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The interstellar medium regulated by supernova remnants and bursts of star formation

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Summary. The time variation of the interstellar medium regulated by supernova remnants is studied by considering the interchange processes among six components: a warm gas, a general ambient gas, a hot gas, small clouds, molecular clouds and giant molecular clouds. Two interesting results are found. One is that the interstellar medium does not always attain a steady state such as a two- or three-phase model, but may exhibit a cyclic phase-change like a limit cycle or go to a runaway state. The other is that bursts of star formation from molecular clouds are expected if the gravitational instability of a cloud ensemble is considered. These results will lead us to a new picture of the evolution and structure of galaxies.

1 Introduction

In the past decade, our picture of the interstellar medium (ISM) has changed substantially due to the discovery of giant molecular clouds and a hot gas component. Much of the mass of the ISM is in giant molecular clouds, and much of the volume of interstellar space is occupied by a hot gas component, so investigations of the structure and evolution of the ISM must take account of these components. A steady-state model of the ISM with a hot ($\geqslant 10^{5.5}$ K), tenuous ($\leqslant 10^{-2}\,\mathrm{cm}^{-3}$) gas in addition to a warm (~ 8000 K) gas and cold clouds ($\lesssim 10^2$ K) has been proposed by McKee & Ostriker (1977), while the importance of molecular clouds in star formation has been indicated by Elmegreen & Lada (1977).

Following our previous paper (Habe, Ikeuchi & Tanaka 1981, hereafter referred to as Paper I), we investigate the structure and time variation of the ISM by calculating the interchange processes driven mainly by supernova remnants (SNRs). They are the sweeping-up of ambient gas by SNRs, the fragmentation of dense SNR shells to form cold clouds and the evaporation of clouds within hot SNR cavities. Moreover, taking account of cloud–cloud collisions and the gravitational instability of a cloud ensemble, we can obtain a complete set of equations including the formation and destruction of molecular clouds.

Two interesting results emerge. One is that the two-phase model proposed by Field (1965) or the three-phase model by McKee & Ostriker (1977) are not the only steady states of the ISM. Here, we assert the presence of a new state of the ISM, in which each component changes cyclically like a limit cycle of a non-linear oscillator. One of the authors (SI) has already indicated this possibility by means of simple model equations (Ikeuchi & Tomita 1983).

The other is that bursts of star formation are expected when the gravitational instability of a cloud ensemble is considered. This will give us an alternative picture of the evolution of galaxies.

In Section 2, we outline the interchange processes in the ISM. The details will be given in Appendices A and B. In Section 3, we present numerical results when only four components, namely a hot gas, a warm gas, an ambient gas and cold clouds, are considered. A cyclic phase-change of the ISM is discussed. In Section 4, we include the formation of molecular clouds and subsequent star formation, and present numerical results on evolution of the ISM. A model for bursts of star formation is indicated. In the final Section 5, some implications of the results are given.

2 Interchange processes of the interstellar medium

2.1 COMPONENTS OF THE ISM

We suppose that the ISM is composed of the following six components:

WG: warm gas (suffix w), which is formed by ionization of clouds. For simplicity, we fix the temperature to be $T_w = 8000$ K, and assume pressure equilibrium between the warm gas and a general ambient medium.

AM: ambient medium (suffix a), which prevails in interstellar space except for the volume occupied by SNRs and warm gas. We assume this ambient medium to consist of a mixture of pre-existing warm gas and hot gas supplied from old SNRs. Its temperature T_a is calculated by taking account of radiative cooling.

HG: hot gas (suffix h), which corresponds to the hot gas within SNR cavities.

Instead of calculating a detailed mass-spectrum of clouds, we represent the cold component by the following three kinds of clouds:

SC: small clouds (suffix sc), by which we mean diffuse H I clouds with mass $m_{sc} = 42.9\, M_\odot$, density $n_{sc} = 40\,\mathrm{cm}^{-3}$ and size $a = 2$ pc (McKee & Ostriker 1977).

MC: molecular clouds (suffix mc), by which we mean massive clouds with mass $m_{mc} = 10^4\, M_\odot$ and size 12.3 pc.

