Minimal Supersymmetric

Bulk Matter Randall-Sundrum Model

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Abstract

The bulk matter Randall-Sundrum (RS) model is a setup where the Standard Model matter and gauge fields reside in the bulk of 5D warped spacetime while the Higgs field is confined on the infrared brane. The wavefunctions of the 1st and 2nd generation matter particles are localized towards the ultraviolet brane and those of the 3rd generation towards the infrared brane, so that the hierarchical structure of the Yukawa couplings arises geometrically without hierarchy in fundamental parameters. This thesis discusses observing signals of this model in the case where the Kaluza-Klein scale is far above the collider scale, but the model is combined with the minimal supersymmetric Standard Model (MSSM) and supersummetry (SUSY) particles are in the reach of collider experiments. The minimal supersummteric extension of the bulk matter RS model is formulated. Then a general SUSY breaking mass spectrum consistent with the bulk matter model is considered; SUSY breaking sector locates on the IR brane and its effects are mediated to 5D MSSM through a hybrid of gravity mediation, gaugino mediation and gauge mediation. This thesis argues that it is possible to observe unique signatures of the bulk matter RS model through rare decays of "almost SU(2) singlet mass eigenstates" that are induced by flavor-violating gravity mediation contributions to matter soft SUSY breaking terms.

Contents

1 Chapter I : Introduction

The development of particle physics today is motivated by "hierarchy problems", that is, large difference in magnitudes among fundamental physical quantities. If a theory with hierarchical quantities gives a consistent description of Nature, theorists still consider the situation "unnatural" and assume an underlying new physics that explains the hierarchy with non-hierarchical parameters.

There are two well-known hierarchy problems in particle physics. One is the "gauge hierarchy problem", which concerns the fact that, in the Standard Model (SM), the mass of Higgs boson is sensitive to higher mass scales; if the Higgs boson couples to heavy particles, its mass receives radiative corrections from loop diagrams involving these particles, and the amount of the corrections to the Higgs mass squared are proportional to the mass squareds of the heavy particles. If SM is valid upto the Planck scale, it is natural to have heavy particles at intermediate mass scales between TeV and the Planck that couple to the Higgs boson. On the other hand, the unitarity of $WW \to WW$ cross section requires that the Higgs mass be below 1 TeV. Hence there should be an unnatural cancellation between the tree-level mass and the radiative corrections in order to derive the Higgs mass below 1 TeV. In other words, the gauge hierarchy problem is a problem of unnaturally large radiative corrections. If the gauge hierarchy problem is really a problem, the solution lies in physics at TeV scale because only new particles or mechanisms at that scale can explain why the radiative corrections to the Higgs mass are at most $O(100)$ GeV.

The second hierarchy problem concerns the SM fermion masses, which range from the electron mass, 511 keV, to the top quark mass, 175 GeV. This hierarchy is translated into the hierarchy of the Yukawa coupling constants of SM. This Yukawa coupling hierarchy problem is of different nature from the gauge hierarchy problem because radiative corrections to a Yukawa coupling constant are proportional to the constant itself. So a small Yukawa coupling never receives large radiative corrections. The essence of the Yukawa hierarchy problem is that, if all the Yukawa coupling constants have the same origin in the UV completion of SM, it is unnatural to have such difference in magnitudes. In contrast to the gauge hierarchy problem, the solution need not lie in TeV scale physics. It is probable that the Yukawa coupling hierarchy reflects physics at an intermediate scale between TeV and the Planck scales, where we cannot directly access through experiments.

The bulk matter Randall-Sundrum (RS) model [2] is a viable candidate for the solution to the Yukawa hierarchy problem. It is an extension of the original RS model [1], which provides a geometrical solution to the gauge hierarchy problem. In RS models, the 5th dimension compactified on the orbifold S^1/Z_2 and two 3-branes at the orbifold points are introduced.

With Anti-deSitter (AdS) curvature, the spacetime metric is warped in the 5th dimension so that any parameters with mass dimension scale down along this direction. In the original model, SM fields are confined on the infrared (IR) brane where the effective Planck scale is at TeV, and only gravity propagates in 5D spacetime (bulk). Since the cut off scale of the SM sector is "warped" down to TeV scale, the radiative corrections to the Higgs boson do not surpass $O(100)$ GeV. The authors of [2] extended the original model to the bulk matter model by confining only the Higgs boson on the IR brane while putting other SM fields in the bulk. The zeroth Kaluza-Klein (KK) modes of the bulk fields are then identified with the 4D SM fields. This model is still free from the gauge hierarchy problem. Moreover the 5D Dirac mass terms for bulk fermions (bulk mass) naturally give rise to the Yukawa coupling hierarchy; the wave functions of the zero-modes of bulk fermions are proportional to the exponential of the values of their bulk masses times y. Therefore the geometrical overlap between these wave functions and the IR brane has exponential hierarchy even if the bulk masses are $O(1)$ times the AdS curvature. The magnitudes of the Yukawa couplings depend on this geometrical overlap because they come from contact terms among fermions in the bulk and the Higgs field on the IR brane. In this way, the hierarchical structure of the Yukawa couplings arises with "natural", non-hierarchical values of the fundamnetal parameters.

Although its elegant solution to both the gauge hierarchy problem and the Yukawa hierarchy problem, the bulk matter RS model suffers from severe constraints from the experimental data of flavor physics. The most stringent constraint comes from ϵ_K of K^0-$\bar{K}^0$ mixing; in the bulk matter model, the couplings of the KK excitations of bulk gluon to SU(2) doublet SM quarks induce flavor-violating interactions. In particular, the exchange of 1st KK gluon between d_L and s_L contributes to K^0-$\bar{K}^0$ mixing at tree level and leads to inconsistency with the experimental data. The current lower bound on the mass of 1st KK gluon is 21 TeV [4]. The effective Planck scale on the IR brane is of the same magnitude as that mass. Therefore, with this bound, the bulk matter RS model fails in solving the gauge hierarchy problem. After all, considering the different nature of the gauge hierarchy and the Yukawa hierarchy problems, it is more likely that their solutions come from different new physics at different scales.

This thesis deals with the bulk matter RS model whose IR scale is at an intermediate scale between TeV and the Planck scales. This model explains the Yukawa hierarchy with non-hierarchical values of bulk masses, whereas radiative corrections to the Higgs field are suppressed at $O(100)$ GeV scale through some other machanism. The purpose of the thesis is to extract signatures that are unique to the bulk matter RS model and are observable at near-future experiments. In general, there cannot be such signatures because the KK scale ($\sim$ IR scale) is assumed far above TeV scale. However, if the entire model contains new source of

flavor violation that is independent of the Yukawa couplings, it is possible to test predictions of the bulk matter model by measuring the structure of the new flavor-violating terms and comparing it with that of the Yukawa coupling constants. This is because, in the bulk matter model, the 5D profiles of the SM matter wave functions control the hierarchy of any flavor-violating terms. The model thus predicts a relation between the Yukawa coupling hierarchy and the hierarchy of the new flavor-violating terms.

As an example of new source of flavor violation, we here consider TeV-scale supersymmetry (SUSY). SUSY-breaking soft mass terms that arise from contact interactions between matter superfields and F-term SUSY breaking (gravity mediation [5]) provide new source of flavor violation that is independent of the Yukawa couplings. SUSY matter particles at TeV scale are accessible by the LHC and the future ILC [14] and CLIC [15]. Flavor mixings of SUSY particles reflect flavor-violating gravity mediation contributions, which are predicted to be connected with the Yukawa coupling hierarchy in the bulk matter RS model. Therefore, by producing SUSY matter particles and measuring their flavor compositions, one can observe signatures unique to the bulk matter model. Additionally, the bulk matter RS model combined with TeV-scale SUSY is a natural setup because the latter solves the gauge hierarchy problem instead of the RS spacetime.

In this thesis, I first formulate the minimal supersymmetric bulk matter RS model where the bulk matter RS model is combined with the minimal supersymmetric Standard Model (MSSM), by extending the superfield formalism to 5D spacetime. I consider a general combination of SUSY breaking mediation mechanisms that are consistent with the bulk matter RS framework; SUSY breaking sector locates on the IR brane and its effects are mediated to MSSM in the bulk through contact terms on the IR brane (gravity mediation [5]), renormalization group evolutions below the KK scale (gaugino mediation [6]) and gauge interactions with messenger fields on the IR brane (gauge mediation [7]). (Anomaly mediation contributions [8] are suppressed at least by the warp factor compared to gaugino mediation ones and hence are negligible.) Based on the data on the Yukawa coupling constants, the Cabbibo-Kobayashi-Maskawa (CKM) matrix and the neutrino oscillations, I determine the 5D profiles of the matter superfields so that the hierarchical structure of the Yukawa couplings arises from non-hierarchical parameters. These profiles in turn determine the magnitudes of gravity mediation contributions, which are the key prediction of the bulk matter RS model.

Next I discuss how the gravity mediation contributions can be extracted from observable quantities related to soft SUSY breaking terms. Gravity mediation intrinsically violates flavor while gaugino and gauge mediation mainly generates flavor-universal terms. Therefore flavor-

violating soft SUSY breaking terms reflect the gravity mediation contributions. One thus can indirectly measure the gravity-mediation-origined terms through flavor-violating interactions of SUSY matter particles. One obstacle is that the Yukawa coupling constants themselves induce flavor-violating soft SUSY breaking terms through renormalization group (RG) evolutions, as in models with minimal flavor violation. However, I will show that it is possible to distinguish gravity mediation contributions from the RG effects of the Yukawa couplings by focusing on SU(2) singlet SUSY matter particles, for which flavor violation of gravity mediation contributions can be significantly larger than that of the RG effects. Thus the flavor-mixing terms for SU(2) singlet SUSY particles well reflect the flavor structure of gravity mediation contributions. In conclusion, the predictions of the bulk matter RS model on gravity mediation can be tested by measuring the flavor compositions of "almost SU(2) singlet" SUSY particle mass eigenstates.

Finally I discuss how such flavor compositions can be studied experimentally. I provide three promising channels of experiments that are feasible at the ILC. One is that SU(2) singlet smuon mixes with stau through gravity-mediation-origined soft terms and one measures the branching ratio of "almost singlet smuon mass eigenstate" decaying into SM tau and another SUSY particle. Another channel is that SU(2) singlet smuon mixes with selectron and one measures the branching ratio of "almost singlet smuon mass eigenstate" decaying into SM electron and another SUSY particle. The third is that SU(2) singlet scharm mixies with stop and one measures the branching ratio of "almost singlet scharm mass eigenstate" decaying into SM top and another SUSY particle. The bulk matter RS model predicts these branching ratios and can be tested through their measurements.

This thesis is organized as follows. In Chapter II, I review the superfield formalism and the minimal supersymmetric Standard Model. In Chapter III, I review the bulk matter Randall-Sundrum model. In Chapter IV, I combine MSSM with the bulk matter RS model and formulate the minimal supersymmetric bulk matter RS model. The 5D profiles of the matter superfields are then fixed to solve the Yukawa hierarchy problem. In Chapter V, I discuss the general property of the SUSY particle mass spectrum with emphasis on the structure of flavor-violating terms. In Chapter VI, I show how the bulk matter RS model predicts the flavor compositions of SUSY particles and how these predictions differ from those of other frameworks. In Chapter VII, I describe the experimental methods to test the predictions of the bulk matter RS model and distinguish the model from others. Chapter VIII is devoted for summary and outlook.

2 Chapter II : Minimal Supersymmetric Standard Model

The minimal supersymmetric Standard Model (MSSM) is the $\mathcal{N} = 1$ extension of the Standard Model. I first describe the superfield formalism and then adopt it to write the MSSM Lagrangian.

2.1 Superfield Formalism

Let us consider 4D $\mathcal{N} = 1$ SUSY. In the superfield formalism, two coordinates $\theta_\alpha, \bar{\theta}^{\dot{\alpha}}$ that are Weyl spinors are introduced, where $\alpha, \dot{\alpha}$ denote spinor indices. $(\theta)^2, (\bar{\theta})^2$ are defined as

$$\theta^2 \equiv \theta^\alpha \theta_\alpha , \quad \bar{\theta}^2 \equiv \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} . \tag{1}$$

Superfield derivatives, $\frac{\partial}{\partial \theta^\alpha}, \frac{\partial}{\partial \theta^{\dot{\alpha}}}$, satisfy

$$\frac{\partial}{\partial \theta^\alpha} \theta^\beta = \delta_\alpha^\beta , \quad \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} \bar{\theta}^{\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}} . \tag{2}$$

Superfield integrals are given by

$$\int \mathrm{d}\theta^\alpha \, (a + b \, \theta^\beta) = b \, \delta^{\alpha\beta} ,$$

$$\int \mathrm{d}^2\theta \equiv \int \frac{1}{4} \mathrm{d}\theta_\alpha \mathrm{d}\theta^\alpha , \quad \int \mathrm{d}^2\bar{\theta} \equiv \int \frac{1}{4} \mathrm{d}\bar{\theta}^{\dot{\alpha}} \mathrm{d}\bar{\theta}_{\dot{\alpha}} ,$$

$$\int \mathrm{d}^2\theta \, \theta^2 = 1 , \quad \int \mathrm{d}^2\bar{\theta} \, \bar{\theta}^2 = 1 , \tag{3}$$

where a, b are c-numbers.

The generators of SUSY transformations, $Q_\alpha, \bar{Q}^{\dot{\alpha}}$, can be represented as

$$iQ_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu)_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}} \, \partial_\mu , \quad i\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\beta (\sigma^\mu)_{\beta\dot{\alpha}} \, \partial_\mu . \tag{4}$$

It follows that the covariant derivatives that are invariant under the SUSY transformations are given by

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}} \partial_\mu , \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\beta (\sigma^\mu)_{\beta\dot{\alpha}} \partial_\mu . \tag{5}$$

Chiral superfield, $\phi(x, \theta, \bar{\theta})$, is defined as

$$\bar{D}_{\dot{\alpha}} \phi = 0 . \tag{6}$$

ϕ can be decomposed into scalars, s, F, and a Weyl spinor, ψ, in the following way:

$$\phi(x, \theta, \bar{\theta}) \;=\; s(y) \;+\; \sqrt{2}\theta^\alpha \psi_\alpha(z) \;+\; \theta^2 F(z) \;, \tag{7}$$

where z is defined as $z^\mu \equiv x^\mu + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}$.

From (4, 7), we see that the action for ϕ_is (i labels different chiral superfields) that is invariant under SUSY transformations takes the following general form: $(\mathrm{d}^4\theta \equiv \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta})$

$$S \;=\; \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta \; f(\phi^\dagger, \phi) \;+\; \int \mathrm{d}^2\theta \; W(\phi) \;+\; \int \mathrm{d}^2\bar{\theta} \; (W(\phi))^\dagger \right] \;, \tag{8}$$

where $W(\phi)$ is a holomorphic function of ϕ, and $f(\phi^\dagger, \phi)$ itself is a real superfield.

Next we introduce Abelian gauge superfield, $V(x, \theta, \bar{\theta})$. This is a real superfield. The gauge transformation can be written as

$$V \;\rightarrow\; V \;+\; \Lambda \;+\; \Lambda^\dagger \;, \tag{9}$$

where Λ is a chiral superfield. When we take the Wess-Zumino gauge, $V(x, \theta, \bar{\theta})$ can be decomposed into a vector, v_μ, a scalar, $D(x)$, and a Weyl spinor, λ, in the following way:

$$V(x, \theta, \bar{\theta}) \;=\; \theta^\alpha(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}} \; v_\mu(x) \;+\; i\theta^2\bar{\theta}_{\dot{\beta}}\bar{\lambda}^{\dot{\beta}}(x) \;-\; i\bar{\theta}^2\theta^\alpha\lambda_\alpha(x) \;+\; \frac{1}{2}\theta^2\bar{\theta}^2 D(x) \;. \tag{10}$$

We define SUSY "field strength" as follows:

$$W_\alpha \;\equiv\; -\frac{1}{4}\bar{D}_{\dot{\beta}}\bar{D}^{\dot{\beta}}D_\alpha \, V \;, \quad \bar{W}_{\dot{\beta}} \;\equiv\; -\frac{1}{4}D^\alpha D_\alpha \bar{D}_{\dot{\beta}} \, V \;. \tag{11}$$

The SUSY invariant action for an Abelian gauge superfield is given by

$$S \;=\; \int \mathrm{d}^4x \left[\int \mathrm{d}^2\theta \; \frac{1}{16g^2} W^\alpha W_\alpha \;+\; \int \mathrm{d}^2\bar{\theta} \; \frac{1}{16g^2} \bar{W}_{\dot{\beta}} \bar{W}^{\dot{\beta}} \right] \;, \tag{12}$$

where g indicates the gauge coupling constant. The Lagrangian is normalized so that, after the rescaling: $V \rightarrow 2gV$, the kinetic term becomes canonical.

For a non-Abelian gauge superfield, we write

$$V \;\equiv\; V^a(T^a) \;, \quad \Lambda \;\equiv\; \Lambda^a(T^a) \;,$$

where T^as denote the generators of the gauge group and a denotes an index of the adjoint representation. The gauge transformation then becomes

$$V \;\rightarrow\; e^{\Lambda^\dagger} \, e^V \, e^{-\Lambda} \;. \tag{13}$$

The SUSY "field strength" becomes

$$W_\alpha \equiv -\frac{1}{4}\bar{D}_{\dot\beta}\bar{D}^{\dot\beta}(e^V D_\alpha e^{-V}) , \quad \bar{W}_{\dot\beta} \equiv \frac{1}{4}D^\alpha D_\alpha(e^{-V}\bar{D}_{\dot\beta}e^V) , \tag{14}$$

which is invariant under the gauge transoformation. The SUSY and gauge invariant action for a non-Abelian gauge field is given by

$$S = \int \mathrm{d}^4x \left[\int \mathrm{d}^2\theta \, \frac{1}{8g^2}\mathrm{tr}\{ W^\alpha W_\alpha \} + \int \mathrm{d}^2\bar{\theta} \, \frac{1}{8g^2}\mathrm{tr}\{ \bar{W}_{\dot\beta}\bar{W}^{\dot\beta} \} \right] , \tag{15}$$

The Lagrangian is normalized so that, after the rescaling: $V \to 2gV$, the kinetic term becomes canonical.

