to show him in verses the result of their observation. The smartest student among all the students (all the students including himself believed so), Jinshu, wrote a verse on a wall to meet everybody's expectation after a long hesitation:

This body(being) is the Bodhi-tree;

The mind (being, the observer in the ground state) is like a mirror;

Be attentive to keep it clean

And lets no dust dull its reflective power.

Enou(638-713A.D.), who was a mere cleaning man of the university, heard of the verse and asked one of the students to write his own verse next to it. He did not know how to write. Enou's lines read:

The Bodhi is not like a mirror;

The mind is not like the mirror bright;

As there is nothing from the beginning,

To where does the dust attach?

The latter succeeded the professor.

We all know that one of the characteristics natures of a mirror is reflection. A clean and perfectly polished mirror reflects light well. But it requires at least one more object to interact to show their existence of their natures. The state, $||\bigcirc\rangle\rangle$ is like a vacuum state; there is "nothing" noticeable unless an operator, such as $O_{(K_L\text{or}K_S)}$, acted upon.

There are other description of $||\bigcirc\rangle\rangle$ in different wordings some of which we refer to documents[16]. These other expressions may be easier for some people to understand it.

The state of observers at the moment of observation, $||\bigcirc \gg$ which is not familiar for the most of physicists has been described by daily language in the examples above and all the documents[16] and books above while physics has mathematics as its language. This does not imply that the state $||\bigcirc \gg$ is vague or ambiguous. At the moment one realize $||\bigcirc \gg$, one is sure of it and agrees with the descriptions of $||\bigcirc \gg$ made by many other professors in their field.

As we see in the above example, it has some techniques for expression:

(1) Negation — A is not like B, A is not like C, A is not like D, so on upon describing A when B,C,and D are known.

$$A \neq B, A \neq C, A \neq D, \text{ etc}$$

(2) Similarity — A is like E, A is like F, A is like G, so on upon describing A when E,F,and G are known.

$$A \sim E, A \sim F, A \sim G, \text{ etc}$$

(3) More like than — A is more like E than F, A is more not likely B than C

$$E \sim A \sim\sim F, C \neq A \neq\neq B$$

(4) technical words — To understand meaning of some technical terms are as difficult as for non-physics majors to understand meaning of the technical words in quantum mechanics such as eigenstates, non-commutative, phase, etc.

4 Summary

We include "person" in "measurer" questioned by Bell[1] and we limit "person" which is relevant for quantum mechanics as the observer at the moment of observation. We study(observe) observers at the moment of observation and find that

there exists the state of observers common to all observers which has characteristic similar to the ground state of atoms and to the vacuum state in quantum field theory. Expressing the state of observers at the moment of observation by $||\bigcirc \gg$, we introduced observation operators such as $O_{(K_L \text{ or } K_S)}$, which act on both the wave function of the system under experiment(observation) and the state of observers. Observation operators brings the system and $||\bigcirc \gg$ into the real world in which we measure(observe) and recognize. Observation is nothing but recognition as expressed by the first and the last of the equation (7).

To observe the ground state, $||\bigcirc \gg$, one has to stay in that state as long as possible. Since the observer is an organic living system, there are abundant resources of disturbances from the environment as well as one's own living system that makes observation of the state, $||\bigcirc \gg$, more difficult. But for many physicists, it is not hard to realize its existence as described in previous sections. Needless to say that it is possible for everyone to realize it: However anyone who wish to convince oneself of its existence, and observe $||\bigcirc \gg$, can do that by trainings and learning for years. Namely the existence of the ground state can be reproducible by anyone: the results obtained by observation can be confirmed by anyone who is willing to go through studying for years and doing experiments by him- or her-self under professors. The state of observers, $||\bigcirc \gg$ is also universal in the same meaning as the universality of the laws in physics. Namely, the laws governing the ground state of observers, $||\bigcirc \gg$ are at work for everybody regardless whether one likes the laws or not, notices the laws or not, or understands the laws or not. In that respect, the laws for the ground state of observers are universal exactly as the laws in physics should be. This reproducibility and universality makes the ground state of observers (at the

moment of observation) understandable for everybody globally regardless of race and gender.

We described the observer at the time of observation in an explicit example: In the example of two entangled neutral K mesons described in the introduction, the objects being observed under the physics experiment are either K_L or K_S . At each observation by an observer, the observer, $|\!|\!\circ\rangle\rangle$, recognizes one of them at each time, appeared in physical plane (see fig.1) following the quantum mechanics' predictions. We expressed observations by operators such as $O_{(K_L \text{ or } K_S)}$ for finding either K_L or K_S and the state of observer by $|\!|\!\circ\rangle\rangle$. The operators act on both of the wave function of two neutral kaons (Equation (2)) and the observer at the moment of observation, $|\!|\!\circ\rangle\rangle$, in unphysical plane and selects one of the states of the wave function and brings it to the physical plane where a K_L or a K_S appears and comes to recognition by the observer, denoted by $|\!|K_L\rangle\rangle_2$ or $|\!|K_S\rangle\rangle_2$.

