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学位論文題目 The systematic construction of multi scalar-tensor theory

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# 博士論文の要旨

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Modern cosmology increasingly relies on effective field theories that extend General Relativity by adding one or more scalar degrees of freedom. Such scalar–tensor theories provide a unified language for inflation, dark energy, and modified gravity, but they also face a fundamental consistency constraint: if higher-derivative interactions are introduced naively, the theory typically propagates an extra ghostly degree of freedom (the Ostrogradsky instability). A central problem is therefore to identify and organize the “healthy” classes of scalar–tensor theories in which the dynamical content is controlled and the equations of motion can be written without introducing pathological modes. In the single-field case, Horndeski theory occupies a distinguished position: it is commonly presented as the most general covariant scalar–tensor theory leading to second-order field equations. By contrast, systematic multiple-field extensions remain incomplete. While the general form of the equations of motion for two-field (bi-Horndeski) theories has been derived, a corresponding general action has not been established, and for three or more fields neither a general action nor a general set of equations of motion is available. This situation motivates the search for a new organizing principle that does not start from a fully constructed action, yet can recover known single-field results and offer a practical route toward multi-field generalizations. This thesis develops such a principle by re-characterizing Horndeski theory through symmetry and closure properties under field redefinitions, rather than through an explicit enumeration of all allowed Lagrangian terms. The key observation is that disformal transformations—metric redefinitions involving the scalar field and its first derivatives—play a structural role in scalar–tensor theories analogous to conformal transformations in Brans–Dicke–type models. In particular, invertible disformal transformations map healthy theories to healthy theories, and they partition the space of scalar–tensor theories into “orbits” under these redefinitions. Building on this viewpoint, I propose a new definition of Horndeski theory based on two mild axioms: (i) the theory is closed under invertible pure disformal transformations, and (ii) the theory contains a specified “minimal Horndeski theory” that serves as an anchor selecting the appropriate disformal orbit. The minimal theory is taken to be the sum of a canonical kinetic term, the Einstein–Hilbert term, and a characteristic derivative coupling to the Einstein tensor. The second axiom is essential: in more general ghost-free frameworks (such as DHOST), multiple inequivalent theory classes can be individually closed under invertible disformal transformations, so closure alone does

not uniquely single out Horndeski. The anchor condition resolves this ambiguity by fixing which closed class is meant.

A main result is that these axioms are sufficiently restrictive to recover the standard single-field Horndeski action uniquely up to boundary terms. In this perspective, “having the most general second-order equations of motion” becomes not the defining principle but a non-trivial property that is recovered once the disformal-closure definition is imposed. Concretely, starting from the minimal anchor theory and systematically generating all terms required by invariance under invertible pure disformal transformations, one reconstructs the familiar Horndeski structure (equivalently, the generalized Galileon action in covariant form) modulo total derivatives. The thesis provides an explicit constructive strategy—essentially an algorithmic procedure—for building the action within this orbit, clarifying how the conventional Horndeski Lagrangians arise mechanically from the closure requirement rather than from an a priori classification of equations of motion.

The advantage of this reformulation is that it suggests a direct and scalable way to attempt multiple-field extensions. Instead of trying to derive or guess a complete multi-field action (a task that becomes rapidly intractable) or deriving the most general multi-field equations of motion and then integrating them back to an action, one can extend the two axioms themselves: generalize “pure disformal transformations” to “pure multi-disformal transformations” acting on the metric with multiple scalar fields, and generalize the minimal anchor theory to an appropriate multi-field minimal theory. This leads to a new definition of multi-Horndeski (and, in particular, bi-Horndeski) theories as the classes closed under invertible pure multi-disformal transformations and containing the multi-field minimal anchor. Within this framework, I formulate a central conjecture: that the multi-field theory class defined in this way yields the most general equations of motion involving up to second-order derivatives of the metric and multiple scalar fields. The conjecture is motivated by the single-field reconstruction result and by the expectation that closure under multi-disformal transformations organizes the allowed derivative structures in a way that naturally enforces the requisite constraints.