Finally, we couple a gauge superfield with a chiral superfield that is in the fundamental representation of the gauge group. This chiral superfield receives the following gauge transformation:

$$\phi \to e^{-\Lambda}\phi . \tag{16}$$

Gauge invariance requires that the action for the chiral superfield, (8), be rewritten as

$$S = \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta \, f(\phi^\dagger e^V \phi) + \int \mathrm{d}^2\theta \, W(\phi) + \int \mathrm{d}^2\bar{\theta} \, (W(\phi))^\dagger \right] . \tag{17}$$

where $f(\phi^\dagger e^V \phi)$ is now a function of $\phi^\dagger e^V \phi$.

2.2 Field Content and Lagrangian

In MSSM, the SM fermions are identified with the fermionic components of matter chiral superfields, the Higgs bosons are with the scalar components of Higgs chiral superfields, and the SM gauge bosons are with the vector components of gauge superfields. Due to the holomorphicity of SUSY theories, the Higgs sector is extended to have two Higgs doublets in order to give masses to both up-type and down-type fermions. We assume R-parity to forbid baryon number or lepton number violating renormalizable superpotential. The superfield content of MSSM is listed in Table 1, where i denotes a flavor index.

Notation	U(1)$_Y$	SU(2)$_L$	SU(3)$_C$	R-parity
Q_i	1/6	2	3	$-$
U_i	$-2/3$	1	$\bar{3}$	$-$
D_i	1/3	1	$\bar{3}$	$-$
L_i	$-1/2$	2	1	$-$
E_i	1	1	1	$-$
H_u	1/2	2	1	$+$
H_d	$-1/2$	2	1	$+$

Notation	Description	R-parity
B	$U(1)_Y$ gauge superfield	$+$
W	$SU(2)_L$ gauge superfield	$+$
G	$SU(3)_C$ gauge superfield	$+$

In the SUSY limit, the MSSM Lagrangian consists of the kinetic terms for B, W, G, the Kaehler potentials for Q, U, D, L, E, H_u, H_d and the following superpotential:

$$W_{MSSM} = \mu H_u H_d + (Y_u)_{ij} H_u U_i Q_j + (Y_d)_{ij} H_d D_i Q_j + (Y_e)_{ij} H_d E_i L_j \tag{18}$$

For MSSM to be a realistic model, the scalar components of Q, U, D, L, E and the fermionic components of B, W, G must be heavy enough to evade the experimental bounds, which means that SUSY must be broken at low energies. To solve the gauge hierarchy problem, this SUSY breaking must be soft, that is, all SUSY breaking terms must have positive mass dimension. The general form of soft SUSY breaking Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{soft} = \ & -(m_Q^2)_{ij} Q_i^\dagger Q_j - (m_U^2)_{ij} U_i^\dagger U_j - (m_D^2)_{ij} D_i^\dagger D_j - (m_L^2)_{ij} L_i^\dagger L_j - (m_E^2)_{ij} E_i^\dagger E_j \\ & + (A_u)_{ij} H_u U_i Q_j + (A_d)_{ij} H_d D_i Q_j + (A_e)_{ij} H_d E_i L_j \\ & - M_1 \tilde{B}\tilde{B} - M_{W2} \tilde{W}\tilde{W} - M_3 \tilde{G}\tilde{G} \\ & - m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - B\mu H_u H_d - \text{h.c.} \ , \end{aligned} \tag{19}$$

where Q, U, D, L, E, H_u, H_d here denote the scalar components of the corresponding chiral superfields and $\tilde{B}, \tilde{W}, \tilde{G}$ denote the fermionic components of the corresponding gauge superfields.

Summing the supersymmetric Lagrangian and the soft SUSY breaking Lagrangian, we obtain the Higgs potential that can break the electroweak symmetry appropriately. The Higgs bosons develop vacuum expectation values (VEV) as

$$<H_u> = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad <H_u> = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \tag{20}$$

where $v_u^2 + v_d^2 \simeq (174 \text{ GeV})^2$. $\tan\beta$ is defined as $\tan\beta \equiv v_u/v_d$.

2.3 Mass Eigenstates of SUSY Particles

SUSY particles mix with each other and form mass eigenstates.

The charged Higgsinos mix with the charged Winos and result in chargino mass eigenstates. The mass matrix is given by

$$\mathcal{L} \supset -\frac{1}{2} (\tilde{W}^+, \tilde{H}_u^+) \begin{pmatrix} M_2 & g_L v_u \\ g_L v_d & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix} + \text{transpose}. \tag{21}$$

The mass eigenstates are denoted by $\chi_1^\pm, \chi_2^\pm$.

The neutral Higgsinos mix with $\tilde{B}, \tilde{W}_3$ and result in neutralino mass eigenstates. The mass matrix is given by

$$\mathcal{L} \supset -\frac{1}{2} \begin{pmatrix} \tilde{B}, & \tilde{W}_3, & \tilde{H}_u^0, & \tilde{H}_d^0 \end{pmatrix} \begin{pmatrix} M_1 & 0 & g_Y v_u/\sqrt{2} & -g_Y v_d/\sqrt{2} \\ 0 & M_2 & -g_L v_u/\sqrt{2} & g_L v_d/\sqrt{2} \\ g_Y v_u/\sqrt{2} & -g_L v_u/\sqrt{2} & 0 & -\mu \\ -g_Y v_d/\sqrt{2} & g_L v_d/\sqrt{2} & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}_3 \\ \tilde{H}_u^0 \\ \tilde{H}_d^0 \end{pmatrix} . \quad (22)$$

The mass eigenstates are denoted by $\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0$.

With the electroweak symmetry breaking, SU(2) doublet matter SUSY particles mix with SU(2) singlets through the A-terms and the F-terms of the Higgs superfields. For example, the up sector of Q_i doublet, denoted by Q_{ui}, mixes with $U_j^\dagger$ in the following way:

$$\mathcal{L} \supset - \begin{pmatrix} Q_{ui}^\dagger, & U_j \end{pmatrix} \begin{pmatrix} (m_Q^2)_{ik} & -v_u(A_u^\dagger)_{il} - \mu v_d(Y_u^\dagger)_{il} \\ -v_u(A_u)_{jk} - \mu^* v_d(Y_u)_{jk} & (m_U^{\dagger 2})_{jl} \end{pmatrix} \begin{pmatrix} Q_{uk} \\ U_l^\dagger \end{pmatrix} , \quad (23)$$

where i, j, k, l are flavor indices. We have similar expressions for the mixings of the down sector of Q_i doublet with $D_j^\dagger$ and of the down sector of L_i doublet with $E_j^\dagger$.

3 Chapter III : Bulk Matter Randall-Sundrum Model

In Randall-Sundrum (RS) model [1], we introduce the 5th dimension, denoted by y, that is compactified on the orbifold $S^1/Z_2 : -\pi R \leq y \leq \pi R$ with y and $-y$ indentified. Two 3-branes are put at $y = 0$ and $y = \pi R$. The former brane is called "UV brane" and the latter "IR brane". The metric is given by

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}y^2 , \tag{24}$$

where k indicates the Anti-deSitter curvature. We assume that k is of the same magnitude as the 5D Planck scale, M_5. Then, if $e^{-kR\pi} << 1$, we have the following formula for the 4D reduced Planck mass, M_* :

$$M_*^2 = \frac{M_5^3}{k}(1 - e^{-2kR\pi}) \simeq \frac{M_5^3}{k} , \tag{25}$$

which implies that $k \sim M_5 \sim M_*$. The effective Planck scale on the UV brane is M_5 and that on the IR brane is $M_5 e^{-kR\pi}$.

In the bulk matter RS model, the SM matter fields as well as the gauge fields are identified with the zero-modes of 5D fields in the bulk. The Higgs field is considered as a 4D field confined on the IR brane.

3.1   5D Profiles of Bulk Gauge Fields

We first discuss 5D bulk gauge fields. The 5D gauge fields are Z_2-even because they have zero-modes only when they are so. We decompose them into Kaluza-Klein (KK) modes, and study their 5D profiles (y-dependence). We take the 5D unitary gauge, $A_5(x, y) = 0$, in the following.

The action for a 5D gauge field is given by

$$S_{gauge} = \int \mathrm{d}^4 x \int \mathrm{d}y \, e^{-4k|y|} \left[-\frac{1}{4} \, e^{2k|y|}\eta^{\mu\nu} \, e^{2k|y|}\eta^{\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a \, + \, \frac{1}{2} \, e^{2k|y|}\eta^{\mu\nu} \left(\partial_y A_\mu^a\right)\left(\partial_y A_\nu^a\right) \right] ,$$
$$\tag{26}$$

where

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

and ∂_μs denote a 4D partial derivative on flat background.

The equation of motion for the bulk gauge field is then given by

$$\left[\,\Box\;+\;\partial_y\;(\;e^{-2k|y|}\partial_y\;)\right]\;A_\mu^a(x,y)\;\;=\;\;0\;.\tag{27}$$

The boundary conditions for $A_\mu^a(x,y)$ are as follows :

$$\partial_y A_\mu^a(x,y)|_{y=0}\;\;=\;\;\partial_y A_\mu^a(x,y)|_{y=\pi R}\;\;=\;\;0\;.$$

Then $A_\mu^a(x,y)$ can be decomposed into canonically normalized KK modes, $A_\mu^{a\,(n)}(x)$, in the following way :

$$A_\mu^a(x,y)\;\;=\;\;\sum_{n=0}\,A_\mu^{a\,(n)}(x)\;f^{(n)}(y)\tag{28}$$

with

$$\begin{aligned}f^{(0)}(y)\;&=\;\;\frac{1}{2\pi R}\;,\\f^{(n)}(y)\;&=\;\;\frac{1}{N_n}\,e^{k|y|}\;\left[\;J_1(\frac{m_n}{k}e^{k|y|})\;+\;\alpha_n Y_1(\frac{m_n}{k}e^{k|y|})\;\right]\;,\end{aligned}\tag{29}$$

where N_n,α_n are defined by

$$\begin{aligned}N_n^2\;&\equiv\;\;\int_{-\pi R}^{\pi R}\mathrm{d}y\;e^{2k|y|}\;\left[\;J_1(\frac{m_n}{k}e^{k|y|})\;+\;\alpha_n Y_1(\frac{m_n}{k}e^{k|y|})\;\right]^2\;,\\\alpha_n\;&\equiv\;\;-\frac{J_0(\frac{m_n}{k})}{Y_0(\frac{m_n}{k})}\;.\end{aligned}\tag{30}$$

m_n indicates the mass of the n-th KK mode, and is obtained from the relation :

$$J_0(\frac{m_n}{k}e^{k\pi R})\;+\;\alpha_n Y_0(\frac{m_n}{k}e^{k\pi R})\;\;=\;\;0\;.\tag{31}$$

The masses of the first several KK modes, $m_1,m_2,...$, are of the same order as $ke^{-k\pi R}$.

Note that the zero-mode has a flat profile (no y-dependence). This is due to the gauge principle.

3.2   5D Profiles of Bulk Fermions

We next discuss 5D bulk fermions. A 5D fermion is a Dirac fermion, which can be decomposed into the Weyl components as $\frac{1\pm\gamma_5}{2}\Psi=\psi_{R/L}$. One of the Weyl components is Z_2-even and the other is Z_2-odd. Only the Z_2-even component has zero-mode, which can be identified with a

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SM fermion. Below we assume that $\psi_L(x, y)$ is Z_2-even. The formula for the opposite case is obtained in a similar manner.

The action for the bulk fermion is given by

$$S_{fermion} = \int d^4x \int dy \, e^{-4k|y|} \left[\frac{i}{2} \bar{\Psi} e^{k|y|} \gamma^\mu \partial_\mu \Psi + \frac{i}{2} \bar{\Psi} i\gamma_5 \partial_y \Psi + \text{h.c.} + \text{sgn}(y) ck \, \bar{\Psi}\Psi \right] ,$$
(32)

where Ψ is a 5D Dirac fermion and c indicates its Dirac mass term normalized by the AdS curvature k. The equations of motion for ψ_R, ψ_L are given by

$$i\bar{\sigma}^\mu \partial_\mu \psi_R + e^{-k|y|} \partial_y \psi_L - e^{-k|y|} \text{sgn}(y)(2-c)k\psi_L = 0 ,$$
$$i\sigma^\mu \partial_\mu \psi_L - e^{-k|y|} \partial_y \psi_R + e^{-k|y|} \text{sgn}(y)(2+c)k\psi_R = 0 .$$
(33)

The boundary conditions for ψ_R, ψ_L are as follows :

$$\psi_R(x, y=0) = \psi_R(x, y=\pi R) = 0 ,$$
$$\{ \partial_y - \text{sgn}(y)(2-c)k \} \psi_L(x,y)|_{y=+0} = \{ \partial_y - \text{sgn}(y)(2-c)k \} \psi_L(x,y)|_{y=\pi R-0} = 0 .$$
(34)

Then $\psi_L(x, y), \psi_R(x, y)$ can be decomposed into canonically normalized KK modes as follows :

$$\psi_R(x, y) = \sum_{n=1} \psi_R^{(n)}(x) f_R^{(n)}(y) ,$$
$$\psi_L(x, y) = \sum_{n=0} \psi_L^{(n)}(x) f_L^{(n)}(y)$$
(35)

with

$$f_L^{(0)}(y) = \frac{1}{N_L^0} e^{(2-c)k|y|} ,$$
$$f_R^{(n)}(y) = \frac{1}{N_R^n} e^{(5/2)k|y|} \left[J_{1/2-c}(\frac{m_n^R}{k} e^{k|y|}) + \beta_n^R Y_{1/2-c}(\frac{m_n^R}{k} e^{k|y|}) \right] \quad (n \geq 1) ,$$
$$f_L^{(n)}(y) = \frac{1}{N_L^n} e^{(5/2)k|y|} \left[J_{1/2+c}(\frac{m_n^L}{k} e^{k|y|}) + \beta_n^L Y_{1/2+c}(\frac{m_n^L}{k} e^{k|y|}) \right] \quad (n \geq 1) , \tag{36}$$

where

$$(N_L^0)^2 = 2\frac{e^{(1-2c)k\pi R} - 1}{k(1-2c)} \, ,$$

$$(N_R^n)^2 = \int_{-\pi R}^{\pi R} dy \; e^{2k|y|} \left[J_{1/2-c}(\frac{m_n^R}{k}e^{k|y|}) + \beta_n^R Y_{1/2-c}(\frac{m_n^R}{k}e^{k|y|}) \right]^2 ,$$

$$(N_L^n)^2 = \int_{-\pi R}^{\pi R} dy \; e^{2k|y|} \left[J_{1/2+c}(\frac{m_n^L}{k}e^{k|y|}) + \beta_n^L Y_{1/2+c}(\frac{m_n^L}{k}e^{k|y|}) \right]^2 , \qquad (37)$$

$$\beta_{Rn} = -\frac{J_{1/2-c}(\frac{m_n^R}{k})}{Y_{1/2-c}(\frac{m_n^R}{k})} \, ,$$

$$\beta_{Ln} = -\frac{J_{-1/2+c}(\frac{m_n^L}{k})}{Y_{-1/2+c}(\frac{m_n^L}{k})} \, . \qquad (38)$$

The mass of the n-th KK mode of ψ_R, m_n^R, and that of ψ_L, m_n^L, are derived from the following relations :

$$J_{1/2-c}(\frac{m_n^R}{k}e^{k\pi R}) + \beta_n^R Y_{1/2-c}(\frac{m_n^R}{k}e^{k\pi R}) = 0 \, ,$$

$$J_{-1/2+c}(\frac{m_n^L}{k}e^{k\pi R}) + \beta_n^L Y_{-1/2+c}(\frac{m_n^L}{k}e^{k\pi R}) = 0 \, . \qquad (39)$$

The masses of the first several KK modes are of the same order as $ke^{-k\pi R}$.

3.3 5D Profiles of Bulk Scalars

Finally we discuss 5D bulk scalars. Although the bulk matter RS model contains no bulk scalars, this discussion will be useful when we supersymmetrize it.