We showed that one way to express person(observer) at the moment of observation, in a fusible expression for physics, is $|\!|\!\circ\rangle\rangle$, the ground state of observers, and that the existence of the ground state has been confirmed years by millions of researchers in other field, the schools of zenists. Observation operators which act on both the wave function of the objects and the ground state of observers are introduced. They project one of the eigenstates of the objects and transform the ground state to a state of recognition as shown in the equation (6), (7), and (8).

Answers to questions such as whether there is any energy and momentum transfer or not from the objects under observation to the ground state, $|\!|\!\circ\rangle\rangle$ upon

observation or vice viser, or whether there is time dependence for the states and operators or not (within this representation), etc are matters for the future research.

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References

- [1] J. S. Bell, *Speakable and unspeakable in quantum mechanics*, (Cambridge University Press, 1987)

- [2] There is an argument on this uncertainty relation whether it is a principle of quantum mechanics or not. See K. Fujikawa and K. Umetsu, Progress of Theoretical Physics, 120, 797 (2008); K. Fujikawa, *Power of Equations*,in p22 (*Mathematical Sciences* 2009), ed. by M. Wadachi

- [3] P. A. M. Dirac, *The Principle of Quantum Mechanics*, (Oxford University Press, 1958)

- [4] A.Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935)

- [5] For instance, see P. H. Eberhard, Nuclear Physics B398 ,155 (1993) and G. C. Ghirardi, R. Grassi, and T. Weber, Proc. *Workshop on Physics and detectors for DAPHNE* ed. by G. Pancheri (Frascati) (1991) and references therein
- [6] Marco Genovese, Physics Report 413, 319 (2005) and references therein; H. J. Lipkin, Physical Review 176, 1715 (1968); F. Selleri, (1986) Lett. Nuovo Cimento 36; I. I. Bigi, Nuclear Physics 24A, 24 (1991); N. D. Meriniss, Am. J. Phys. 66, 920 (1998) S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. 28, 938 (1972); E. S. Fry and R. C. Thompson, Phys. Rev. Lett. 37, 465 (1976) ; A. Aspect, A. Dalibard, and G. Roger, Phys. Rev. Lett. 49 1804 (1982) ; etc.
- [7] J. S. Bell, Physics 1, 195 (1965); F. Clauser, M. A. Shimony and R. A. Holt, Phys. Rev. Lett. 23 (1969); F. Uchiyama, Phys. Lett. 231A 295 (1997); B.C. Heismayer, Foundation of Physics Lett. 14, 200 (2001). For an explanation of Bell's inequality relationship, see J. J. Sakurai and San Fu Tuan, *Modern Quantum Mechanics*, (Addison-Wesley Publishing Co.1979)
- [8] E. Schoreninger, *Mind and Matter*, (Cambridge University Press, 1958)
- [9] Shinichiro Tomonaga, *What is Physics*, (Misuzu Shobou, 1983)
- [10] Needless to say, this ground state is a state of anyone, which can be realized by anyone who wishes to do so. Its existence is as universal as gravity acting equally on any massive "objects" regardless of being organic or inorganic, being noticed or not noticed. However this ground state is not easily noticeable when one is engaging busily in doing things, in actions (including speaking, writing, and listening), in thoughts, and in sensations. It is only realizable in calmness.

But anyone in a profession which requires concentration will have an experience being in this ground/vacuume state.

[11] Here, we do not imply that , $||\bigcirc \gg$,is a vector in a Hilbert space, though we use a similar looking notation. $||\bigcirc \gg$ is not a vector. It is nothing to do with the Hilbert space in which the states of a physical system being observed in physics are described. Here we use this vector-like looking notation for a state of the observer at the moment of observation to indicate that some aspects of that is similar to vacuum state in the field theory. Similarly speaking, in the present paper, the operators connecting physical and unphysical spaces, such as $O_{cypresstree}$ and $O_{(K_L or K_S)}$ are acting like projection operators in physics but not projection operators. This is because, imagewisely $||\bigcirc \gg$ is more like a vacuum, full-empty, in physics than a state containing all conceivable states.

[12] We thank P.H.Eberhard for calling attention to this similarity

[13] The observation by observer 2 and then by observer 1 :

$$\begin{aligned}
& \psi^* O^{(1)}_{(K_L or K_S)} O^{(2)}_{(K_L or K_S)} \psi ||\bigcirc \gg_2 ||\bigcirc \gg_1 \\
= & \psi^* O^{(1)}_{(K_L or K_S)} O^{(2)}_{(K_L or K_S)} (\psi_{12} - \psi_{21}) ||\bigcirc \gg_2 ||\bigcirc \gg_1 \\
= & \psi^* O^{(1)}_{(K_L or K_S)} \frac{1}{\sqrt{2}} (|K_L\rangle_1 ||K_S \gg_2 ||\bigcirc \gg_1 - |K_S\rangle_1 ||K_L \gg_2 ||\bigcirc \gg_1) \\
= & \frac{1}{2} (||K_L \gg_2 ||K_S \gg_1 + ||K_S \gg_2 ||K_L \gg_1) \\
= & \frac{1}{2} (K_L^2 K_S^1 + K_L^1 K_S^2) ||\bigcirc \gg_1 ||\bigcirc \gg_2 \tag{10}
\end{aligned}$$