GMC: giant molecular clouds (suffix gmc), by which we mean giant clouds with mass $m_{gmc} = 3 \times 10^5\, M_\odot$ – these are the clouds associated with active star-formation.

The schematic picture of the ISM composed of these six components is shown in Fig. 1. As usual, the mass, volume, number density, mass density, temperature and specific internal energy of each component are denoted as M, V, n, ρ, T and I, respectively, with a corresponding suffix.

2.2 FINAL FATE OF SNRs

Before considering the interchange processes among the six components of the ISM, it is important to note that the final fate of SNRs depends upon the density n_a and temperature T_a of the ambient medium as indicated in Paper I. In Fig. 2, we illustrate this for a supernova rate $S = 10^{-13}$ SN pc^{-3} yr^{-1} and a space density of small clouds $N_{sc} = 10^{-4}$ pc^{-3}. The

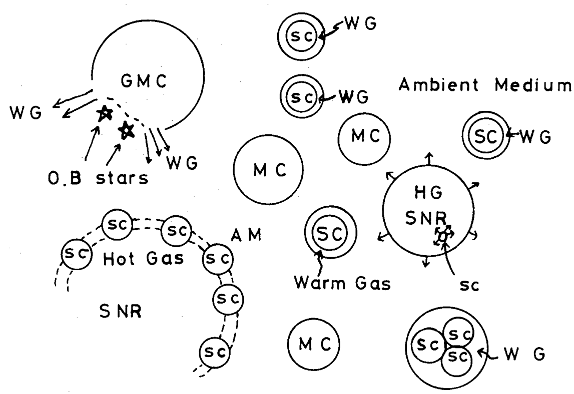

Figure 1. Schematic picture of mutual interchange processes in the ISM.

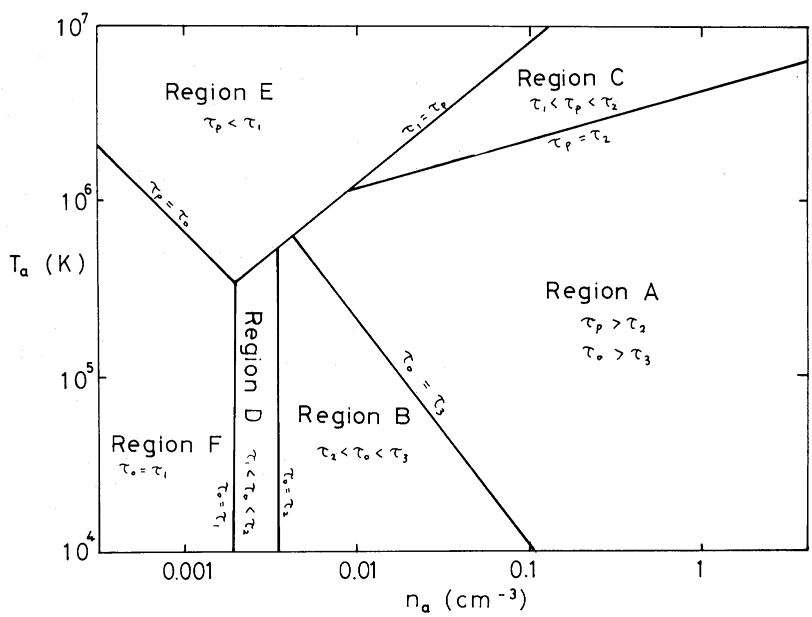

Figure 2. Regions of ambient matter, in which SNRs have different fates. Here we take $S = 10^{-13}$ SN pc^{-3} yr^{-1} and $N_{sc} = 10^{-4}$ pc^{-3}.

evolution of a normal SNR is divided (Spitzer 1978; McKee & Ostriker 1977) into the evaporation-dominated stage, the Sedov–Taylor stage and the snowplough stage. The expansion laws and transition times between them are summarized in Appendix A. On the other hand, an SNR can be said to 'die' when either the pressure of the shock front becomes equal to the pressure of the ambient medium or when the overlapping of many SNRs occurs. The final fate of an SNR is therefore covered by one of the following six cases, which are denoted as A, B, C, D, E and F in the n_a–T_a plane.