The action for the bulk scalar is given by

$$S_{scalar} = \int d^4x \int dy \; e^{-4k|y|} \left[e^{2k|y|}\eta^{\mu\nu}\partial_\mu\Phi^\dagger\partial_\nu\Phi \; - \; (\partial_y - \text{sgn}(y)ck)\Phi^\dagger(\partial_y - \text{sgn}(y)ck)\Phi \right] \, . \qquad (40)$$

The equation of motion for $\Phi(x,y)$ is given by

$$\left[\Box \; - \; e^{-2k|y|}\partial_y^2 \; + \; 4e^{-2k|y|}\text{sgn}(y)k\partial_y \; + \; e^{-2k|y|}(c^2 - 4c)k^2 \right] \Phi(x,y) = 0 \, . \qquad (41)$$

If Φ is Z_2-odd, the boundary condition is

$$\Phi(x, y = 0) = \Phi(x, y = \pi R) = 0 \, . \qquad (42)$$

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If Φ is Z_2-even, the boundary condition is

$$(\partial_y - \text{sgn}(y)ck)\Phi(x, y)|_{y=+0} = (\partial_y - \text{sgn}(y)ck)\Phi(x, y)|_{y=\pi R - 0} = 0 . \tag{43}$$

The KK decomposition for a Z_2-odd Φ is given by

$$\Phi(x, y) = \sum_{n=1} \phi^{(n)}(x) f_{odd}^{(n)}(y) , \tag{44}$$

with

$$f_{odd}^{(n)}(y) = \frac{1}{N_n^{odd}} e^{2k|y|} \left[J_{c-2}(\frac{m_n}{k}e^{k|y|}) + \zeta_n^{odd} Y_{c-2}(\frac{m_n}{k}e^{k|y|}) \right] , \tag{45}$$

where

$$(N_n^{odd})^2 = \int_{-\pi R}^{\pi R} dy\, e^{2k|y|} \left[J_{c-2}(\frac{m_n}{k}e^{k|y|}) + \zeta_n^{odd} Y_{c-2}(\frac{m_n}{k}e^{k|y|}) \right]^2 , \tag{46}$$

$$\zeta_n^{odd} = -\frac{J_{c-2}(\frac{m_n}{k})}{Y_{c-2}(\frac{m_n}{k})} , \tag{47}$$

and the mass of the n-th KK mode, m_n, is obtained from

$$J_{c-2}(\frac{m_n}{k}e^{k\pi R}) + \zeta_n^{odd} Y_{c-2}(\frac{m_n}{k}e^{k\pi R}) = 0 . \tag{48}$$

The KK decomposition for a Z_2-odd Φ is given by

$$\Phi(x, y) = \sum_{n=0} \phi^{(n)}(x) f_{even}^{(n)}(y) , \tag{49}$$

with

$$f_{even}^{(0)}(y) = \frac{1}{N_0^{even}} e^{ck|y|} ,$$

$$f_{even}^{(n)}(y) = \frac{1}{N_n^{even}} e^{2k|y|} \left[J_{c-2}(\frac{m_n}{k}e^{k|y|}) + \zeta_n^{even} Y_{c-2}(\frac{m_n}{k}e^{k|y|}) \right] , \tag{50}$$

where

$$(N_0^{even})^2 = 2\frac{e^{(2c-2)k\pi R} - 1}{k(2c - 2)} ,$$

$$(N_n^{even})^2 = \int_{-\pi R}^{\pi R} dy\, e^{2k|y|} \left[J_{c-2}(\frac{m_n}{k}e^{k|y|}) + \zeta_n^{even} Y_{c-2}(\frac{m_n}{k}e^{k|y|}) \right]^2 , \tag{51}$$

$$\zeta_n^{even} = -\frac{J_{c-1}(\frac{m_n}{k})}{Y_{c-1}(\frac{m_n}{k})} , \tag{52}$$

and the mass of the n-th KK mode, m_n, is obtained from

$$J_{c-1}(\frac{m_n}{k}e^{k\pi R}) + \zeta_n^{even} Y_{c-1}(\frac{m_n}{k}e^{k\pi R}) = 0 . \tag{53}$$

The masses of the first several KK modes are of the same order as $ke^{-k\pi R}$.

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4 Chapter IV : Bulk Matter RS Model with 5D MSSM

4.1 Setup

We introduce the same spacetime as in Chapter III; the 5th dimension is compactified on the orbifold $S^1/Z_2 : -\pi R \leq y \leq \pi R$ with y and $-y$ indentified, and the metric is given by (7). The UV brane locates at $y = 0$ and the IR brane at $y = \pi R$.

In the rest of the thesis, we assume that the effective Planck scale on the IR brane, $M_5 e^{-kR\pi}$ ($\sim ke^{-kR\pi}$) , is at an intermediate scale between M_* and TeV. In particular, it is assumed far above 21 TeV. Since the most severe constraint on the IR scale of the bulk matter RS model comes from the data on the $K^0 - \bar{K}^0$ mixing, which require it to be larger than 21 TeV [4], my model is free from any constraint on the bulk matter RS model itself. At the same time, it is hopeless to observe the effects of the Kaluza-Klein excitations through near-future experiments.

Let us consider 5D MSSM [3] where the 4D $\mathcal{N} = 1$ Higgs superfields are confined on the IR brane, and the 5D $\mathcal{N} = 1$ gauge superfields and matter hypermultiplets live in the bulk. In the following, we use the 4D superfield formalism extended with the 5th dimension y. We introduce a chiral superfield, X, on the IR brane whose F-component, F_X, develops vev to break 4D $\mathcal{N} = 1$ SUSY there. We consider both cases where there are one to several messenger pairs on the IR brane and there is no messenger pair at all. (It is easy to extend the model to cases where the messengers live in the bulk.) The gauge symmteries of the messenger pairs are not specified. SU(2) doublet squark, singlet up-type squark, singlet down-type squark, doublet slepton, singlet charged slepton hypermultiplets are denoted by Q_i, U_i, D_i, L_i, E_i, respectively, with i being flavor index. The up-type Higgs and the down-type Higgs superfields are denoted by H_u, H_d, respectively.

An off-shell 5D $\mathcal{N} = 1$ gauge superfield consists of a 5D gauge field A_M ($M = 0, 1, 2, 3, 5$), two 4D Weyl spinors λ_1, λ_2, a real scalar Σ, a real auxiliary field D and a complex auxiliary field F, all of which transform as the adjoint reprsentation of some gauge group. They combine to form one 4D $\mathcal{N} = 1$ gauge superfield V and one 4D $\mathcal{N} = 1$ chiral superfield χ that are

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\bar{\theta}^2\theta^2 D ,$$

$$\chi = \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F .$$

By $Z_2 : y \to -y$ symmetry, they transform as

$$V \to V , \quad \chi \to -\chi .$$

Assuming the invariance of the theory under the Z_2 symmtery, we obtain the following action for 5D $\mathcal{N} = 1$ gauge superfields:

$$
S_{5D\,gauge} = \int dy \int d^4x \, e^{-4k|y|} \left[\frac{1}{16(g_5^a)^2} \int d^2\theta e^{k|y|} \, \text{tr} \left\{ (e^{\frac{3}{2}k|y|}W^{a\,\alpha})(e^{\frac{3}{2}k|y|}W_\alpha^a) + \text{h.c.} \right\} \right.
$$
$$
\left. + \frac{1}{4(g_5^a)^2} \int d^4\theta e^{2k|y|} \, \text{tr} \left\{ (\sqrt{2}\partial_y + \chi^{a\,\dagger})e^{-V}(-\sqrt{2}\partial_y + \chi^a)e^V - (\partial_y e^{-V})(\partial_y e^V) \right\} \right],
$$
(54)

where a indicates an index of the adjoint representations of gauge groups and $W^{a\,\alpha}$ denotes the field strength of V^a in 4D flat spacetime. When the unitary gauge, $A_5^a = 0$, is chosen, only V^a has the massless mode in 4D picture. This mode has no dependence on y and will be written as $V_0(x, \theta, \bar{\theta})$.

A 5D $\mathcal{N} = 1$ hypermultiplet is expressed in terms of two 4D $\mathcal{N} = 1$ chiral superfields Φ, Φ^c that are in conjugate representaions of some gauge group. We assume that the former is Z_2-even and the latter Z_2-odd. Taking the basis of diagonal bulk masses, we have the following action for 5D $\mathcal{N} = 1$ hypermultiplets:

$$
S_{5D\,chiral} = \int dy \int d^4x e^{-4k|y|} \left[\int d^4\theta e^{2k|y|} \, (\Phi_i^\dagger e^{-V}\Phi_i + \Phi_i^c e^V \Phi_i^{c\,\dagger}) \right.
$$
$$
\left. + \int d^2\theta e^{k|y|} \, \Phi_i^c \{\partial_y - \chi/\sqrt{2} - (3/2 - c_i) \, \text{sgn}(y) \, k\}\Phi_i + \text{h.c.} \right], \quad (55)
$$

where i is a flavor index and c_i denotes the 5D bulk mass in unit of AdS curvature k. Only Φ_i has the massless mode in 4D picture, which will be written as $\phi_i(x + i\theta\sigma\bar{\theta}, \theta)e^{(3/2 - c_i)k|y|}$. This is shown by decomposing Φ_i, Φ_i^c into scalar and spinor components:

$$
\Phi_i = s_i + \sqrt{2}\theta e^{-\frac{k}{2}|y|}\psi_i + \theta^2 e^{-k|y|}F_i , \quad \Phi_i^c = s_i^c + \sqrt{2}\theta e^{-\frac{k}{2}|y|}\psi_i^c + \theta^2 e^{-k|y|}F_i^c .
$$

After integrating out F_i, F_i^c, the action contains the following term:

$$
S_{5D\,chiral} \supset \int dy \int d^4x e^{-4k|y|} \left[\{\partial_y - (3/2 - c_i) \, \text{sgn}(y) \, k\}s_i^\dagger \{\partial_y - (3/2 - c_i) \, \text{sgn}(y) \, k\}s_i \right.
$$
$$
+ \{\partial_y + (3/2 - c_i) \, \text{sgn}(y) \, k\}s_i^{c\,\dagger} \{\partial_y + (3/2 - c_i) \, \text{sgn}(y) \, k\}s_i^c
$$
$$
\left. + (\psi_i^c e^{\frac{k}{2}|y|})\{\partial_y - (2 - c_i) \, \text{sgn}(y) \, k\}(\psi_i e^{\frac{k}{2}|y|}) + \text{h.c.} \right]. \quad (56)
$$

The term involving s_i corresponds to the action (40) in section 3.3 with c replaced by $3/2 - c_i$. On the other hand, the term involving ψ_i, ψ_i^c corresponds to the action (32) in section 3.2 with c replaced by $-(2 - c_i)$ and Ψ replaced by $\Psi e^{\frac{k}{2}|y|}$. Therefore the zero-modes of s_i and ψ_i are respectively given by

$$
s_i^{(0)}(x + i\theta\sigma\bar{\theta}) \, e^{(3/2 - c_i)k|y|} , \quad \psi_i^{(0)}(x + i\theta\sigma\bar{\theta}) \, e^{(3/2 - c_i)k|y|} . \quad (57)
$$

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We thus proved that the zero-mode of the chiral superfield Φ_i can be written in the form $\phi_i(x + i\theta\sigma\bar{\theta}, \theta)e^{(3/2-c_i)k|y|}$.

We write down the 4D effective action for the fields in the bulk in terms of the massless modes:

$$
S_{4D\,eff.} = \int \mathrm{d}^4x \left[\frac{2\pi R}{4g_5^{a\,2}} \int \mathrm{d}^2\theta \ W^{a\alpha}W_\alpha^a \ + \ \text{h.c.} \right.
$$
$$
\left. + \ \int \mathrm{d}^4\theta \ 2\frac{e^{(1-2c_i)kR\pi} - 1}{(1-2c_i)k} \ \phi_i^\dagger \ e^{-V}\phi_i \right] ,
$$
(58)

where the dimensionful 5D gauge coupling, g_5^a, is connected to 4D gauge coupling g_4^a by the relation: $g_5^a = \sqrt{2\pi R}g_4^a$. ϕ_i represents the zero-mode of each of Q_i, U_i, D_i, L_i, E_i.

We also introduce an IR-brane-localized action. Below are the parts of the action relevant to the topic of this paper.

MSSM term:

$$
S_{IR} \supset \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta \ e^{-2kR\pi} \left\{ H_u^\dagger e^{-V} H_u \ + \ H_d^\dagger e^{-V} H_d \right\} \right.
$$
$$
+ \ \int \mathrm{d}^2\theta \ e^{-3kR\pi} \left\{ e^{(3-c_i-c_j)kR\pi} \frac{(y_u)_{ij}}{M_5} H_u U_i Q_j \ + \ e^{(3-c_k-c_l)kR\pi} \frac{(y_d)_{kl}}{M_5} H_d D_k Q_l \right\} \ + \ \text{h.c.}
$$
$$
\left. + \ \int \mathrm{d}^2\theta \ e^{-3kR\pi} \ e^{(3-c_m-c_n)kR\pi} \frac{(y_e)_{mn}}{M_5} H_d E_m L_n \ + \ \text{h.c.} \right] .
$$
(59)

Gaugino mass term:

$$
S_{IR} \supset \int \mathrm{d}^4x \left[\int \mathrm{d}^2\theta \ d_a \frac{X}{M_5} W^{a\alpha} W_\alpha^a \ + \ \text{h.c.} \right] .
$$
(60)

Matter soft SUSY breaking mass term:

$$
S_{IR} \supset \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta \ e^{-2kR\pi} \ e^{(3-c_i-c_j)kR\pi} \left\{ d_{Q1\,ij} \frac{X+X^\dagger}{M_5^2} Q_i^\dagger Q_j \ + \ d_{Q2\,ij} \frac{X^\dagger X}{M_5^3} Q_i^\dagger Q_j \right\} \right]
$$
$$
+ \ (Q \ \to \ U, \ D, \ L, \ E) .
$$
(61)

A-term-generating term:

$$
S_{IR} \supset \int \mathrm{d}^4x \left[\int \mathrm{d}^2\theta \ e^{-3kR\pi} \left\{ e^{(3-c_i-c_j)kR\pi} \frac{(a_u)_{ij}}{M_5^2} X H_u U_i Q_j \ + \ e^{(3-c_k-c_l)kR\pi} \frac{(a_d)_{kl}}{M_5^2} X H_d D_k Q_l \right. \right.
$$
$$
\left. \left. + \ e^{(3-c_m-c_n)kR\pi} \frac{(a_e)_{mn}}{M_5^2} X H_d E_m L_n \right\} \ + \ \text{h.c.} \right] .
$$
(62)

Messenger term:

$$
S_{IR} \supset \sum_I \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta \ e^{-2kR\pi} \left\{ \Xi_I^\dagger e^{-V} \Xi_I + \bar{\Xi}_I^\dagger e^V \bar{\Xi}_I \right\} \right.
$$
$$
\left. + \ \int \mathrm{d}^2\theta \ e^{-3kR\pi} \left\{ M_{mess\,I} \ \Xi_I \bar{\Xi}_I \ + \ \lambda_{mess\,I} \ X \ \Xi_I \bar{\Xi}_I \right\} \ + \ \text{h.c.} \right] ,
$$
(63)

where $M_{mess\,I}$ indicates the SUSY conserving mass for the messenger pair $\Xi_I, \bar{\Xi}_I$. Note that we did not necessarily assume the existence of messengers. In that case, only gaugino mediation gives rise to flavor-conserving soft masses, as is realized in the model [13].

In addition, the terms for the Higgs superfields exist on the IR brane. We simply assume that μ-term and $B\mu$-term are somehow derived at TeV scale.

We normalize X, H_u, H_d, Q_i, U_i, D_i, L_i, E_i, Ξ_I, $\bar{\Xi}_I$ to make their kinetic terms in the 4D effective theory canonical. This is done by the following rescaling:

$$X \rightarrow \tilde{X} = e^{-kR\pi}X, \quad H_u \rightarrow \tilde{H}_u = e^{-kR\pi}H_u, \quad H_d \rightarrow \tilde{H}_d = e^{-kR\pi}H_d,$$

$$\phi_i \rightarrow \tilde{\phi}_i = \sqrt{2\frac{e^{(1-2c_i)kR\pi} - 1}{(1-2c_i)k}}\, \phi_i,$$

$$\Xi_I \rightarrow \tilde{\Xi}_I = e^{-kR\pi}\Xi_I, \quad \bar{\Xi}_I \rightarrow \tilde{\bar{\Xi}}_I = e^{-kR\pi}\bar{\Xi}_I. \tag{64}$$

Then the MSSM term, the gaugino mass term, the matter soft SUSY breaking mass term, the A-term-generating term and the messenger term become as follows:

$$
\begin{aligned}
S_{IR} \supset & \int \mathrm{d}^4x \Bigg[\int \mathrm{d}^4\theta \left\{ \tilde{H}_u^\dagger e^{-V} \tilde{H}_u + \tilde{H}_d^\dagger e^{-V} \tilde{H}_d \right\} \\
& + \int \mathrm{d}^2\theta \left\{ \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} (y_u)_{ij}\, \tilde{H}_u \tilde{U}_i \tilde{Q}_j \right. \\
& + \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} (y_d)_{kl}\, \tilde{H}_d \tilde{D}_k \tilde{Q}_l \\
& + \left. \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} (y_e)_{mn}\, \tilde{H}_d \tilde{E}_m \tilde{L}_n \right\} + \text{h.c.} \Bigg].
\end{aligned} \tag{65}
$$

The gaugino mass term will be

$$S_{IR} \supset \int \mathrm{d}^4x \left[\int \mathrm{d}^2\theta\, d_a \frac{\tilde{X}}{M_5 e^{-kR\pi}} W^{a\alpha} W_\alpha^a + \text{h.c.} \right]. \tag{66}$$

The matter soft mass term will be

$$
\begin{aligned}
S_{IR} \supset & \int \mathrm{d}^4x \Bigg[\int \mathrm{d}^4\theta \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} \times \\
& \left\{ d_{Q1\,ij} \frac{\tilde{X}+\tilde{X}^\dagger}{M_5 e^{-kR\pi}} \tilde{Q}_i^\dagger \tilde{Q}_j + d_{Q2\,ij} \frac{\tilde{X}^\dagger \tilde{X}}{M_5^2 e^{-2kR\pi}} \tilde{Q}_i^\dagger \tilde{Q}_j \right\} \Bigg] \\
& + (\tilde{Q} \rightarrow \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}).
\end{aligned} \tag{67}
$$