Here the $\frac{1}{2}$ in the last two expressions is from usual normalization of the wave function ψ

- [14] This English expression of $||\bigcirc \gg$ is by D.T. Suzuki. We tried to locate D.T.Suzuki's original paper in which this English expression appears but failed. See Akizuki, Ryoumin, *Suzuki Zengaku to Nishida Tetsugaku (in Japanese)*, (Shunjuu-sha, page 73, 1971) in where this English expression is referred to D.T.Suzuki.
- [15] There are tens of books writing about this episode. For an example, Yamanaka Minetaro, *Zen to wa Nanika) (in Japanese)*, (shibun-dou, page 80, 1957)
- [16] The book, *Mumonkan*, contains 48 "questions and answers" on the vacuum/ground state, $||\bigcirc \gg$, from various observers' verses collected by professor Mumon who attaches his comments to each one of the verses. There are huge number of lecture books on *Mumonkan*. Some of them are given here: Yamada Mumon and Takahashi Shinkichi, *Mumonkan* (in Japanese), (Houzoukan, 1958); Yamamoto Genpho, *Mumonkan Teishou* (in Japanese), (Daihourin-kaku, 1960) Osaka Kohryu, *Teishou Mumonkan* (in Japanese), (Daizou shuppan 1989); Nishimura Eshin, *Mumonkan* (in Japanese), (Iwanami, 2004); Zenkei Shibayama, *Mumonkan* (in English) trans. Sumiko Kudo, (Harper and Row, New York, 1974)

Figure Captions

Fig.1 shows figuratively observation of K_L at $t = t_4$ and $X = -X_4$ by observer 2,

$||\bigcirc \gg$, operator, $O^2_{(K_L \text{ or } K_S)}$) projects the entangled state ψ in

unphysical plane onto physical plane upon measurement: Positive and negative directions of the vertical coordinate distinguishing K_S and K_L . For the sake of visual assistance, white and black squares with finite widths for K_S and K_L are drawn respectively.

On the left, at $t' = 0$, a Φ decays into K_L and K_S and the entangled wave functions of K_L and K_S start to travel into opposite directions for $t' < t'_4$. On the righthand side, It shows that an observation by observer 2 at $(t = t_4, X = -X_4)$ changes the wave function ψ to ψ_{12} and observer 2 recognizes a K_L , $||K_L \gg_2$ while K_S wave function, $|K_S \gg$ keeps traveling in unphysical sheet till observer 1 measures it. We choose for times, $t = t'$ and for coordinates, $X = X'$ for simplicity.

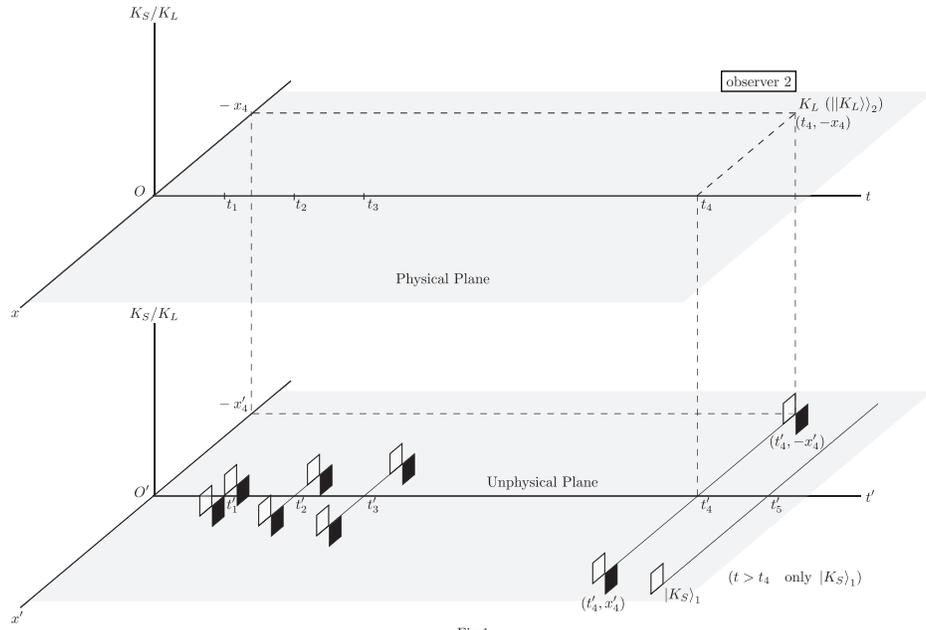


Fig.1