As an application and non-trivial consistency check, I analyze the emergence of “antisymmetric structures” that are known to be specific to multi-field theories. In previous work on extended multi-Galileon and multi-field generalizations, Erwan Allys identified terms in flat space that exhibit antisymmetry in internal field indices, and Shingo Akama and Tsutomu Kobayashi constructed covariant counterparts (often referred to as the Allys–Akama–Kobayashi, or AAK, terms). These terms do not arise from a straightforward multi-field substitution in the single-field Horndeski action, yet they are believed to be necessary ingredients to capture the full structure of healthy multi-field scalar–tensor dynamics. I show that such antisymmetric terms appear naturally within the present disformal-closure approach: when the minimal multi-field

anchor and the multi-disformal closure condition are implemented (in a tractable setup focusing on the sector up to the quartic order in the usual Horndeski hierarchy), the resulting Lagrangian contains the AAK-type structures with the characteristic antisymmetry in internal indices. This provides concrete evidence that the new definition is not merely a rephrasing of known single-field results but genuinely encodes multi-field-specific features that have proven difficult to generate systematically by other approaches. Moreover, it indicates that the disformal-closure viewpoint has the potential to account for key features captured by the known bi-Horndeski equations of motion, while offering a more constructive route toward the corresponding action.

Overall, the thesis contributes a conceptual and practical reorganization of Horndeski theory and its multi-field prospects. Conceptually, it shifts the emphasis from “second-order equations” as a starting definition to “closure under invertible disformal transformations” plus an anchor condition that selects the correct orbit in the space of ghost-free theories. Practically, it yields (i) a reconstruction of the standard single-field Horndeski action up to boundary terms from these axioms, (ii) a blueprint for defining and exploring multi-field analogues by extending the axioms rather than attempting a direct brute-force construction, and (iii) explicit evidence—via the appearance of AAK-type antisymmetric terms—that the approach naturally generates structures known to be essential in multi-field settings. This framework opens a path toward a systematic multi-field scalar–tensor theory program in which disformal geometry and orbit structure guide the construction, classification, and comparison of candidate actions and their equations of motion, and it suggests concrete next steps toward establishing or refuting the conjectured completeness of the resulting multi-field class.

Results of the Doctoral Thesis Defense

博士論文審査結果

Name in Full

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Title

論文題目

The systematic construction of multi scalar-tensor theory

片山友貴君の学位論文は、一般相対性理論に新たな自由度や相互作用を加えた一般化重力理論を対象とし、従来知られている単一場のホルンデスキ理論を多成分場へ拡張するための基礎的枠組みを提案したものである。

現代宇宙論においては、一般相対論を拡張したスカラー-テンソル理論が、インフレーションやダークエネルギー、修正重力を統一的に記述する理論として重要な役割を果たしている。しかし、高階微分相互作用を安易に導入すると、オストログラツキー不安定性に対応するゴースト自由度が出現するため、健全な理論クラスを体系的に特定することが不可欠である。単一場の場合には、2階の運動方程式を導く最も一般的な理論としてホルンデスキ理論が確立されているが、多成分場への体系的拡張はいまだ未完成であり、特に一般作用の構成が大きな未解決課題となっている。

本論文では、作用の全列挙から出発する従来の方法に代えて、場の再定義に対する閉性という観点からホルンデスキ理論を再定義する新たな原理を提案した。鍵となるのは、スカラー場とその一次微分を含む計量の再定義であるディスフォーマル変換であり、可逆な純ディスフォーマル変換が健全な理論を再び健全な理論へと写す点に着目している。この性質に基づき、(i) 可逆な純ディスフォーマル変換の下で閉じていること、(ii) 正しい理論クラスを選択するための「最小ホルンデスキ理論」を含むこと、という二つの公理によってホルンデスキ理論を定義した。最小理論として、標準的な運動項、アインシュタイン-ヒルベルト項、およびアインシュタインテンソルへの微分結合を含む理論を採用することにより、これらの公理が、境界項を除いて通常の単一場ホルンデスキ作用を一意に再構成することを示した。この枠組みにおいては、「最も一般的な2階の運動方程式を持つ」という性質は定義そのものではなく、閉性条件から導かれる結果として理解される。

さらに本論文では、この考え方を多成分場理論へ拡張する道筋を提示した。純ディスフォーマル変換を多成分場へ一般化し、対応する多成分場の最小理論を採用することで、多成分場ホルンデスキ理論を定義できることを提案している。その有効性の検証として、従来単純な多場化では得られなかった内部指標に関する反対称構造 (AAK 項) が自然に生成されることを示し、多成分場特有の力学構造を適切に捉えられることを明らかにした。

以上の研究内容は、博士論文として十分な新規性と独創性を有している。論文は英語で執筆されており、また海外における長期研究滞在の経験も有することから、博士の学位取得に必要な英語能力を備えていると認められる。以上により、本委員会は本論文が博士論文として求められる水準を十分に満たしていると評価し、博士論文本審査を合格と判定することで全員一致した。