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The A-term-generating term will be

$$
\begin{aligned}
S_{IR} \;\supset\; \int \mathrm{d}^4 x \;\Bigg[\int \mathrm{d}^2\theta \;\Bigg\{ & \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}}\sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}}\frac{k}{M_5}\frac{(a_u)_{ij}}{M_5 e^{-kR\pi}}\;\tilde{X}\tilde{H}_u\tilde{U}_i\tilde{Q}_j \\
+ & \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}}\sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}}\frac{k}{M_5}\frac{(a_d)_{kl}}{M_5 e^{-kR\pi}}\;\tilde{X}\tilde{H}_d\tilde{D}_k\tilde{Q}_l \\
+ & \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}}\sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}}\frac{k}{M_5}\frac{(a_e)_{mn}}{M_5 e^{-kR\pi}}\;\tilde{X}\tilde{H}_d\tilde{E}_m\tilde{L}_n \Bigg\} \;+\; \text{h.c.} \Bigg] \;.
\end{aligned}
$$
$$(68)$$

The messenger term will be

$$
\begin{aligned}
S_{IR} \;\supset\; \sum_I \int \mathrm{d}^4 x \;\Bigg[& \int \mathrm{d}^4\theta \;\{\; \tilde{\Xi}_I^\dagger e^{-V}\tilde{\Xi}_I + \tilde{\bar{\Xi}}_I^\dagger e^{V}\tilde{\bar{\Xi}}_I \;\} \\
+ & \int \mathrm{d}^2\theta \;\{\; M_{mess\,I}e^{-kR\pi}\,\tilde{\Xi}_I\tilde{\bar{\Xi}}_I \;+\; \lambda_{mess\,I}\tilde{X}\,\tilde{\Xi}_I\tilde{\bar{\Xi}}_I \;\} \;+\; \text{h.c.} \Bigg] \;.
\end{aligned}
$$
$$(69)$$

We introduce light neutrino masses by writing an IR-scale-suppressed higher dimensional operators or by adopting the seesaw mechanism [9]. In either case, we have the following term for light neutrino masses:

$$
\begin{aligned}
S_{IR} \;\supset\; & \int \mathrm{d}^4 x \int \mathrm{d}^2\theta \; e^{-3kR\pi}\, e^{(3-c_p-c_q)kR\pi}\,(Y_\nu)_{pq}\frac{L_p H_u L_q H_u}{M_{seesaw}} \;+\; \text{h.c.} \\
=\; & \int \mathrm{d}^4 x \int \mathrm{d}^2\theta \;\sqrt{\frac{1-2c_p}{2\{1-e^{-(1-2c_p)kR\pi}\}}}\sqrt{\frac{1-2c_q}{2\{1-e^{-(1-2c_q)kR\pi}\}}}\,(Y_\nu)_{pq}\,\frac{\tilde{L}_p\tilde{H}_u\tilde{L}_q\tilde{H}_u}{M_{seesaw}e^{-kR\pi}} \;+\; \text{h.c.} \,,
\end{aligned}
$$
$$(70)$$

where M_{seesaw} indicates the mass scale relevant to the light neutrino mass. To maintain the generality of the model, we hereafter consider cases with SU(2) singlet neutrinos. Their Majorana masses in the *4D effective* theory are assumed to be around a common scale, denoted by M_{Maj}, which is lower than $M_{mess\,I}e^{-kR\pi}$ or $M_5 e^{-kR\pi}$. The results of this paper can be extended to cases without singlet neutrinos by dropping terms containing M_{Maj}.

Now the MSSM Yukawa coupling constants are expressed as :

$$
\begin{aligned}
(Y_u)_{ij} &= \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}}\sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}}\frac{k}{M_5}(y_u)_{ij}\,, \\
(Y_d)_{kl} &= \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}}\sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}}\frac{k}{M_5}(y_d)_{kl}\,, \\
(Y_e)_{mn} &= \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}}\sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}}\frac{k}{M_5}(y_e)_{mn}\,,
\end{aligned}
$$
$$(71)$$

and the neutrino mass matrix m_ν is given by :

$$(m_\nu)_{pq} = \sqrt{\frac{1 - 2c_p}{2\{1 - e^{-(1-2c_p)kR\pi}\}}} \sqrt{\frac{1 - 2c_q}{2\{1 - e^{-(1-2c_q)kR\pi}\}}} \ (Y_\nu)_{pq} \ \frac{v_u^2}{M_{seesaw}e^{-kR\pi}} \ . \qquad (72)$$

The geometrical factor $\sqrt{(1-2c) / (2\{1 - e^{-(1-2c)kR\pi}\})}$ generates hierarchical couplings without fundamental hierarchy [2]; for $c < 1/2$, it is approximated by $\sqrt{1/2 - c}$ and is $O(1)$, whereas for $c > 1/2$, it is approximated by $\sqrt{c - 1/2} \ e^{-(c-1/2)kR\pi}$ and is exponentially suppressed. Note that this factor cannot be larger than $O(1)$. We assume that the components of 5D coupling matrices y_u, y_d, y_e are all $O(1)$ and that the hierarchy of the Yukawa coupling constants stems solely from the following terms:

$$\sqrt{\frac{1 - 2c_i}{2\{1 - e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1 - 2c_j}{2\{1 - e^{-(1-2c_j)kR\pi}\}}} \ .$$

This is how the bulk matter RS model explains the Yukawa coupling hierarchy.

We further assume that the components of Y_ν are $O(1)$. Note that Y_ν arises by integrating out singlet Majorana neutrinos. If the components of 5D neutrino Dirac coupling are $O(1)$, it is possible to take the value of M_{seesaw} such that $(Y_\nu)_{ij} \sim O(1)$ holds, regardless of the 5D disposition of singlet neutrino fields and the flavor structure of the Majorana mass term. Hence this is a natural assumption in the bulk matter RS model, in which all 5D couplings are considered $O(1)$. With this assumption, the hierarchy of the light neutrino mass matrix (72) arises only from the geometrical factors of SU(2) doublet lepton fields.

We hereafter use the following notations:

$$\alpha_i \equiv \sqrt{\frac{1 - 2c_{qi}}{2\{1 - e^{-(1-2c_{qi})kR\pi}\}}} \qquad \text{with} \ \ i = 1, 2, 3 \qquad (73)$$

for SU(2) doublet quark superfields with flavor index i, and β_i, γ_i, δ_i, ϵ_i for SU(2) singlet up-type quark, singlet down-type quark, doublet lepton and singlet charged lepton, respectively. Then the hierarchical structures of the up-type quark Yukawa matrix Y_u, the down-type Yukawa matrix Y_d and the charged lepton Yukawa matrix Y_e (in the basis of diagonal 5D bulk masses) are expressed as:

$$(Y_u)_{ij} \sim \beta_i \alpha_j \ , \quad (Y_d)_{ij} \sim \gamma_i \alpha_j \ , \quad (Y_e)_{ij} \sim \epsilon_i \delta_j \ , \qquad (74)$$

and that of the neutrino mass matrix m_ν is expressed as:

$$(m_\nu)_{ij} \sim \delta_i \delta_j \ \frac{v_u^2}{M_{seesaw}e^{-kR\pi}} \ . \qquad (75)$$

4.2 Determination of the Geometrical Factors

The magnitudes of the geometrical factors, $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$, can be almost determined by the data on SM fermion masses, CKM matrix and neutrino oscillations. The sole exception is the absolute scale of δ_is, of which we only know the relative scales between different flavors. In this subsection, we will estimate these factors. The values that correspond to the model must be given at the KK scale, $M_5 e^{-kR\pi}$, where the 5D theory is connected to the 4D effective theory. However, as is seen from [10], RG evolutions change the Yukawa coupling constants by at most 2 and the CKM matrix components by at most 1.2 through evolving from $\sim 10^{15}$ GeV to the electroweak scale. Also the neutrino mass matrix is affected only by $O(1)$ through RG evolutions [11]. Therefore we may estimate the magnitudes of $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$ from the data at low energies.

We first derive the model's predictions on the eigenvalues of the Yukawa coupling matrices and the components of CKM matrix. Let us diagonalize the Yukawa matrices:

$$
\begin{aligned}
V_u Y_u U_u^\dagger &= \text{diag} , \\
V_d Y_d U_d^\dagger &= \text{diag} , \\
V_e Y_e U_e^\dagger &= \text{diag} .
\end{aligned}
$$

For successful diagonalization of the hierarchical Yukawa matrices, the unitary matrices, $U_u, U_d, V_u, V_d, U_e, V_e$, need to have the following structure:

$$
U_u \sim U_d \sim \begin{pmatrix} 1 & 0 & 0 \\ \alpha_1/\alpha_2 & 1 & 0 \\ \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 \end{pmatrix} , \quad V_u \sim (\alpha \to \beta) , \quad V_d \sim (\alpha \to \gamma) ,
$$

$$
U_e \sim (\alpha \to \delta) , \quad V_e \sim (\alpha \to \epsilon) \tag{76}
$$

which leads to

$$
\begin{aligned}
V_u Y_u U_u^\dagger &\sim \text{diag} (\beta_1 \alpha_1, \ \beta_2 \alpha_2, \ \beta_3 \alpha_3) , \\
V_d Y_d U_d^\dagger &\sim \text{diag} (\gamma_1 \alpha_1, \ \gamma_2 \alpha_2, \ \gamma_3 \alpha_3) , \\
V_e Y_e U_e^\dagger &\sim \text{diag} (\epsilon_1 \delta_1, \ \epsilon_2 \delta_2, \ \epsilon_3 \delta_3) .
\end{aligned}
\tag{77}
$$

The hierarchical structure of CKM matrix U_{CKM} is given by

$$
U_{CKM} = U_u U_d^\dagger \sim \begin{pmatrix} 1 & \alpha_1/\alpha_2 & \alpha_1/\alpha_3 \\ \alpha_1/\alpha_2 & 1 & \alpha_2/\alpha_3 \\ \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 \end{pmatrix} .
\tag{78}
$$

We next list the experimental data on CKM matrix and the neutrino mass matrix. The absolute values of the CKM matrix components, $|U_{CKM}|$, at the electroweak scale has been

measured to be [12]

$$|U_{CKM}[M_W]|$$
$$= \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415 + 0.0010 - 0.0011 \\ 0.00874 + 0.00026 - 0.00037 & 0.0407 \pm 0.0010 & 0.999133 + 0.000044 - 0.000043 \end{pmatrix} .$$

We approximate this matrix by the following formula:

$$|U_{CKM}| \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \text{with} \quad \lambda = 0.22 . \tag{79}$$

To discuss the neutrino mass matrix, we adopt the tri-bi-maximal mixing matrix:

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

and the following data on neutrino mass squared differences [12]:

$$\Delta m_{21}^2 = 7.59 \pm 0.20 \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{32}^2| = 2.43 \pm 0.13 \times 10^{-3} \text{ eV}^2.$$

We assume that the mass of the lightest neutrino is negligible. Then the neutrino mass matrix, U_{MNS} diag $(m_{\nu1},\ m_{\nu2},\ m_{\nu3})\ U_{MNS}^\dagger$, is given by

$$U_{MNS} \text{ diag } (m_{\nu1},\ m_{\nu2},\ m_{\nu3})\ U_{MNS}^\dagger = \begin{pmatrix} 0.29 & 0.29 & 0.29 \\ 0.29 & 2.8 & -2.2 \\ 0.29 & -2.2 & 2.8 \end{pmatrix} \times 10^{-11} \text{ GeV}$$

for normal hierarchy ,

$$\tag{80}$$

$$U_{MNS} \text{ diag } (m_{\nu1},\ m_{\nu2},\ m_{\nu3})\ U_{MNS}^\dagger = \begin{pmatrix} 4.9 & 0.026 & 0.026 \\ 0.026 & 2.5 & 2.5 \\ 0.026 & 2.5 & 2.5 \end{pmatrix} \times 10^{-11} \text{ GeV}$$

for inverted hierarchy .

$$\tag{81}$$

We now compare the predictions of the model with the data and estimate the magnitudes of $\alpha_i,\ \beta_i,\ \gamma_i,\ \delta_i,\ \epsilon_i$. For Yukawa eigenvalues, we simply have

$$\beta_1\alpha_1 \sim m_u/v\sin\beta , \quad \beta_2\alpha_2 \sim m_c/v\sin\beta , \quad \beta_3\alpha_3 \sim m_t/v\sin\beta , \tag{82}$$

$$\gamma_1\alpha_1 \sim m_d/v\cos\beta , \quad \gamma_2\alpha_2 \sim m_s/v\cos\beta , \quad \gamma_3\alpha_3 \sim m_b/v\cos\beta , \tag{83}$$

$$\epsilon_1\delta_1 \sim m_e/v\cos\beta , \quad \epsilon_2\delta_2 \sim m_\mu/v\cos\beta , \quad \epsilon_3\delta_3 \sim m_\tau/v\cos\beta , \tag{84}$$

25

where the mass values can be approximated by their pole values. Since the top Yukawa coupling is ~ 1, we have $\alpha_3 \beta_3 \sim 1$, which leads to

$$\alpha_3 \sim 1 , \quad \beta_3 \sim 1 . \tag{85}$$

Comparing (78) with (79), we find that putting

$$\alpha_1 \sim \lambda^3 , \quad \alpha_2 \sim \lambda^2 \tag{86}$$

works. We then have

$$\beta_1 \sim \lambda^{-3} \, m_u/v \sin \beta , \quad \beta_2 \sim \lambda^{-2} \, m_c/v \sin \beta , \tag{87}$$

$$\gamma_1 \sim \lambda^{-3} \, m_d/v \cos \beta , \quad \gamma_2 \sim \lambda^{-2} \, m_s/v \cos \beta , \quad \gamma_3 \sim m_b/v \cos \beta . \tag{88}$$

Next compare the matrix (75) with the neutrino mass matrix. For normal hierarchy case, it is possible to reproduce the hierachical structure of the neutrino mass matrix by assuming the relation:

$$3\delta_1 \sim \delta_2 \sim \delta_3 , \tag{89}$$

and the ratio up to 3 among the components of 5D coupling Y_ν. In contrast, for inverted hierarchy case, ~ 200 ratio is required among the 5D coupling components no matter how we choose δ_is, which makes it difficult to naturally explain the hierarchy of the neutrino mass matrix. The situation gets worse if we consider non-negiligible mass of the lightest neutrino. In conclusion, the bulk matter RS model favors the normal hierarchy of neutrino masses and implies the relation (89) for δ_is. We estimate ϵ_is assuming the relation (89); we obtain

$$\epsilon_1 \sim 3 \, \delta_3^{-1} \, m_e/v \cos \beta , \quad \epsilon_2 \sim \delta_3^{-1} \, m_\mu/v \cos \beta , \quad \epsilon_3 \sim \delta_3^{-1} \, m_\tau/v \cos \beta . \tag{90}$$

The magnitude of δ_3 is a free parameter because we do not specify the scale of M_{seesaw}.

5   Chapter V : Flavor-Violating Soft SUSY Breaking Terms

In this model, flavor-conserving soft SUSY breaking terms arise from RG contributions of gaugino masses below the KK scale (gaugino mediation) and gauge interactions with messenger superfields (gauge mediation). On the other hand, flavor-violating terms arise from contact interactions with SUSY breaking sector on the IR brane (gravity mediation) and RG contributions of the Yukawa couplings. Of particular importance are the gravity mediation contributions, which have a flavor structure unique to the bulk matter RS model. In this section, we separately estimate the gravity mediation contributions and the Yukawa coupling contributions to flavor-violaing soft SUSY breaking terms.

Remember that there are two scales of soft SUSY breaking terms, namely gravity mediation scale and gauge mediation scale. Assuming that the messenger masses and couplings are around the same orders, we define the following two mass parameters:

$$M_{grav} \equiv \frac{|<F_{\tilde{X}}>|}{M_5 e^{-kR\pi}} , \tag{91}$$

$$M_{gauge} \equiv \frac{1}{16\pi^2} \frac{\lambda_{mess} |<F_{\tilde{X}}>|}{M_{mess} e^{-kR\pi}} , \tag{92}$$

where M_{mess} represents the typical scale of the SUSY conserving messenger masses $M_{mess\,I}$, and λ_{mess} the typical scale of the messenger couplings to SUSY breaking sector $\lambda_{mess\,I}$. Note that Yukawa RG contributions to flavor-violating terms depend on both M_{grav} and M_{gauge}, whereas gravity mediation contributions do only on M_{grav}.

5.1   Flavor-Violating Gravity Mediation Contributions

Let us estimate the magnitudes of gravity mediation contributions in the bulk matter RS model.

From (67), we obtain the following formulae for gravity-mediation-origined soft SUSY breaking matter mass terms at the scale $M_5 e^{-kR\pi}$:

For SU(2) doublet squarks, we have

$$(m_Q^2)_{ij} = (-d_{Q2ij} + d_{Q1ij}^2) \frac{k}{M_5} \alpha_i \alpha_j M_{grav}^2 . \tag{93}$$

By substituting (U, β), (D, γ), (L, δ), (E, ϵ) into (Q, α) in the above formula, we obtain similar expressions for SU(2) singlet up-type squarks, down-type squarks, SU(2) doublet sleptons and singlet charged sleptons.

Assuming that the 5D couplings d_{*2ij}, d_{*1ij} are $O(1)$, we obtain the following estimates on the magnitudes at the scale $M_5 e^{-kR\pi}$:

$$(m_Q^2)_{ij} \;\sim\; \alpha_i \alpha_j \; M_{grav}^2 \;. \tag{94}$$

and similar formulae with (U, β), (D, γ), (L, δ), (E, ϵ) replacing (Q, α) in the above formula.