Region A. Pressure equilibrium with the ambient medium occurs at the snowplough stage.

Region B. Overlapping occurs at the snowplough stage.

Region C. Pressure equilibrium occurs at the Sedov stage.

Region D. Overlapping occurs at the Sedov stage.

Region E. Pressure equilibrium occurs at the evaporation-dominated stage.

Region F. Overlapping occurs at the evaporation-dominated stage.

It is easily understood that, if the ambient medium is in Regions C, D, E or F, the cold, dense shall at the shock front is not formed. This classification depends sensitively upon the supernova rate S and the space density of clouds N_{sc}.

2.3 INTERCHANGE PROCESSES

The interchange processes among the above six components of the ISM are illustrated in Fig. 3. The processes I–X and the corresponding interchange rates, discussed in more detail in Appendix B, are as follows:

I. α_s^A: sweeping-up of ambient medium by SNRs to form hot cavities (α_s^{AH}) and cold dense shells (α_s^{AC}). We assume that the dense shell is broken-up into small clouds soon after the death of an SNR.

II. α_s^W: sweeping-up of warm gas by SNRs to form hot cavities (α_s^{WH}) and cold shells (α_s^{WC}).

III. α_s^H: sweeping-up of hot gas in SNR cavities, in which new supernovae exploded, to form cold shells.

IV. α_{evp}: evaporation of small clouds within hot cavities. We adopt the classical evaporation rate.

V. α_{cc}: formation rate of MC from SC by mutual encounters.

VI. α_{cc}^M: formation rate of GMC from MC by the gravitational instability of an ensemble of MCs. The treatment adopted here is summarized in Appendix B.

VII. α_{ion}: ionization by the UV flux of the surfaces of SC to form warm gas, assumed to be proportional to the rate of supernova explosions.

VIII. α_{mix}: mixing of hot gas in SNR cavities with the ambient medium after the destruction of SNR shells.

IX. α_{SF}: star formation rate from giant molecular clouds.

X. α_e: erosion of giant molecular clouds to produce warm gas, by young stars formed at the boundaries of clouds.

The processes XI–XIV, shown by dashed lines in Fig. 3, are not considered in the present paper, since they are less important within the time-scale ($\lesssim 10^9$ yr) here:

XI. α_{SF}: star formation from small clouds.

XII. α_{ML}: mass loss from stars.

XIII. α_{esc}: gas escape from a galaxy.

XIV. α_{inf}: infall of halo gas.

These will be included in a succeeding paper on the evolution of galaxies.

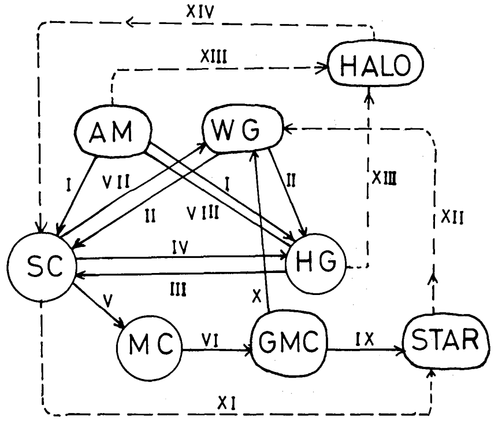

Figure 3. Interchange processes among six components of the ISM. The processes I–XIV are described in the text, but processes shown by dashed lines, and the halo gas component, are not considered in the present paper.

2.4 TIME VARIATION OF EACH COMPONENT

Summing the interchange processes I–X, the fundamental equations for time variations of the masses of respective components are

$$\frac{dM_{\rm w}}{dt} = -\alpha_{\rm s}^{\rm WH} - \alpha_{\rm s}^{\rm WC} + \alpha_{\rm ion} + \alpha_{\rm e}, \tag{2.1}$$

$$\frac{dM_{\rm a}}{dt} = -\alpha_{\rm s}^{\rm AH} - \alpha_{\rm s}^{\rm AC} + \alpha_{\rm mix}, \tag{2.2}$$

