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（様式３）

博士論文の要旨

氏　　名
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論文題目
　Numerical studies on topological aspects of gauge theories

We can explore the topological nature of quantum field theories via topological terms. Recently, gauge theories with a theta term have been studied by 't Hooft anomaly matching. In particular, there is a constraint on the phase structure of the 4D SU(N) pure gauge theory by a 't Hooft anomaly involving the CP and center symmetries at $\theta = \pi$. The constraint is consistent with the well-known scenario at large N, where the theory at $\theta = \pi$ is confined with spontaneously broken CP at low temperature and then has a transition to deconfined phase with restored CP at a finite temperature. However, it is highly nontrivial whether or not this structure persists for small N since there are various ways to satisfy the anomaly matching condition. For instance, the theory for small N at low temperature may be deconfined or gapless as well as spontaneously broken CP. Therefore it is an interesting challenge to investigate the phase structure by first-principle calculation at the smallest N i.e. $N = 2$.

The effect of the theta term is genuinely non-perturbative. The theory with a theta term should be analyzed by non-perturbative calculations based on the lattice gauge theory. However, the Monte Carlo simulation of the theory including the theta term is difficult due to the sign problem. The aim of this work is to develop a method to simulate gauge theories with a theta term avoiding the sign problem. We first focus on the complex Langevin method (CLM), which is one of the approaches allowing us to avoid the sign problem. The advantage of this method is that the calculation cost of the CLM depends linearly on the system size, so that one can easily apply it to theories in higher dimensions. However, the CLM has the drawback that it possibly gives wrong results depending on the system and the parameter region. Fortunately, a practical criterion for correct convergence was proposed in the recent study. Thus, the CLM is thought to be a useful method as long as the criterion is satisfied.

As a first step, We applied the CLM to 2D U(1) gauge theory with a theta term. This model is suitable for the testing ground since it can be solved analytically with finite lattice spacing and finite volume on an arbitrary manifold. We found that a naive implementation of the CLM on the periodic lattice fails. We clarified that the configurations which appear when the topology change occurs during the simulation necessarily result in a large drift force. In fact, frequent appearance of the large drift force violates the criterion for correct convergence. If one avoids this problem by simply

approaching the continuum limit, the topology change never occurs during the simulation. It causes the other problem of the ergodicity.

In order to circumvent this problem, we introduced a puncture on the lattice, namely removing a plaquette from the action. It makes the manifold non-compact, so that the 2π periodicity of θ is lost. We obtained the exact results for this punctured model and proved that it is equivalent to the infinite volume limit of the original model for $|\theta| < \pi$. Thus, we can still extract the information of the original model from the punctured model. Rather surprisingly, we find that the CLM works and reproduces the exact results for the punctured model even for large θ. The topology change can occur freely thanks to the degrees of freedom around the puncture. As approaching the continuum limit, the large drift force does not appear because the problematic plaquette (puncture) is removed from the action.

Next, I analyzed 4D SU(2) gauge theory with a theta term by using the CLM. This work is motivated by the prediction for the phase structure of the theory at $\theta = \pi$ by the 't Hooft anomaly matching. The methods free from the sign problem, such as the CLM, allow us to calculate observables for $\theta \neq \pi$ directly. We applied the CLM to the theory with the topological charge naively defined by the clover leaf formula and found that it works in the high temperature region. However, the topological property of the theory is unclear since the topological charge on the lattice is contaminated by the short-range fluctuations. The topological charge does not approach an integer. In fact, the situation is different from the case of 2D where the topological property is clear even on the finite lattice.

Then, we introduced the stout smearing in the CLM in order to suppress the contamination of the topological charge. It successfully eliminates the short- range fluctuations, and the topological charge becomes close to an integer. However, we found that there is a relation between the topology change and the large drift force. They are correlated with each other as the case of 2D. Thus, we need to modify the boundary condition of the 4D lattice to avoid the large drifts.

We also developed the TRG method to study gauge theories avoiding the sign problem. The TRG can be alternative method for the case where the CLM does not work. In this work, we focused on higher-rank gauge group, namely U(N) and SU(N) for $N \geq 3$. The TRG is indeed powerful method for various systems in 2D. The computational cost increases only logarithmically with the system size, and so one can easily access the infinite volume limit. However, extension to higher dimensional theories is not straightforward. One need to avoid rapid growth of the calculation cost due to the increasing rank of tensors. Recently, some algorithms have been proposed to deal with this problem, which makes the TRG applicable to quantum field theories in 4D. On the other hand, the application to gauge theories is so far limited to U(1) and SU(2) gauge groups. In view of this situation, we studied the TRG of U(N) and SU(N) gauge theories in 2D, which is analytically solvable. We used the character expansion to rewrite the group integral as a sum over discrete indices. Unlike in U(1) and SU(2), it is nontrivial to restrict the number of representations to be used in constructing the tensor network. We proposed a practical way to truncate the sum and confirmed its efficiency, which allow us to take the large-N limit. We expect that this method is useful also for more nontrivial gauge theories in higher dimensions.

For the theory in 2D, the structure of the tensor is simple, so that the TRG algorithm can be completed exactly. Thanks to this property, we can precisely investigate the large- N behavior of the singular values of the tensor. We found that the singular-value spectrum has a definite profile in the large-N limit. Based on this fact, we proposed a novel interpretation of the Eguchi-Kawai reduction in terms of the TRG. We also found a new type of volume independence in the large-N limit of 2D U(N) gauge theory with the theta term in the strong coupling region.

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Results of the doctoral thesis defense

博士論文審査結果

Name in Full
氏　　名　松本　祥

Title
論文題目　Numerical studies on topological aspects of gauge theories

松本祥氏の学位論文は、ゲージ理論のトポロジカルな側面に関して、同氏が博士課程在学中に行ってきた一連の数値的な研究をまとめたものである。特に、このような数値計算においては、ゲージ場の配位がインスタントン数でラベルされるトポロジカル・セクターに分かれ、異なるセクター間の遷移が抑制される。そのため、素朴なモンテカルロ計算では、異なるセクターに属するゲージ配位を正しい重みでサンプルすることができない、という「トポロジー凍結問題」が一般に発生する。さらに、こうした計算を θ 真空に対して行うためには、インスタントン数 n をもつ配位に対して、$e^{in\theta}$ という位相因子を分配関数の重みとして入れる必要があり、いわゆる符号問題と呼ばれる大きな困難が生じることも知られている。

松本氏は、符号問題の解決法として、最近大きく進展している複素ランジュバン法を用いることにより、2 次元の U(1)ゲージ理論における θ 真空の数値シミュレーションに成功した。さらに、この方法を拡張して、4 次元の SU(2)ゲージ理論における θ 真空の数値シミュレーションにも取り組んだ。その過程で、トポロジー凍結を克服するアイディアとして、ゲージ配位の微細な揺らぎを平均化する smearing と呼ばれる手法をシミュレーションに取り入れ、比較的粗い格子上の計算でも、ゲージ配位のトポロジーを正しく扱う方法を開発した。さらに、θ を純虚数に解析接続することにより、符号問題を回避した計算を実行することにより、$\theta = \pi$ で予想される CP 対称性の自発的破れと、高温においてその対称性が回復していることを間接的に示唆する証拠を得た。この結果は、トホーフトによるアノマリー・マッチング条件に基づく理論的な予測を検証する一つの可能性を示すものであり、重要なものと言える。

これらの内容は学術的に十分な価値が認められるものであり，博士論文の内容として必要な水準を満たしていると判断し、審査委員会は全会一致で松本氏の博士論文審査を合格と判定した。