Next we estimate the magnitudes of the A-terms that are induced by gravity mediation. The terms (68) directly contribute to the A-terms. Furthermore, since the Higgs superfields can couple to SUSY breaking sector in the following way:

$$\int \mathrm{d}^4\theta \left[d_{uA} \frac{\tilde{X}}{M_5 e^{-kR\pi}} \tilde{H}_u^\dagger \tilde{H}_u \;+\; d_{dA} \frac{\tilde{X}}{M_5 e^{-kR\pi}} \tilde{H}_d^\dagger \tilde{H}_d \;+\; \mathrm{h.c.} \right] \;,$$

the A-terms also arise from the Higgs F-terms via (65). Hence gravity-mediation-origined A-terms at the scale $M_5 e^{-kR\pi}$ are given by

$$\begin{aligned}
A_{uij} &= -d_{uA} \, (y_u)_{ij} \, \beta_i \alpha_j \, \frac{k}{M_5} \, M_{grav} \;+\; (a_u)_{ij} \, \beta_i \alpha_j \, \frac{k}{M_5} \, M_{grav} \;, \\
&= -d_{uA} \, (Y_u)_{ij} \, M_{grav} \;+\; (a_u)_{ij} \, \beta_i \alpha_j \, \frac{k}{M_5} \, M_{grav} \;, \tag{95}
\end{aligned}$$

$$A_{dij} = -d_{dA} \, (Y_d)_{ij} \, M_{grav} \;+\; (a_d)_{ij} \, \gamma_i \alpha_j \, \frac{k}{M_5} \, M_{grav} \;, \tag{96}$$

$$A_{eij} = -d_{dA} \, (Y_e)_{ij} \, M_{grav} \;+\; (a_e)_{ij} \, \epsilon_i \delta_j \, \frac{k}{M_5} \, M_{grav} \;. \tag{97}$$

Assuming that the components of 5D couplings d_{*A}, $(a_*)_{ij}$ are $O(1)$, we obtain the following estimates on the magnitudes at the scale $M_5 e^{-kR\pi}$:

$$A_{uij} \;\sim\; (Y_u)_{ij} \, M_{grav} \;+\; \beta_i \alpha_j \, M_{grav} \;, \tag{98}$$

$$A_{dij} \;\sim\; (Y_d)_{ij} \, M_{grav} \;+\; \gamma_i \alpha_j \, M_{grav} \;, \tag{99}$$

$$A_{eij} \;\sim\; (Y_e)_{ij} \, M_{grav} \;+\; \epsilon_i \delta_j \, M_{grav} \;. \tag{100}$$

5.2   RG Contributions

Let us estimate the magnitudes of the flavor-violating soft SUSY breaking terms that arise from the RG equations involving the Yukawa couplings. In doing so, we take the specific flavor basis where Y_u or Y_d and Y_e are diagonal.

We first study how Y_u, Y_d, Y_e-diagonal bases change through RG evolutions. Define the following unitary matrices U_*:

$$U_U Y_u U_{Qu} = (diag.) \,,$$
$$U_D Y_d U_{Qd} = (diag.) \,,$$
$$U_E Y_e U_L = (diag.) \,.$$

Note that U_*s depend on the renormalization scale because the Yukawa matrices receive RG corrections. We will calculate how U_*s vary through RG evolutions. We have the RG equation below:

$$
\begin{aligned}
\mu \frac{\mathrm{d}}{\mathrm{d}\mu}(U_U Y_u U_{Qu}) &= (\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_U) U_U^\dagger (U_U Y_u U_{Qu}) + U_U (\mu \frac{\mathrm{d}}{\mathrm{d}\mu} Y_u) U_{Qu} + (U_U Y_u U_{Qu}) U_{Qu}^\dagger (\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_{Qu}) \\
&= (\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_U) U_U^\dagger (U_U Y_u U_{Qu}) \\
&+ \frac{1}{16\pi^2} U_U \{ Y_u Y_d^\dagger Y_d + 3 Y_u Y_u^\dagger Y_u + 3\mathrm{tr}[Y_u^\dagger Y_u] Y_u + \mathrm{tr}[Y_D^\dagger Y_D] Y_u \\
&\quad - (\frac{13}{15} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2) Y_u \} U_{Qu} \\
&+ (U_U Y_u U_{Qu}) U_{Qu}^\dagger (\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_{Qu}) \,,
\end{aligned}
\tag{101}
$$

where Y_D is neutrino Dirac coupling which appears if there exist singlet neutrinos lighter than the KK scale, from which the RG equations are calculated. We hereafter adopt GUT normalization for g_1. From (48), we see that, to keep $U_U Y_u U_{Qu}$ diagonal through RG evolutions, it is sufficient to have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_U = 0 \,, \tag{102}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_{Qu} = -\frac{1}{16\pi^2} (\mathrm{off-diagonal\ components\ of\ } Y_d^\dagger Y_d) \, U_{Qu} \,. \tag{103}$$

In a similar manner, we obtain the following sufficient conditions for other U_*s:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_D = 0 \,, \tag{104}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_{Qd} = -\frac{1}{16\pi^2} (\mathrm{off-diagonal\ components\ of\ } Y_u^\dagger Y_u) \, U_{Qd} \,, \tag{105}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_E = 0 \,, \tag{106}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} U_L = -\frac{1}{16\pi^2} (\mathrm{off-diagonal\ components\ of\ } Y_D^\dagger Y_D) \, U_L \,. \tag{107}$$

29

Now that we know how Y_u, Y_d, Y_e-diagonal bases change through RG evolutions, we estimate the RG contributions to the A-terms in these bases. Below is the list of the MSSM RG equations for the A-terms:

$$
\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_U A_u U_{Qu}) =\ & 3U_U A_u Y_u^\dagger Y_u U_{Qu} + 3U_U Y_u Y_u^\dagger A_u U_{Qu} \\
& + (U_U A_u U_{Qu})(\text{diagonal part of } U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu}) + 2U_U Y_u Y_d^\dagger A_d U_{Qu} \\
& + 2(\ 3\mathrm{tr}[Y_u^\dagger A_u] - \frac{13}{15}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2} - \frac{16}{3}g_3^2 M_{1/2}^{a=3}\ )(U_U Y_u U_{Qu}) \\
& + (\ 3\mathrm{tr}[Y_u^\dagger Y_u] - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\ )(U_U A_u U_{Qu}) \\
& + \mathrm{tr}[Y_D^\dagger Y_D](U_U A_u U_{Qu}) + \mathrm{tr}[Y_D^\dagger A_D](U_U Y_u U_{Qu})\ ,
\end{aligned}
\tag{108}
$$

$$
\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_D A_d U_{Qd}) =\ & 3U_D A_d Y_d^\dagger Y_d U_{Qd} + 3U_D Y_d Y_d^\dagger A_d U_{Qd} \\
& + (U_D A_d U_{Qd})(\text{diagonal part of } U_{Qd}^\dagger Y_u^\dagger Y_u U_{Qd}) + 2U_D Y_d Y_u^\dagger A_u U_{Qd} \\
& + 2(\ 3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{7}{15}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2} - \frac{16}{3}g_3^2 M_{1/2}^{a=3}\ )(U_D Y_d U_{Qd}) \\
& + (\ 3\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\ )(U_D A_d U_{Qd})\ ,
\end{aligned}
\tag{109}
$$

$$
\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_E A_e U_L) =\ & 3U_E A_e Y_e^\dagger Y_e U_L + 3U_E Y_e Y_e^\dagger A_e U_L \\
& + 2(\ 3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{9}{5}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2}\ )(U_E Y_e U_L) \\
& + (\ 3\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{9}{5}g_1^2 - 3g_2^2\ )(U_E A_e U_L) \\
& + (U_E A_e U_L)(\text{diagonal part of } U_L^\dagger Y_D^\dagger Y_D U_L) + 2U_E Y_e Y_D^\dagger A_D U_L\ ,
\end{aligned}
\tag{110}
$$

where Y_D and A_D respectively indicate neutrino Dirac coupling and its corresponding A-term.

Note that the magnitudes of the components of the Yukawa couplings in each basis are given by (δ_{ij} is the ordinary Kronecker's delta):

$$
\begin{aligned}
(U_U Y_u U_{Qu})_{ij} &\sim \beta_i \alpha_i\, \delta_{ij}\ , & (U_U Y_d U_{Qu})_{ij} &\sim \gamma_i \alpha_j\ , \\
(U_D Y_u U_{Qd})_{ij} &\sim \beta_i \alpha_j\ , & (U_D Y_d U_{Qd})_{ij} &\sim \gamma_i \alpha_i\, \delta_{ij}\ , \\
(U_E Y_e U_L)_{ij} &\sim \epsilon_i \delta_i\, \delta_{ij}\ , & (U_E Y_D U_L)_{ij} &\sim \zeta_i \delta_j\ ,
\end{aligned}
$$

where ζ_is indicate the geometrical factors for singlet neutrinos and satisfy $\zeta_i \leq 1$. Note also that the A-terms receive RG corrections which are proportional to the corresponding Yukawa couplings and to the gaugino masses. We write these terms by M_u, M_d, M_e, M_D respectively for A_u, A_d, A_e, A_D. They depend on both M_{grav} and M_{gauge}.

With these ingredients, we estimate the RG contributions to those parts of A-terms which are not proportional to the corresponding Yukawa couplings, or equivalently the off-diagonal

components of $(U_U A_u U_{Qu})$, $(U_D A_d U_{Qd})$, $(U_E A_e U_L)$. In the right hand sides of (108-110), the second lines determine the magnitudes of the RG contributions. We thus obtain the following estimates $(i \neq j)$:

$$\Delta(U_U A_u U_{Qu})_{ij} \sim 2 \beta_i (\alpha_i)^2 (\gamma_3)^2 \alpha_j \times \frac{1}{16\pi^2} \int d(\ln\mu) \, M_d \,, \tag{111}$$

$$\Delta(U_D A_d U_{Qd})_{ij} \sim 2 \gamma_i (\alpha_i)^2 (\beta_3)^2 \alpha_j \times \frac{1}{16\pi^2} \int d(\ln\mu) \, M_u \,, \tag{112}$$

$$\Delta(U_E A_e U_L)_{ij} \sim 2 \epsilon_i (\delta_i)^2 (\zeta_3)^2 \delta_j \times \frac{1}{16\pi^2} \int d(\ln\mu) \, M_D \,. \tag{113}$$

Since M_u, M_d, M_D depend on M_{grav} and M_{gauge}, so do the magnitudes of the RG contributions above.

Let us estimate the RG contributions to soft SUSY breaking matter mass terms. Below are those parts of the MSSM RG equations that give rise to flavor-violating soft SUSY breaking

mass terms:

$$16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_{Qu}^\dagger m_Q^2 U_{Qu}) \supset U_{Qu}^\dagger Y_u^\dagger Y_u m_Q^2 U_{Qu} + U_{Qu}^\dagger m_Q^2 Y_u^\dagger Y_u U_{Qu}$$

$$+ \quad 2U_{Qu}^\dagger Y_u^\dagger m_U^2 Y_u U_{Qu} + 2(U_{Qu}^\dagger Y_u^\dagger Y_u U_{Qu})m_{H_u}^2$$

$$+ \quad (\text{diagonal parts of } U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})(U_{Qu}^\dagger m_Q^2 U_{Qu})$$

$$+ \quad (U_{Qu}^\dagger m_Q^2 U_{Qu})(\text{diagonal parts of } U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})$$

$$+ \quad 2U_{Qu}^\dagger Y_d^\dagger m_D^2 Y_d U_{Qu} + 2(U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})m_{H_d}^2$$

$$+ \quad 2U_{Qu}^\dagger A_u^\dagger A_u U_{Qu} + 2U_{Qu}^\dagger A_d^\dagger A_d U_{Qu} , \tag{114}$$

$$16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_U m_U^2 U_U^\dagger) \supset 2U_U Y_u Y_u^\dagger m_U^2 U_U^\dagger + 2U_U m_U^2 Y_u Y_u^\dagger U_U^\dagger$$

$$+ \quad 4U_U Y_u m_Q^2 Y_u^\dagger U_U^\dagger + 4(U_U Y_u Y_u^\dagger U_U^\dagger)m_{H_u}^2$$

$$+ \quad 4U_U A_u A_u^\dagger U_U^\dagger , \tag{115}$$

$$16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_D m_D^2 U_D^\dagger) \supset 2U_D Y_d Y_d^\dagger m_D^2 U_D^\dagger + 2U_D m_D^2 Y_d Y_d^\dagger U_D^\dagger$$

$$+ \quad 4U_D Y_d m_Q^2 Y_d^\dagger U_D^\dagger + 4(U_D Y_d Y_d^\dagger U_D^\dagger)m_{H_d}^2$$

$$+ \quad 4U_D A_d A_d^\dagger U_D^\dagger , \tag{116}$$

$$16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_L^\dagger m_L^2 U_L) \supset U_L^\dagger Y_e^\dagger Y_e m_L^2 U_L + U_L^\dagger m_L^2 Y_e^\dagger Y_e U_L$$

$$+ \quad 2U_L^\dagger Y_e^\dagger m_E^2 Y_e U_L + 2(U_L^\dagger Y_e^\dagger Y_e U_L)m_{H_d}^2$$

$$+ \quad 2U_L^\dagger A_e^\dagger A_e U_L$$

$$+ \quad (\text{diagonal parts of } U_L^\dagger Y_D^\dagger Y_D U_L)(U_L^\dagger m_L^2 U_L)$$

$$+ \quad (U_L^\dagger m_L^2 U_L)(\text{diagonal parts of } U_L^\dagger Y_D^\dagger Y_D U_L)$$

$$+ \quad 2U_L^\dagger Y_D^\dagger m_N^2 Y_D U_L + 2(U_L^\dagger Y_D^\dagger Y_D U_L)m_{H_u}^2 + 2U_L^\dagger A_D^\dagger A_D U_L , \tag{117}$$

$$16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_E m_E^2 U_E^\dagger) \supset 2U_E Y_e Y_e^\dagger m_E^2 U_E^\dagger + 2U_E m_E^2 Y_e Y_e^\dagger U_E^\dagger$$

$$+ \quad 4U_E Y_e m_L^2 Y_e^\dagger U_E^\dagger + 4(U_E Y_e Y_e^\dagger U_E^\dagger)m_{H_d}^2$$

$$+ \quad 4U_E A_e A_e^\dagger U_E^\dagger . \tag{118}$$

We first focus on the differences among the diagonal components of different flavors. From (114), the difference between the components $(U_{Qu}^\dagger m_Q^2 U_{Qu})_{ii}$ and $(U_{Qu}^\dagger m_Q^2 U_{Qu})_{jj}$ that arises through RG evolutions is given by $(i > j)$:

$$\Delta\{ (U_{Qu}^\dagger m_Q^2 U_{Qu})_{ii} - (U_{Qu}^\dagger m_Q^2 U_{Qu})_{jj} \} \sim 2(\alpha_i)^2(\beta_i)^2 \frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(m_Q^2 + m_U^2 + m_{H_u}^2 + M_u^2)$$

$$+ \quad 2(\alpha_i)^2(\gamma_3)^2 \frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(m_Q^2 + m_D^2 + m_{H_d}^2 + M_d^2) , \tag{119}$$

where we neglected the terms proportional to $(\alpha_j)^2$ because they are smaller than those proportional to $(\alpha_i)^2$. Similarly, the difference between $(U_L^\dagger m_L^2 U_L)_{ii}$ and $(U_L^\dagger m_L^2 U_L)_{jj}$ is given, from (117), by

$$
\begin{aligned}
\Delta\{ (U_L^\dagger m_L^2 U_L)_{ii} - (U_L^\dagger m_L^2 U_L)_{jj} \} \quad \sim \quad & 2\,(\delta_i)^2(\epsilon_i)^2\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_L^2 + m_E^2 + m_{H_d}^2 + M_e^2\,) \\
+ \quad & 2\,(\delta_i)^2(\zeta_3)^2\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_L^2 + m_N^2 + m_{H_u}^2 + M_D^2\,)\,.
\end{aligned}
$$
(120)

On the other hand, the differences among the diagonal components of SU(2) singlet soft mass terms follow different formulae. From (115, 116, 118), we have

$$
\Delta\{ (U_U^\dagger m_U^2 U_U)_{ii} - (U_U^\dagger m_U^2 U_U)_{jj} \} \quad \sim \quad 4\,(\beta_i)^2(\alpha_i)^2\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_U^2 + m_Q^2 + m_{H_u}^2 + M_u^2\,)\,,
$$
(121)

$$
\Delta\{ (U_D^\dagger m_D^2 U_D)_{ii} - (U_D^\dagger m_D^2 U_D)_{jj} \} \quad \sim \quad 4\,(\gamma_i)^2(\alpha_i)^2\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_D^2 + m_Q^2 + m_{H_d}^2 + M_d^2\,)\,,
$$
(122)

$$
\Delta\{ (U_E^\dagger m_E^2 U_E)_{ii} - (U_E^\dagger m_E^2 U_E)_{jj} \} \quad \sim \quad 4\,(\epsilon_i)^2(\delta_i)^2\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_E^2 + m_L^2 + m_{H_d}^2 + M_e^2\,)\,.
$$
(123)

We next study the off-diagonal components. In (114), terms $2U_{Qu}^\dagger Y_d^\dagger m_D^2 Y_d U_{Qu}$, $2(U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})m_{H_d}^2$, $2U_{Qu}^\dagger A_d^\dagger A_d U_{Qu}$ generate off-diagonal components, whose magnitudes are given by ($i \neq j$)

$$
\Delta(U_{Qu}^\dagger m_Q^2 U_{Qu})_{ij} \quad \sim \quad 2\,\alpha_i(\gamma_3)^2\alpha_j\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_D^2 + m_{H_d}^2 + M_d^2\,)\,.
$$
(124)

