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（様式３）

博士論文の要旨

氏　　名　　　石川　力

論 文 題 目　Spectral sum of hadronic correlation functions from lattice Quantum Chromodynamics

Quantum Chromodynamics (QCD) describes the strong interaction among quarks and gluons. Quarks are the elementary particles that constitute hadrons. Therefore, QCD plays an important role even in various electroweak precision tests where hadrons appear.

An important property of QCD is the asymptotic freedom. The coupling of QCD is scale-dependent and becomes smaller at higher energy scales. This property is unique to QCD. Other interactions of the Standard Model do not have it. The scale dependence restricts the region where perturbative calculation can be safely applied. At low energies, quarks become strongly coupled to gluons, and the perturbative calculation breaks down. Therefore, it is difficult to investigate hadron physics directly from QCD without using the effective theory.

Lattice QCD has become a standard tool to study quarks and gluons. It provides a fully nonperturbative calculation that does not rely on perturbative expansions. Since Wilson proposed the lattice gauge theory to demonstrate quark confinement in 1974, lattice QCD has achieved a lot of success. Measuring two- and three-point correlation functions, one can determine physical quantities such as decay constants and form factors from hadronic matrix elements. These parameters are important inputs for phenomenological studies of the Standard Model. Lattice QCD yields the better understandings of the nonperturbative nature of QCD.

Apart from lattice QCD, another approach to the nonperturbative physics of QCD has been evolved, which is based on the analyticity of the correlation functions and quark-hadron duality. This approach enables us to link quarks in the Euclidean domain to hadrons in the physical energy domain. The spectral sum of hadronic correlation functions, such as the vacuum polarization function $\Pi(q^2)$, of the form,

$$\int ds\, e^{-s/M^2}\, \mathrm{Im}\,\Pi(s)$$

has often been introduced since the seminal work of Shifman, Vainshtein, and Zakharov in the late 1970's. The spectral sum is equivalent to the Borel transform of the function $\Pi(q^2)$. The integral over invariant mass squared s smears out contributions of individual resonances so that one can use perturbative treatment of QCD with quarks and gluons as fundamental degrees of freedom, as far as the Borel mass M, a parameter to control the typical energy scale, is sufficiently large. The integral is a quantity effectively defined in the space-like momentum region, and there would be no issue of the violation of the quark-hadron duality.

The integral suppresses the contributions from the energy region above M and thus, is more sensitive to low-lying hadronic states. If one can find a window where M^2 is large enough to use perturbative expansion of QCD with nonperturbative corrections included by operator product expansion (OPE) and at the same time sufficiently small to be sensitive to lowest-lying hadronic states, the spectral sum may be used to obtain constraints on the parameters of low-lying hadrons, such as their masses and decay constants. This method, called the QCD sum rule, has been widely applied to estimate masses, decay constants, and other properties of hadronic states in various channels. However, an important question of how well the perturbative QCD with some nonperturbative corrections included through OPE can represent the spectral sum is yet to be addressed, especially when the correlation function is not always fully available from the experimental data, *e.g.* due to a limitation of accessible kinematical region.

In this thesis, we propose a method to use lattice QCD to compute the Borel transform of the vacuum polarization function appearing in the QCD sum rule. We construct the spectral sum corresponding to the Borel transform from two-point functions computed on the Euclidean lattice. As a proof of principle, we compute the $s\bar{s}$ correlators at three lattice spacings and take the continuum limit. We confirm that the method yields results that are consistent with the operator product expansion in the large Borel mass region. The method provides a ground on which the OPE analyses can be directly compared with nonperturbative lattice computations.

We find a good agreement between the lattice data and OPE in the region of M > 1.0 GeV. The OPE is truncated at the order of $1/M^6$. Since the OPE involves unknown condensates, this comparison can be used to determine these parameters, provided that the lattice data are sufficiently precise. As the first example, we attempt to extract the gluon condensate, which appears in OPE at the order $1/M^4$. The size of the error is comparable to those of previous phenomenological estimates. With more precise lattice data in various channels, one would be able to determine the condensates of higher dimensions as well, which have not been determined well solely from phenomenological inputs.

Our work provides a technique to relate two major tools to study nonperturbative aspects of QCD, *i.e.* the QCD sum rule and the lattice QCD. There would be a number of applications, for which new insights into the QCD phenomenology are expected.

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（様式８・別紙１）

Results of the doctoral thesis defense

博士論文審査結果

Name in Full
氏　名　石川　力

Title
論文題目　Spectral sum of hadronic correlation functions from lattice Quantum Chromodynamics

石川氏の博士論文は、QCD 和則（及びそこでの計算手法である摂動展開と演算子積展開）の妥当性を格子ゲージ理論に基づいた非摂動計算により検証する方法に関する研究である。QCD 和則とは、QCD の摂動計算からハドロンの諸性質を抽出する枠組みであり、相関関数の解析性に基づいているが、摂動展開と演算子積展開から不定性が生じうる。この研究では、QCD 和則の計算でしばしば導入される（運動量空間の）相関関数のボレル和を、摂動的に扱える高エネルギー領域と非摂動的な低エネルギーの全体にわたって格子 QCD の数値シミュレーションを用いて計算し、QCD 和則の解析の妥当性を検証している。

ボレル和は、スペクトル表示で書かれた相関関数の重み付き積分の形であらわされるが、過去にこの量が格子 QCD で計算された例はない。これは、スペクトル関数を求めるのが不良設定問題に相当し、大きな誤差を生ずるためである。この問題を避けるため、石川氏は、格子上の相関関数の時間依存性を用いてスペクトル関数の重み付き積分を評価する手法を開発し、その有効性を実証した。スペクトル関数自体を計算する必要がないために、大きな誤差の問題は起こらない。

格子 QCD 計算の結果は QCD 和則で用いられる摂動展開および演算子積展開の評価を、高エネルギー領域でよく再現しており、その手法を肯定的に検証したと言える。低エネルギーでは演算子積展開の不定性が非常に大きくなるが、この領域でも格子 QCD 計算では精密な結果が得られる。また、この手法の応用として、演算子積展開にあらわれるグルーオン凝縮の値を評価した。

この研究は、博士論文として必要な新規性を十分に備えている。実際、この研究に関する査読付き論文はすでに出版済みである。博士論文は、この論文の研究を中心として、そこにいたる前提知識などの簡潔なまとめを含んで書かれており、石川氏が、この分野の研究に必要な知識を十分に持っていることが示されている。

以上のことから、審査委員会の全員一致で石川氏の博士論文を合格と判断した。