$$\frac{dM_{\rm h}}{dt} = -\alpha_{\rm s}^{\rm H} - \alpha_{\rm mix} + \alpha_{\rm s}^{\rm WH} + \alpha_{\rm s}^{\rm AH} + \alpha_{\rm evp}, \tag{2.3}$$

$$\frac{dM_{\rm sc}}{dt} = -\alpha_{\rm evp} - \alpha_{\rm cc} - \alpha_{\rm ion} + \alpha_{\rm s}^{\rm WC} + \alpha_{\rm s}^{\rm AC} + \alpha_{\rm s}^{\rm H}, \tag{2.4}$$

$$\frac{dM_{\rm mc}}{dt} = -\alpha_{\rm cc}^{\rm M} + \alpha_{\rm cc}, \tag{2.5}$$

$$\frac{dM_{\rm gmc}}{dt} = -\alpha_{\rm SF} - \alpha_{\rm e} + \alpha_{\rm cc}^{\rm M}. \tag{2.6}$$

If the mass of stars M_* is included, the total mass $M_{\rm t} = \Sigma_{\rm i} M_{\rm i}$ (i = w, a, h, sc, mc, gmc, *) is conserved.

The specific internal energies of the ambient medium $I_{\rm a}$ and the hot gas in SNR cavities $I_{\rm h}$ are calculated as

$$\frac{d}{dt}(I_{\rm a} M_{\rm a}) = -I_{\rm a}(\alpha_{\rm s}^{\rm AH} + \alpha_{\rm s}^{\rm AC}) + I_{\rm h}\alpha_{\rm mix} - L(n_{\rm a}, T_{\rm a}), \tag{2.7}$$

and

$$\frac{d}{dt}(I_{\rm h} M_{\rm h}) = -I_{\rm h}(\alpha_{\rm s}^{\rm H} + \alpha_{\rm mix}) + H_{\rm SN}, \tag{2.8}$$

respectively, where $L(n_{\rm a}, T_{\rm a})$ is the radiative loss rate of the ambient medium, and $H_{\rm SN}$ is the energy injection rate due to supernovae exploded in SNR cavities. They are also summarized in Appendix A.

3 A four-component model of the ISM

In the first place, we examine the time variation of the ISM when MC and GMC are not taken into account. The interchange processes among the other four components are mainly driven by SNRs, and their typical time-scales are 10^6 yr, while the time-scales associated with molecular clouds such as cloud–cloud collisions, star formation and erosion are $\sim 10^{7-8}$ yr. Therefore, as long as we are concerned with the evolution of the ISM within a time-scale of $\sim 10^7$ yr, the principal network of interchange processes is restricted to four components and we set $\alpha_{\rm cc} = \alpha_{\rm cc}^{\rm M} = \alpha_{\rm SF} = \alpha_{\rm e} = 0$. Furthermore, within this time-scale we can set the supernova rate S and the total mass of the four components $\overline{M} = \Sigma_{\rm i} M_{\rm i}$ (i = w, a, h, sc) to be constant. This model is similar to that studied in Paper I, but here we examine a more extended range of parameters.