Similarly, we have

$$
\Delta(U_L^\dagger m_L^2 U_L)_{ij} \quad \sim \quad 2\,\delta_i(\zeta_3)^2\delta_j\,\frac{1}{16\pi^2}\int \mathrm{d}(\ln\mu)\,(\,m_N^2 + m_{H_u}^2 + M_D^2\,)\,.
$$
(125)

On the other hand, RG contributions to the off-diagonal components of $(U_U^\dagger m_U^2 U_U)$ arise from those of $(U_{Q_u}^\dagger m_Q^2 U_{Q_u})$ and $(U_U A_u U_{Qu})$ via terms $4U_U Y_u m_Q^2 Y_u^\dagger U_U^\dagger$, $4U_U A_u A_u^\dagger U_U^\dagger$ in (115). From (111), (124), (115), we obtain the following estimate on the magnitudes of the off-diagonal components ($i \neq j$):

$$
\begin{aligned}
\Delta(U_U^\dagger m_U^2 U_U)_{ij} \quad \sim \quad & 8\,\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j\,\left(\frac{1}{16\pi^2}\right)^2\int \mathrm{d}(\ln\mu)\int \mathrm{d}(\ln\mu')\,(\,m_D^2 + m_{H_d}^2 + M_d^2\,) \\
+ \quad & 16\,\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j\,\left(\frac{1}{16\pi^2}\right)^2\int \mathrm{d}(\ln\mu)\,\left(M_u\int \mathrm{d}(\ln\mu')M_d\right)\,.
\end{aligned}
$$
(126)

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In the same way, we obtain the following estimates on the RG contributions to the off-diagonal components of $(U_D^\dagger m_D^2 U_D)$, $(U_E^\dagger m_E^2 U_E)$:

$$
\begin{aligned}
\Delta(U_D^\dagger m_D^2 U_D)_{ij} \sim\ & 8\,\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_j)^2\gamma_j \left(\frac{1}{16\pi^2}\right)^2 \int d(\ln\mu) \int d(\ln\mu')\ (\,m_U^2 + m_{H_u}^2 + M_u^2\,) \\
+\ & 16\,\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_j)^2\gamma_j \left(\frac{1}{16\pi^2}\right)^2 \int d(\ln\mu) \left(M_d \int d(\ln\mu') M_u\right)\ , \quad (127)
\end{aligned}
$$

$$
\begin{aligned}
\Delta(U_E^\dagger m_E^2 U_E)_{ij} \sim\ & 8\,\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_j)^2\epsilon_j \left(\frac{1}{16\pi^2}\right)^2 \int d(\ln\mu) \int d(\ln\mu')\ (\,m_N^2 + m_{H_u}^2 + M_D^2\,) \\
+\ & 16\,\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_j)^2\epsilon_j \left(\frac{1}{16\pi^2}\right)^2 \int d(\ln\mu) \left(M_e \int d(\ln\mu') M_D\right)\ . \quad (128)
\end{aligned}
$$

Finally, we briefly discuss whether this model gives a realistic mass spectrum consistent with the bounds on flavor-violating processes.

For cases without messenger fields, i.e. when gaugino mediation is the only source of soft SUSY breaking masses, the paper [13] showed that there exist mass spectra below TeV scale that satisfy all experimental bounds. However, ~ 0.1 suppression on the term $(A_e)_{21}$ relative to its natural scale ($\sim \epsilon_2\delta_1 M_{grav}$) is required to evade the bound on $\mu \to e\gamma$ branching ratio. Other soft SUSY breaking terms are less constrained.

If there are 1 to several messenger pairs, the resultant mass spectra are more likely to evade the experimental bounds because gauge mediation contributes solely to flavor-universal soft SUSY breaking terms.

6   Chapter VI : Signatures of the Model

In the previous section, we saw that the bulk matter RS model combined with 5D MSSM predicts a unique flavor structure of gravity mediation contributions to flavor-violating soft terms. We here discuss the ways to observe this structure through future collider experiments.

Focus on the flavor compositions of SUSY matter particle mass eigenstates. Due to flavor-violating soft mass terms $(m_*^2)_{ij}$ and flavor-violating A-terms $(A_*)_{ij}$, SUSY particles of different flavors mix in one mass eigenstate, whose flavor composition reflects the relative size of the flavor-violating terms. Since sparticles of different flavors decay into different SM particles (plus the lightest or the next-to-lightest SUSY particle), one can measure the flavor composition by detecting the decay products of that mass eigenstate, counting the event numbers of different decay modes and calculating their ratios. These ratios are connected to the structure of flavor-violating soft SUSY breaking terms and make it possible to experimentally test the predictions of the bulk matter RS model.

Below we formulate the relation between flavor-violating terms and sparticle flavor mixings. In the first subsection, we interpret the predictions of the bulk matter RS model in terms of the flavor mixing ratios of sparticle mass eigenstates. In the next subsection, we look into the predictions of models other than the bulk matter RS model and discuss whether or not it is possible to distinguish different models.

Consider the situation where sparticle "a" with soft SUSY breaking mass m_a^2 mixes with sparticle "b" with soft mass m_b^2 through mixing term Δm^2. The mass matrix in the basis of (a, b) is given by

$$\begin{pmatrix} m_a^2 & \Delta m^2 \\ \Delta m^2 & m_b^2 \end{pmatrix} .$$

The mass eigenstates are derived by diagonalizing the matrix above. If $|m_a^2 - m_b^2| >> 2|\Delta m^2|$ holds, the mixing ratios of "a" and "b" in the two mass eigenstates are approximately given by

$$|m_a^2 - m_b^2| \; : \; |\Delta m^2| , \qquad |\Delta m^2| \; : \; |m_a^2 - m_b^2| .$$

6.1   Predictions of the Bulk Matter RS Model

The bulk matter RS model predicts a nontrivial structure of flavor-violating soft SUSY breaking terms, given by (94, 98-100, 119-128). This structure can be translated into the flavor

composition of each SUSY particle mass eigenstate. One subtlety is that the flavor-violating terms contain two different SUSY breaking mass scales, namely, the IR-scale-suppressed gravity mediation scale, M_{grav}, and the gauge mediation scale, M_{gauge}; flavor-violating gravity mediation contributions depend solely on M_{grav}, whereas RG contributions are proportional to the net soft SUSY breaking mass scale that depends both on M_{grav} and M_{gauge}. The relative size of these scales affects the predictions on the flavor compositions. We consider three cases with $M_{grav} \gtrsim M_{gauge}$, $M_{grav} < M_{gauge}$ and $M_{grav} << M_{gauge}$, whose precise definitions will be given each time. These cases lead to different predictions.

6.1.1 Case with $M_{grav} \gtrsim M_{gauge}$

In this case, flavor-universal soft SUSY breaking masses, m_*^2, and gaugino masses, M_{**}, are of the same magnitude as the gravity mediation scale, M_{grav}. The differences among the diagonal components of different flavors come from the gravity mediation contributions (94) and the RG contributions (119-123). Since we now have $m_*^2 \sim M_{grav}^2$, $M_{**} \sim M_{grav}$, terms (94) surpass terms (119-123). Hence we may make the following approximations for $i > j$ in any flavor basis:

$$(m_Q^2)_{ii} - (m_Q^2)_{jj} \sim \alpha_i^2 M_{grav}^2 \tag{129}$$

and similar formulae with (U, β), (D, γ), (L, δ), (E, ϵ) relacing (Q, α) in the above formula.

In a similar manner, in any flavor basis, the A-terms are approximated by

$$(A_u)_{ij} \supset \beta_i \alpha_j M_{grav}, \qquad (A_d)_{ij} \supset \gamma_i \alpha_j M_{grav}, \qquad (A_e)_{ij} \supset \epsilon_i \delta_j M_{grav}, \tag{130}$$

and the off-diagonal components of soft SUSY breaking mass terms are by $(i \neq j)$

$$(m_Q^2)_{ij} \sim \alpha_i \alpha_j M_{grav}^2 \tag{131}$$

and similar formulae with (U, β), (D, γ), (L, δ), (E, ϵ) relacing (Q, α) in the above formula.

Sparticle Q_i mixes with sparticle Q_j $(j \neq i)$ through the term $(m_Q^2)_{ij}$ and with U_k or D_k $(k \neq i)$ through the A-terms and the VEVs of the Higgs bosons. In this way, there appears a mass eigenstate that consists mainly of Q_i and partly of Q_j and U_k or D_k, which we hereafter call "almost Q_i mass eigenstate". From (129, 131), the mixing ratio of Q_j in "almost Q_i" mass eigenstate is given by

$$\frac{|(m_Q^2)_{ij}|}{|(m_Q^2)_{ii} - (m_Q^2)_{jj}|} \simeq \frac{\alpha_i \alpha_j M_{grav}^2}{(\alpha_i)^2 M_{grav}^2} \sim \frac{\alpha_j}{\alpha_i} \tag{132}$$

for $i > j$, and by

$$\frac{|(m_Q^2)_{ij}|}{|(m_Q^2)_{ii} - (m_Q^2)_{jj}|} \simeq \frac{\alpha_i \alpha_j M_{grav}^2}{(\alpha_j)^2 M_{grav}^2} \sim \frac{\alpha_i}{\alpha_j} \tag{133}$$

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for $i < j$. On the other hand, the mixing ratio of U_j in the up-sector of "almost Q_i mass eigenstate" is given by ($i \neq j$)

$$\frac{v_u \, |(A_u)_{ji}|}{|m_Q^2 - m_U^2|} \quad \sim \quad \frac{v_u \, \beta_j \alpha_i M_{grav}}{M_{susy}^2} \quad \sim \quad \beta_j \alpha_i \, \frac{v_u}{M_{susy}} \, , \tag{134}$$

where we used the fact that the difference between the flavor-universal masses of SU(2) doublet and singlet squarks is of the same magnitude as the soft SUSY breaking mass scale itself, denoted by M_{susy}.

The mixing ratios in other mass eigenstates follow similar formulae. There is a subtlety about the ratio of L_j in "almost L_i mass eigenstate" because we have $3\delta_1 \sim \delta_2 \sim \delta_3$ and the approximation used to derive (132-134) is no longer valid. Actually, the mixing ratio of L_j in "almost L_i mass eigenstate" is $O(1)$ for any i, j.

6.1.2 Case with $M_{grav} << M_{gauge}$

In this sub-subsection, we concentrate on the case where the ratio M_{grav}/M_{gauge} is so small that the RG contributions to flavor-violating soft SUSY breaking terms, (111-113, 119-128), are of the same magnitude as or larger than the gravity mediation contributions, (94, 98-100).

In these cases, the mixing ratio of Q_j in "almost Q_i mass eigenstate" is given, from (119, 124), by ($i > j$)

$$\frac{|(m_Q^2)_{ij}|}{|(m_Q^2)_{ii} - (m_Q^2)_{jj}|} \quad \sim \quad \frac{\alpha_i(\gamma_3)^2\alpha_j}{(\alpha_i)^2(\beta_i)^2 + (\alpha_i)^2(\gamma_3)^2} \quad \sim \quad \frac{\alpha_j}{\alpha_i} \frac{(\gamma_3)^2}{(\beta_i)^2 + (\gamma_3)^2} \, , \tag{135}$$

in the flavor basis where Y_u is diagonalized. Here we used the fact that the integrand of the right hand side of (119) and that of (124) are of the same magnitude. On the other hand, from (111), the mixing ratio of U_j in the up-sector of "almost Q_i mass eigenstate" is given by ($i \neq j$)

$$\frac{v_u|(A_u)_{ji}|}{|m_Q^2 - m_U^2|} \quad \sim \quad 2\beta_j(\alpha_j)^2(\gamma_3)^2\alpha_i \, \frac{v_u}{M_{gauge}} \tag{136}$$

in Y_u-diagonal basis. Here we approximated the difference of flavor-conserving masses of SU(2) doublet squarks and singlet up-type squarks by M_{gauge}. The mixing ratio of D_j in the down-sector of "almost Q_i mass eigenstate" in Y_d-diagonal basis takes a similar expression. The same discussion applies to the mixings in "almost L_i mass eigenstate".

The mixing ratios in "almost U_i mass eigenstate" follow different formulae. From (121, 126), the ratio of U_j is given by ($i > j$)

$$\frac{|(m_U^2)_{ij}|}{|(m_U^2)_{ii} - (m_U^2)_{jj}|} \quad \sim \quad \frac{24 \, \beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j}{4 \, (\beta_i)^2(\alpha_i)^2} \quad \sim \quad 6 \, (\gamma_3)^2(\alpha_j)^2 \, \frac{\beta_j}{\beta_i} \tag{137}$$

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in Y_u-diagonal basis. On the other hand, from (111), the ratio of the up-sector of Q_j in "almost U_i mass eigenstate" is given by ($i \neq j$)

$$\frac{v_u|(A_u)_{ij}|}{|m_Q^2 - m_U^2|} \quad \sim \quad 2\beta_i(\alpha_i)^2(\gamma_3)^2\alpha_j \frac{v_u}{M_{gauge}} \tag{138}$$

in Y_u-diagonal basis. The same discussion applies to the mixings in "almost D_i mass eigenstate" and "almost E_i mass eigenstate".

6.1.3 Case with $M_{grav} < M_{gauge}$ but not $M_{grav} << M_{gauge}$

Consider the case where M_{grav} is slightly smaller than M_{gauge}. Then gravity mediation contributions surpass RG contributions for some of the flavor-violating soft SUSY breaking terms, and the opposite holds for the other terms. In these cases, the mixing ratios of sparticle mass eigenstates generally depend on the unknown ratio M_{grav}/M_{gauge} and the model loses its predictive power.

However, certain mixing ratios are more likely to reflect the gravity mediation contributions. For example, if $M_{grav} \gtrsim \delta_3 M_{gauge}$, as to terms $(m_E^2)_{ii} - (m_E^2)_{jj}$ and $(m_E)_{ij}^2$, the gravity mediation contributions described by (94) are larger than the RG contributions, (126-128). Then the mixing ratio of E_j in "almost E_i mass eigenstate" is the same as in the case with $M_{grav} \gtrsim M_{gauge}$. Focusing on such mixing ratios, it is still possible to observe the signatures of the model.

6.2 Comparison with Other Models

To test the predictions of the bulk matter RS model, we must check whether they contain signatures distinguishable from other models. As an example, we investigate two types of models; one is "minimal flavor violation", in which RG contributions of the Yukawa couplings are the only source of flavor-violating soft SUSY breaking terms. The other is "4D gravity mediation", in which gravity mediation contributes uniformly to all flavor-violating terms. We will compare the predictions of these models with the bulk matter RS model and discuss the ways to distinguish them.

6.2.1 Minimal Flavor Violation

The minimal flavor violation (MFV) scenario leads to the same result as in section 4.1.2, i.e. the bulk matter RS model with $M_{grav} << M_{gauge}$. This is because the argument in section 4.1.2

holds irrespective of gravity mediation contributions. We thus conclude that it is impossible to experimentally distinguish the bulk matter RS model from the MFV scenario when we have $M_{grav} << M_{gauge}$, as in 4.1.2.

In contrast, if $M_{grav} \gtrsim M_{gauge}$, the MFV scenario and the bulk matter RS model have distinctively different predictions on the mixing ratios in "almost U_i, D_i, E_i mass eigenstates" with $i = 1, 2$. This is seen by comparing (132-134) (Q relaced by U, D, E) with (137, 138); the flavor mixings in these mass eigenstates are suppressed at least by $(\alpha_2)^2$ or $(\delta_2)^2$ in "minimal flavor violation" compared to the bulk matter RS model. Therefore it is possible to discriminate the two models by observing the flavor compositions of "almost 1st or 2nd generation SU(2) singlet sparticle mass eigenstates".

6.2.2 4D Gravity Mediation

We here discuss the case where 4D theory description is valid even at the Planck scale, or all matter superfields are confined on the same 4D brane. Then the gravity mediation contributions are of the same magnitude irrespective of flavors. Of particular interest is the situation where the gravity mediation contributions surpass the flavor-violating RG contributions, which is the case when M_{grav} is only slightly smaller than M_{gauge}. In this situation, the differences between diagonal components of soft SUSY breaking masses $(m_*^2)_{ii} - (m_*^2)_{jj}$, and off-diagonal components $(m_*^2)_{ij}$, in any flavor basis are of the same magnitude. Then the mixing ratios in sparticle mass eigenstates are all $O(1)$. It is easy to distinguish this model from the bulk matter RS model, where the mixing ratios of recessive flavors are suppressed by the geometrical factors.

7 Chapter VII : Experimental Studies

In the previous chapter, we saw that the bulk matter RS model has a unique prediction on the flavor compositions of sparticle mass eigenstates that may be distinguishable from other models. In this section, we study how to measure the predicted mixing ratios through collider experiments. We focus on the case where $M_{grav} \simeq M_{gauge}$ holds or M_{grav} is slightly smaller than M_{gauge}, and put emphasis on distinguishing the bulk matter RS model from the MFV scenario.

The basic strategy is to create a specific mass eigenstate(s) of SUSY matter particles, detect its decay products and count the numbers of events of different decay modes. The branching fractions of different modes reflect the flavor composition of that mass eigenstate. There are, however, three challenges for this study.

First, we have to detect small flavor components of sparticle mass eigenstates, which means that we have to observe *rare* decay events in collider experiments. For this purpose, the probability of misidentifying the decay products of the dominant mode as those of a rare mode must be negligibly small. For example, the stau components of "almost smuon mass eigenstates" are detectable because SM tau from the stau components, when we focus on its hadronic decay, leaves a signal different from muon events. However, it is impossible to observe the smuon components of "almost stau mass eigenstates" because SM tau from the dominant stau components may decay into SM muon, which mimics the smuon component signal.

Second, we have to extract the decay products of a *specific* mass eigenstate in order to compare the data with the predictions of the bulk matter RS model. It is thus required to produce only specific mass eigenstates at a collider. This is achieved by lepton colliders, such as the ILC [14] and the CLIC [15], where the center-of-mass energy of a process is fixed. For example, the flavor-mixing ratios in "almost SU(2) doublet smuon mass eigenstate" and in "almost SU(2) singlet smuon mass eigenstate" are predicted to be different. To confirm this prediction, we must produce one of the two eigenstates selectively. If the latter is lighter than the former, we take the center-of-mass energy between their thresholds so that only the latter is created on-shell. We then measure the mixing ratios of the latter eigenstate through its decay products. In conclusion, lepton colliders are essential when studying the flavor compositions of sparticle mass eigenstates.

Finally, we have to focus on "almost SU(2) singlet mass eigenstates" in order to discriminate the bulk matter RS model from the MFV scenario. This is understood by comparing the predictions of the bulk matter RS model, (132-134), with those of the MFV scenario, (135-

138). Remember that we have

$$\gamma_3 \;\sim\; \tan\beta \,\frac{m_b}{v} \,, \qquad \beta_3 \;\sim\; 1$$

and we do not know the magnitude of ζ_3. Hence it can be the case that the mixing ratios of Q_j in "almost Q_i mass eigenstate", and those of L_j in "almost L_i mass eigenstate" are of the same magnitudes for the bulk matter RS model and the MFV scenario. In contrast, the mixing ratios of U_j, D_j, E_j in "almost U_i, D_i, E_i mass eigenstates" are of different magnitudes for the two models because the mixing ratios in the MFV scenario, (137, 138), are suppressed by the factors $(\alpha_1)^2, (\alpha_2)^2, (\delta_1)^2$ or $(\delta_2)^2$ compared to those in the bulk matter RS model, (132-134). We further notice that "almost 3rd generation sparticle mass eigenstates" are not suitable for our study because the 3rd generation sparticles have significant left-right mixing terms due to their large Yukawa couplings. We conclude that observing the rare decays of "almost SU(2) singlet 1st and 2nd generation mass eigenstates" is the only way to distinguish the bulk matter RS model and the MFV scenario.

Taking these points into account, we will discuss three types of experiments that are feasible at future lepton colliders. The first type of experiment deals with the rare decay of "almost SU(2) singlet smuon mass eigenstate" into SM tau, which reflects the mixing of singlet smuon with stau. Another type of experiment deals with the rare decay of "almost SU(2) singlet smuon mass eigenstate" into SM electron or that of "almost SU(2) singlet selectron" into SM muon, which reflects the mixing of singlet smuon and selectron. The other type of experiment deals with the rare decay of "almost SU(2) singlet scharm mass eigenstate" into SM top, which reflects the mixing of singlet scharm with stop. For a concrete discussion, we make assumptions on the SUSY particle mass spectrum in section 5.1. We then look into the three types of experiments in section 5.2., 5.3. and 5.4.

"Almost the lighter stau / stop mass eigenstate" are schematically denoted by $\tilde{\tau}_1$ / $\tilde{t}_1$, and "almost singlet selectron / smuon / scharm mass eigenstate" are by $\tilde{e}_R$ / $\tilde{\mu}_R$ / $\tilde{\mu}_R$.

It is impossible to do these experiments at hadron colliders. This is fundamentally because we need to create "almost SU(2) singlet mass eigenstates" exclusively, without creating their "almost SU(2) doublet" counterparts, in order to discriminate the bulk matter RS model from the MFV scenario. Hadron colliders would necessarily create both eigenstates, and the decay products of the latter would contaminate the signals that allow us to distinguish the two scenarios. It is true that "almost SU(2) singlet eigenstates" are normally lighter than their "almost SU(2) doublet" counterparts, and hence the production cross sections of the latter are lower even at hadron colliders. However, since the two scenarios only predict the orders of magnitudes of the branching ratios of rare events, even small contamination from the latter

41

would make it difficult to test the predictions.

7.1 Assumptions on the Mass Spectrum

In this subsection, we make plausible assumptions on the SUSY particle mass spectrum that are consistent with the bulk matter RS model combined with 5D MSSM.

We assume that squarks are heavier than sleptons and gluino is heavier than Wino and Bino because of their SU(3) charges. Also, Wino is assumed heavier than Bino due to its SU(2) charge. SU(2) doublet squarks are heavier than singlet squarks, and doublet sleptons are than singlet sleptons. Since gauge superfields are flat in the bulk, i.e. they have no y-dependence, they obtain large soft SUSY breaking masses through contact terms on the IR brane. Therefore gluino tends to be heavier than squarks. Wino and Bino are heavier than sleptons but lighter than squarks.

The μ-term is assumed larger than Wino and Bino masses, as is normally the case.

We do not specify the mass order among doublet and singlet squarks and sleptons because gravity mediation contributions may distort the mass spectrum. However, we expect that the masses of the 1st and 2nd generation SUSY particles are almost degenerate because their Yukawa couplings as well as their overlap with the IR brane are small.

Gravitino is always the lightest SUSY particle (LSP) because its mass is given by $\sim$ TeV $\times e^{-kR\pi}$. The next-to-lightest SUSY particle (NLSP) is "almost SU(2) singlet selectron", "almost singlet smuon" or "almost the lighter stau" mass eigenstate. The lifetime of the NLSP satisfies

$$
\begin{aligned}
ct_{NLSP} &\simeq 48\pi \frac{|<F_{\tilde{X}}>|^2}{m_{NLSP}^5} \simeq 48\pi \frac{M_{grav}^2 (M_5 e^{-kR\pi})^2}{(m_{NLSP})^5} \\
&\simeq (1.2 \times 10^{-26})\text{m} \times \left(\frac{M_{grav}}{\text{GeV}}\right)^2 \left(\frac{M_5 e^{-kR\pi}}{\text{GeV}}\right)^2 \left(\frac{300\text{GeV}}{m_{NLSP}}\right)^5 .
\end{aligned}
$$

We assume that the lifetime is enough long that the NLSP reaches the inner detector before it decays.

The order of the sparticle soft SUSY breaking masses is summarized below:

$$
\tilde{H}_u, \tilde{H}_d > \tilde{g} > \tilde{q}_L > \tilde{q}_R > \chi_1^\pm, \chi_2^0 (\fallingdotseq \tilde{W}) > \chi_1^0 (\fallingdotseq \tilde{B}) > \tilde{l}_L > \tilde{l}_R > \psi_{3/2} .
$$

7.2   Type I - Smuon Rare Decay with Stau-like NLSP

Consider the case where "almost the lighter stau mass eigenstate" ($\tilde{\tau}_1$) is the NLSP. Tune the center-of-mass energy of the lepton collider between the thresholds of "almost singlet selectron / smuon mass eigenstates" ($\tilde{e}_R/\tilde{\mu}_R$) and "almost SU(2) doublet selectron / smuon", "almost the heavier stau" mass eigenstates. Then $\tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_1$ are produced on-shell, while other sparticle mass eigenstates are not.

The signal for $\tilde{e}_R$ or $\tilde{\mu}_R$ pair production is a pair of long-lived charged massive particles, which are NLSP $\tilde{\tau}_1$s, plus two pairs of hard SM leptons. Note that, since the masses of $\tilde{e}_R$ and $\tilde{\mu}_R$ are almost degenerate, we cannot detect SM leptons emitted when the heavier one decays into the lighter one. Normally, we have two SM muons or electrons plus two SM taus in these events (we call this "main mode"), e.g.

$$ee \;\rightarrow\; \tilde{\mu}_R\,\tilde{\mu}_R \;\rightarrow\; \mu\,\tau\,\tilde{\tau}_1\,\mu\,\tau\,\tilde{\tau}_1 \tag{139}$$

for the sumon production. However, due to the small stau components in $\tilde{e}_R$ / $\tilde{\mu}_R$, we may also have one SM muon or electron plus three SM taus in these events (we call this "rare mode"), e.g.

$$ee \;\rightarrow\; \tilde{\mu}_R\,\tilde{\mu}_R \;\rightarrow\; \tau\,\tau\,\tilde{\tau}_1\,\mu\,\tau\,\tilde{\tau}_1 \tag{140}$$

for the smuon production. Requiring hadronic decay of SM taus and taking advantage of the tau vertexing, one can reduce the probability of misidentifying a main mode event as a rare mode event to a negligible level.

The branching ratio of the rare mode is proportional to the square of the mixing ratio. The stau component in $\tilde{\mu}_R$ plays a dominant role because the stau component in $\tilde{e}_R$ is much more suppressed. From (132) with (Q, α) replaced by (E, ϵ), and from (134) with $(Q, U, \alpha, \beta, v_u)$ replaced by $(L, E, \delta, \epsilon, v_d)$, the bulk matter RS model predicts that the branching ratio of the rare mode is given by

$$\begin{aligned} Br(\tilde{\mu}_R \;\rightarrow\; \tau\,\tau\,\tilde{\tau}_1) \;&\sim\; \left(\frac{\epsilon_2}{\epsilon_3}\right)^2 + \left(\epsilon_2\delta_3\frac{v_d}{M_{susy}}\right)^2 \\ &\sim\; \left(\frac{m_\mu}{m_\tau}\right)^2 + \left(\frac{m_\mu}{M_{susy}}\right)^2, \end{aligned} \tag{141}$$

where the first term comes from the mixing with singlet stau and the second from the mixing with doublet stau. If $\tilde{e}_R$ is lighter than $\tilde{\mu}_R$, the branching ratio is reduced by 1/2 compared to the opposite case because a half of $\tilde{\mu}_R$s decay into $\tilde{e}_R$s. However, this does not affect the order estimate above. Since we have $M_{susy} >> m_\tau$, the second term is negligible and the branching

ratio becomes

$$Br(\tilde{\mu}_R \ \to \ \tau \ \tau \ \tilde{\tau}_1) \ \sim \ 0.004 \ . \tag{142}$$

Note that the prediction above may change by $O(0.1) - O(10)$ because we only know the magnitudes of the higher-dimensional couplings for soft SUSY breaking terms.

On the other hand, the MFV scenario predicts that the branching ratio of the rare mode is given by

$$Br(\tilde{\mu}_R \ \to \ \tau \ \tau \ \tilde{\tau}_1) \ \sim \ \left\{ 6(\zeta_3)^2(\delta_2)^2 \frac{\epsilon_2}{\epsilon_3} \right\}^2 \ + \ \left\{ 2(\zeta_3)^2(\delta_2)^2 \epsilon_2 \delta_3 \frac{v_d}{M_{susy}} \right\}^2$$
$$\sim \ (\zeta_3 \delta_2)^4 \times 0.1 \ , \tag{143}$$

where we used (137) and (138) with $(Q, U, \alpha, \beta, \gamma, v_u)$ replaced by $(L, E, \delta, \epsilon, \zeta, v_d)$. Although we cannot determine the magnitude of $\zeta_3 \delta_2$, we expect it to be smaller than 0.1; from (72), we have the following flavor-mixing term for SU(2) doublet smuon and selectron:

$$(m_L^2)_{12} \ \sim \ \delta_1(\zeta_3)^2 \delta_2 \ M_{susy}^2 \ \sim \ \frac{1}{3}(\zeta_3 \delta_2)^2 \ M_{susy}^2 \ .$$

For example, with $M_{susy} = 500$ GeV, $m_{\tilde{l}_L}^2 = 500$ GeV, $M_{\tilde{B}} = M_{\tilde{W}} = 750$ GeV, $\mu = 1000$ GeV, $\tan\beta = 10$ and vanshing A-terms, taking $\zeta_3 \delta_2 = 0.1$ would saturate the current bound on $\mu \to e\gamma$ branching ratio, $Br(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [16]. Hence the branching ratio of the rare mode in the MFV scenario satisfies

$$Br(\tilde{\mu}_R \ \to \ \tau \ \tau \ \tilde{\tau}_1) \ \lesssim \ 10^{-5} \ , \tag{144}$$

which may get smaller if the bound on $Br(\mu \to e\gamma)$ improves. We conclude that the branching ratio of the rare mode is distinctively smaller in the MFV scenario than in the bulk matter RS model.

7.3 Type II - NLSP Selectron Rare Decay into Muon, or NLSP Smuon Rare Decay into Electron / Tau

Consider the case where "almost singlet smuon" or "almost singlet selectron2 mass eigenstate ($\tilde{\mu}_R$ or $\tilde{e}_R$) is the NLSP and is long-lived. Tune the center-of-mass energy slightly above the threshold of $\tilde{\mu}_R$ / $\tilde{e}_R$ so that they are produced with a low β (Lorentz velocity). Slow long-lived sleptons may be trapped in the inner detector. According to the paper [17], taking $\beta \lesssim 0.2$ is sufficient to trap 600 GeV or lighter sleptons in the inner detector. We analyze the decay

products of these sleptons to study their flavor compositions.

First study the case where $\tilde{e}_R$ is lighter than $\tilde{\mu}_R$ and is the long-lived NLSP. $\tilde{e}_R$ mainly decays into a SM electron and a gravitino (main mode). However, due to its smuon component, it also decays into a SM muon and a gravitino (rare mode). Hence we expect to observe rare mode events where one of the sparticle pair produced by the collider decays into a SM muon and the other into a SM electron with large vertex separation due to the longevity of $\tilde{e}_R$.

The bulk matter RS model predicts that the branching ratio of the rare mode is given by

$$
\begin{aligned}
Br(\tilde{e}_R \ \to \ \mu \ \psi_{3/2}) \ &\sim \ \left(\frac{\epsilon_1}{\epsilon_2}\right)^2 \ + \ \left(\epsilon_1 \delta_2 \frac{v_d}{M_{susy}}\right)^2 \\
&\sim \ \left(3\frac{m_e}{m_\mu}\right)^2 \ + \ \left(3\frac{m_e}{M_{susy}}\right)^2 \\
&\sim \ 0.0002 \ ,
\end{aligned}
\tag{145}
$$

where we neglected the second terms of the right hand sides because we have

$$
3\frac{m_e}{m_\mu} \ >> \ 3\frac{m_e}{M_{susy}}
$$

in realistic SUSY models.

On the other hand, the MFV scenario predicts that the branching ratio of the rare mode is given by

$$
\begin{aligned}
Br(\tilde{e}_R \ \to \ \mu \ \psi_{3/2}) \ &\sim \ \left\{6(\zeta_3)^2(\delta_1)^2\frac{\epsilon_1}{\epsilon_2}\right\}^2 \ + \ \left\{2(\zeta_3)^2(\delta_1)^2\epsilon_2\delta_1\frac{v_d}{M_{susy}}\right\}^2 \\
&\sim \ (\zeta_3\delta_1)^4 \times 0.03 \ ,
\end{aligned}
\tag{146}
$$

Again, the bound on $\mu \to e\gamma$ branching ratio gives a severe constraint on the value of $\zeta_3\delta_1$, and the branching ratio satisfies

$$
Br(\tilde{e}_R \ \to \ \mu \ \psi_{3/2}) \ < \ 10^{-6}
\tag{147}
$$

for realistic mass spectra.

$\tilde{e}_R$ also decays into a SM tau and a gravitino but the branching ratio is suppressed by the factor $(m_e/m_\tau)^2$ and is thus negligibly small.

Next consider the case where $\tilde{\mu}_R$ is lighter than $\tilde{e}_R$ and is the long-lived NLSP. $\tilde{\mu}_R$ mainly decays into a SM muon and a gravitino (main mode), but also into a SM electron and a gravitino, or into a SM tau and a gravitino (rare modes). The branching ratio of the rare mode

where the sparticle pair decay into a muon and an electron and two gravitinos is the same as (145, 146). The branching ratio of the rare mode where the sparticle pair decay into a tau and a muon and two gravitinos is the same as (142, 143).

7.4   Type III - Scharm Rare Decay into SM Top

7.4.1   Scharm is Lighter than Stop

Consider the case where $\tilde{c}_R$ is lighter than $\tilde{t}_1$. Tune the center-of-mass energy between the thresholds of $\tilde{c}_R$ and $\tilde{t}_1$. Then $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R$, whose masses are almost degenerate, are produced on-shell, while other squark mass eigenstates are not. $\tilde{c}_R$ / $\tilde{u}_R$ / $\tilde{s}_R$ / $\tilde{d}_R$ mainly decay into SM charm / up / strange / down and the lightest neutralino χ_1^0, which is Bino-like (main mode). χ_1^0 promptly decays into several SM leptons and NLSP, e.g. we have

$$ ee \;\rightarrow\; \tilde{c}_R \, \tilde{c}_R \;\rightarrow\; c \, \chi_1^0 \, c \, \chi_1^0 \;\rightarrow\; (c-\text{jet}) \; NLSP \; (c-\text{jet}) \; NLSP \; (\text{SM leptons}) \qquad (148) $$

for scharm pair production. Due to the small stop components, they also decay into SM top and χ_1^0 with a tiny branching ratio (rare mode), e.g. we have

$$ ee \;\rightarrow\; \tilde{c}_R \, \tilde{c}_R \;\rightarrow\; t \, \chi_1^0 \, c \, \chi_1^0 \;\rightarrow\; (\text{top decay products}) \; NLSP \; (c-\text{jet}) \; NLSP \; (\text{SM leptons}) \quad (149) $$

for scharm pair production. The signal of the main mode is two hard jets, two long-lived charged massive particles and several SM leptons. On the other hand, the signal of the rare mode is, when SM top decays hadronically, four hard jets, two long-lived charged massive particles and several SM leptons. We see that the probability of misidentifying a main mode event as a rare mode event is negligibly small if we require the hadronic top decay in rare mode events.

Of the four eigenstates, $\tilde{c}_R$ dominantly contributes to rare mode events because the stop components in the other eigenstates are more suppressed than in $\tilde{c}_R$. On can confirm this by requiring c-tagging for one of the jets in rare mode events.

From (133) with (Q, α) replaced by (U, β) and (134), the bulk matter RS model predicts that the branching ratio of the rare mode is given by

$$
\begin{aligned}
Br(\tilde{c}_R \;\rightarrow\; t \, \chi_1^0) \;&\sim\; \left(\frac{\beta_2}{\beta_3}\right)^2 \;+\; \left(\beta_2 \alpha_3 \frac{v_u}{M_{susy}}\right)^2 \\
&\sim\; \left(\frac{1}{\lambda^2} \frac{m_c}{m_t}\right)^2 \;+\; \left(\frac{1}{\lambda^2} \frac{m_c}{m_t} \; \frac{m_t}{M_{susy}}\right)^2 \\
&\sim\; 0.02 \;,
\end{aligned}
\qquad (150)
$$

where we neglected the second term because we have

$$ m_t < M_{susy} $$

in realistic SUSY models.

From (137) and (138), the MFV scenario predicts that the branching ratio of the rare mode is given by

$$Br(\tilde{c}_R \rightarrow t\, \chi_1^0) \quad \sim \quad \left\{6(\gamma_3)^2(\alpha_2)^2\frac{\beta_2}{\beta_3}\right\}^2 \quad + \quad \left\{2(\gamma_3)^2(\alpha_2)^2\beta_2\alpha_3\frac{v_u}{M_{susy}}\right\}^2$$

$$\lesssim \quad 5 \times 10^{-6} \, , \tag{151}$$

where we used the fact that $\gamma_3 \leq 1$. Comparing (151) with (150), we notice that the bulk matter RS model and the MFV scenario have distinctively different predictions.

7.4.2 Stop is Lighter

Consider the case where $\tilde{t}_1$ is lighter than $\tilde{c}_R$. $\tilde{c}_R$ is still lighter than "almost SU(2) doublet squark" mass eigenstates (including $\tilde{t}_1, \tilde{b}_1$). Tune the center-of-mass energy between the thresholds of $\tilde{c}_R$ and "almost doublet squark" eigenstates. Then $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R, \tilde{b}_1, \tilde{t}_1$ are produced on-shell, while the other squark mass eigenstates are not.

We want to extract the signals of rare mode events where one of the pair of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R$ or $\tilde{d}_R$s decays into a SM top and a neutralino. However these events are contaminated by the events where one of the pair of $\tilde{b}_1$s decays into a SM top and a chargino; the chargino decays into a NLSP and SM leptons, but one charged lepton is mis-detected. b-jet from the other $\tilde{b}_1$ is mis-b-tagged. There is also a contamination from the events where one of the pair of $\tilde{t}_1$s decays into a SM bottom and a chargino, or into a SM charm and a neutralino due to the scharm component in $\tilde{t}_1$. We take advantage of kinematical properties to reject these contaminations.

Tune the center-of-mass energy close to the threshold of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R$ so that they are produced almost at rest. Suppose that one observed three hard jets from the hadronic decay of SM top (t), another hard jet (j) and several SM leptons plus two NLSPs. Further assume that the 3-momenta of t and j are reconstructed successfully. We want to know whether this event comes from the decay of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R$ or from $\tilde{t}_1, \tilde{b}_1$. In the former case, the 3-momenta satisfy

$$|\vec{p}_j| \quad + \quad \sqrt{|-\vec{p}_j|^2 + m_\chi^2} \quad \simeq \quad m_{\tilde{c}_R} \, , \tag{152}$$

$$\sqrt{|\vec{p}_t|^2 + m_t^2} \quad + \quad \sqrt{|-\vec{p}_t|^2 + m_\chi^2} \quad \simeq \quad m_{\tilde{c}_R} \, , \tag{153}$$

where $\vec{p}_j$ and $\vec{p}_t$ respectively denote the 3-momenta of j and t, and m_χ the mass of Bino-like neutralino. In the latter case, however, the above equations hold for specific situations where j and t go in special directions against the initial $\tilde{t}_1$s or $\tilde{b}_1$s because the $\tilde{t}_1, \tilde{b}_1$s are boosted. To

summarize, the rare mode signals of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R$ can be extracted through the discriminants (152, 153).

The branching ratios of the rare mode in the bulk matter RS model and the MFV scenario are the same as in (150, 151).

7.5 Cross Sections

We calculate the cross sections of the rare modes given above and discuss their accessibility at collider experiments.

First focus on the Type I experiment, where one of the pair of "almost singlet smuon or selectron mass eigenstates" decays into two SM taus and a NLSP in case the NLSP is stau-like. The center-of-mass energy is tuned above the threshold of $\tilde{\mu}_R$, namely we take

$$\sqrt{s} \;=\; 2m_{\tilde{\mu}_R} \;+\; 100 \text{ GeV} \;.$$

In Figure 1, we plot the mass of $\tilde{\mu}_R$ vs. the cross section of the rare mode at a e^+e^- collider. We take the branching ratio of the rare mode as

$$Br(\tilde{\mu}_R \;\rightarrow\; \tau\,\tau\,\tilde{\tau}_1) \;=\; \left(\frac{m_\mu}{m_\tau}\right)^2$$

based on (141). We assume that $\tilde{\mu}_R$ is slightly lighter than $\tilde{e}_R$ so that a half of $\tilde{e}_R$s decay into $\tilde{\mu}_R$ and contribute to the rare mode. In calculating the $\tilde{e}_R$ pair production cross section, the Bino mass is assumed to be $1.5m_{\tilde{\mu}_R}$. Also shown is the total cross section of $\tilde{\mu}_R, \tilde{e}_R$ production events.

Next focus on the Type II, where one of the pair of "almost singlet selectron mass eigenstates" decays into a SM muon and a gravitino if it is the NLSP, or one of the pair of "almost singlet smuon mass eigenstates" decays into a SM electron / tau and a gravitino if it is the NLSP. We tune the center-of-mass energy slightly above the threshold of $\tilde{e}_R$ / $\tilde{\mu}_R$ so that their velocities are low enough to trap them inside the inner detector. For simplicity, we take

$$\sqrt{s} \;=\; 2m_{\tilde{e}_R/\tilde{\mu}_R} \;+\; 20 \text{ GeV} \;,$$

In Figure 2, we plot the mass of $\tilde{e}_R$ vs. the cross section of the rare mode if it is the NLSP, and the mass of $\tilde{\mu}_R$ vs. the cross section of the rare mode if it is the NLSP. We take the branching ratios of the rare modes as

$$Br(\tilde{e}_R \;\rightarrow\; \mu\,\psi_{3/2}) \;=\; Br(\tilde{\mu}_R \;\rightarrow\; e\,\psi_{3/2}) \;=\; \left(\frac{1}{3}\frac{m_e}{m_\mu}\right)^2 \;,$$

$$Br(\tilde{\mu}_R \;\rightarrow\; \tau\,\psi_{3/2}) \;=\; \left(\frac{m_\mu}{m_\tau}\right)^2 \;,$$
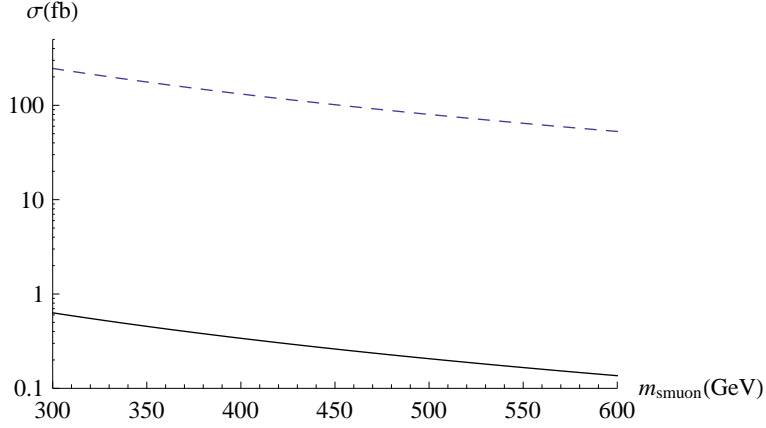
Figure 1: Mass of $\tilde{\mu}_R$ vs. the cross section of the rare mode where a $\tilde{\mu}_R$ is produced and decays into two SM taus and a NLSP stau (the straight line). The center-of-mass energy is taken as $\sqrt{s} = 2m_{\tilde{\mu}_R} + 100$ GeV. The total cross section of $\tilde{\mu}_R$ or $\tilde{e}_R$ production process is also shown (the dashed line).
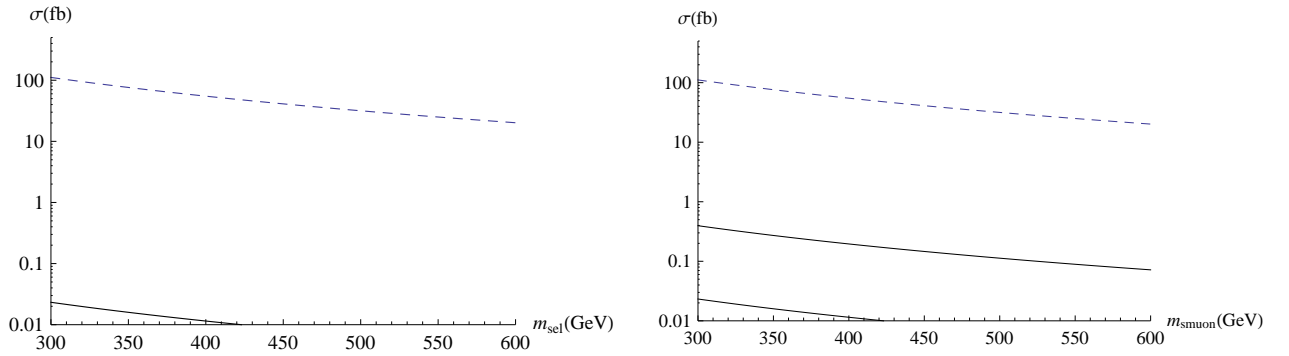

Figure 2: Left: Mass of $\tilde{e}_R$ vs. the cross section of the rare mode where a NLSP $\tilde{e}_R$ is produced and decays into a SM muon and a gravitino (the straight line). Right: Mass of $\tilde{\mu}_R$ vs. the cross section of a rare mode where a NLSP $\tilde{\mu}_R$ is produced and decays into a SM electron and a gravitino (the straight line below), and the other rare mode where a NLSP $\tilde{\mu}_R$ is produced and decays into a SM tau and a gravitino (the straight line above). The total cross sections of $\tilde{e}_R$ (left) and $\tilde{\mu}_R$ (right) production processes are also shown (the dashed lines).

based on (141, 145). Also shown are the total cross sections of $\tilde{e}_R$ and $\tilde{\mu}_R$ production processes.

Finally focus on the Type III, where one of the pair of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R$s decays into a SM top and a neutralino. The center-of-mass energy is tuned above the threshold of $\tilde{c}_R$. First we take

$$\sqrt{s} \;=\; 2m_{\tilde{c}_R} \;+\; 100 \text{ GeV}$$

so that the cross section is nearly maximized. Second we take

$$\sqrt{s} \;=\; 2m_{\tilde{c}_R} \;+\; 10 \text{ GeV}$$

so that $\tilde{c}_R$s are produced almost at rest and the rare mode events are kinematically distinguishable from $\tilde{t}_1, \tilde{b}_1$ production events in case $\tilde{t}_1, \tilde{b}_1$ are lighter. In Figure 3, we plot the mass of $\tilde{c}_R$ vs. the cross section of the rare mode for both cases. The branching ratio of the rare mode is taken as

$$Br(\tilde{c}_R \;\rightarrow\; t\,\chi_1^0) \;=\; \left(\frac{1}{\lambda^2}\frac{m_c}{m_t}\right)^2$$

for both cases, based on (97). Also shown is the total cross section of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R, \tilde{d}_R$ production processes.
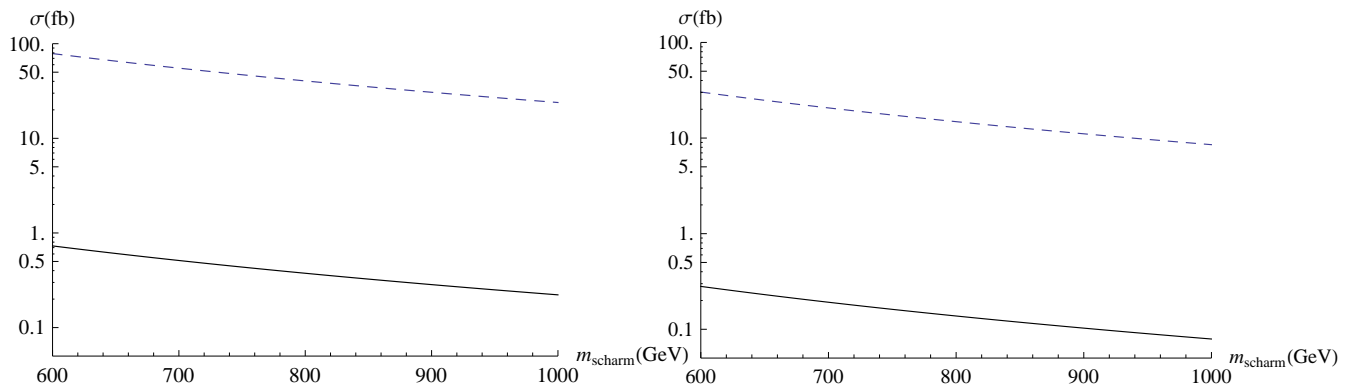

Figure 3: Mass of $\tilde{c}_R$ vs. the cross section of the rare mode where a $\tilde{c}_R$ is produced and decays into a SM top and a neutralino (the straight lines). The center-of-mass energy is taken as $\sqrt{s} = 2m_{\tilde{c}_R} + 100$ GeV in the left and $\sqrt{s} = 2m_{\tilde{c}_R} + 10$ GeV in the right. The total cross section of $\tilde{c}_R, \tilde{u}_R, \tilde{s}_R$ or $\tilde{d}_R$ production processes is also shown (the dashed lines).

Note that we can in principle reject all background events when observing the signals of the rare modes. Therefore detecting several signals is sufficient to confirm the bulk matter RS model. From Figure 1 and 2, we find that one can study the stau component in $\tilde{\mu}_R$ at the ILC with the integrated luminosity of $\sim 100\text{fb}^{-1}$. However, studying the smuon component in $\tilde{e}_R$ or the selectron component in $\tilde{\mu}_R$ requires $\sim 1000\text{fb}^{-1}$ integrated luminosity. From Figure 3, we see that $\sim 100\text{fb}^{-1}$ integrated luminosity is sufficient to study the stop component in $\tilde{c}_R$.

8 Chapter VIII : Summary and Outlook

We discussed observing signals of the bulk matter RS model, especially when the IR scale is far above TeV scale. We saw that this is possible in the case of the minimal supersymmteric extension of the bulk matter RS model where the warped spacetime solely explains the hierarchy of the Yukawa couplings, while SUSY solves the gauge hierarchy problem. There, gravity mediation contributions to soft SUSY breaking terms reflect the 5D disposition of superfields. Hence flavor-violating soft SUSY breaking matter mass terms that arise from gravity mediation exhibit a flavor structure unique to the bulk matter RS model. RG running of the Yukawa coupling constants also contributes to the flavor-violating terms, but its contributions and the gravity mediation contributions are distinguishable if the mass scale of gauge mediation is not much larger than that of gravity mediation. Then the latter contributions can be extracted by investigating the 1st and 2nd generation SU(2) singlet SUSY particles, where the former are further suppressed by the small Yukawa coupling constants. We focused on the flavor compositions of SUSY particle mass eigenstates, which reflect the relative size of flavor-violating soft SUSY breaking terms. We enumerated three modes of collider experiments where one can measure the compositions by observing rare decays of SUSY particles. Predictions on their branching ratios were made based on the bulk matter RS model, and were compared with those of the minimal flavor violation scenario. These predictions will be confirmed or rejected by a future lepton collider whose center-of-mass energy is tuned appropriately.

A lesson of this study is that if new physics at TeV scale contains a flavor-violating sector other than the Yukawa couplings, it is possible to observe signatures of models that explain the Yukawa coupling hierarchy through the flavor structure of the new sector. In the case of this thesis, MSSM contains gravity-mediation-origined soft mass terms, which provide a new source of flavor violation. Gravity mediation and the Yukawa couplings are independent in the original MSSM, but have a correlation if the bulk matter RS model is the origin of the Yukawa coupling hierarchy. Hence we can predict the flavor structure of gravity mediation contributions (up to their orders of magnitudes) from the data on SM, and eventually confirm or reject the bulk matter RS model through a detailed study on SUSY matter particles. This study can be extended to any new physics scenario at TeV scale as long as it couples to matter fields and may violate flavor. In any case, SU(2) singlet muon and charm and their new physics partners play a pivotal role; SU(2) singlets receive less flavor-violating quantum corrections from the SM Yukawa couplings, and thus new flavor-violating terms are easy to extract. Since the 1st and 2nd generation particles only have small Yukawa couplings, we expect that their new SU(2) singlet partners almost do not mix with SU(2) doublets. Muon has much larger Yukawa coupling than electron and is much more sensitive to the origin of the Yukawa coupling

hierarchy. The partner of charm may have a large flavor-violating mixing with top, the only quark whose flavor can be identified with virtually no misidentification rate.

9   Acknowledgements

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