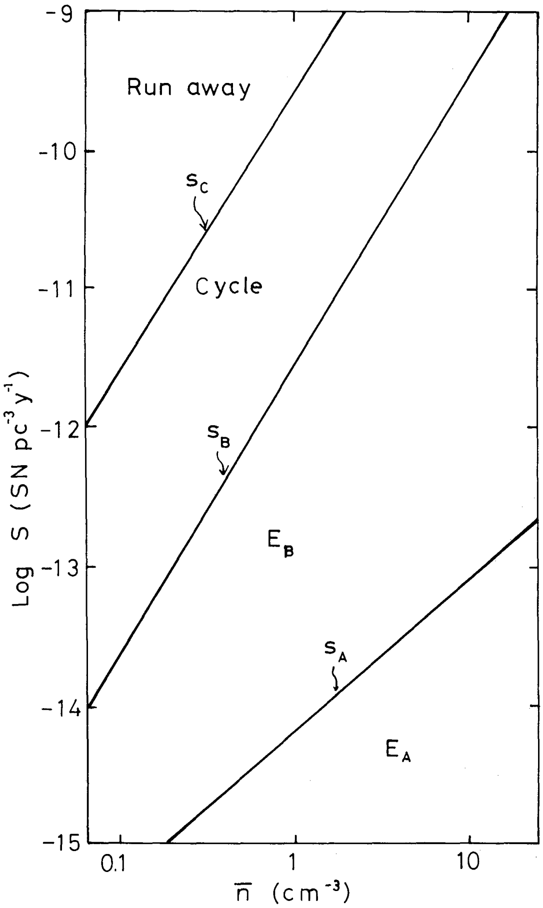

Figure 4. Four different types of evolutionary or steady-state ISM in the plan of supernova rate S and total gas density $\bar{n}$. The dependence of critical supernova rates S_A, S_B and S_C upon the density is described in the text.

The parameters which characterize the regulation processes of the ISM are the supernova rate S, the total mass $\bar{M}$ and the ionization rate ζ of clouds. In place of $\bar{M}$, we take the density $\bar{n}$ divided by an appropriate volume V. As initial conditions, we take the simple ones $n_a = \bar{n}$, $n_w = n_h = 0$, $T_a = 10^4$ K and $n_{sc} = 0$.

As shown in Fig. 4, we find four types of ISM depending upon the parameters $\bar{n}$ and S. The evolutionary characteristics of each type can be summarized as follows:

(a) E_A-type: when the total gas density is high and/or the supernova rate is so small that $S < S_A$, the steady state of the ISM is promptly reached in region A. This type is expected when the sweeping-up of gas is less effective than the supply of gas by ionization. The warm gas therefore always prevails in interstellar space, so that this E_A-type steady state is similar to the two-phase model (Field 1965).

(b) E_B-type: with an increase of the supernova rate to $S_A < S < S_B$ and/or a decrease of the gas density, the volume occupied by young SNRs increases and the gas-sweeping rate exceeds the rate of supply of gas by ionization. The density of the ambient medium then decreases rapidly and its temperature increases because of an efficient supply of hot gas from SNR cavities. The steady state is realized near the boundary between regions B and E, in which the gas-sweeping balances the supply of gas from the evaporation of clouds. This E_B-type steady state agrees well with the three-phase model proposed by McKee & Ostriker (1977).

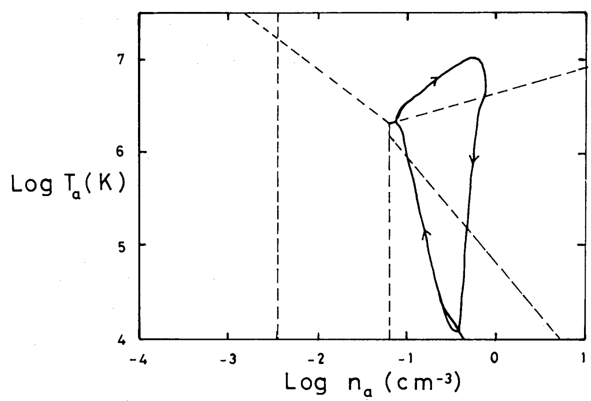

Figure 5. A typical limit-cycle orbit in the n_a–T_a plane for the case $\bar{n} = 1\ \mathrm{cm}^{-3}$ and $S = 2.0 \times 10^{-11}$ SN pc^{-3} yr^{-1}. The dashed lines indicate the regions of ambient matter in which SNRs have different fates, as in Fig. 2.

(c) Cyclic type: with a further increase of the supernova rate to $S_B < S < S_C$, the ambient gas is heated to region C or E, in which the clouds are not formed but continue to be evaporated. The density of the ambient medium therefore increases and hot gas with a temperature $\sim 10^6$ K occupies the whole volume. If $S < S_C$, the radiative cooling of the ambient medium becomes more efficient than shock heating by supernovae, and the ambient gas begins to cool at the stage when almost all the clouds are evaporated. Before long, the interstellar medium returns to region A and small clouds are newly formed. In this way, the phase of the ISM changes cyclically as region A → B → C(E) → A → B →. . . .. In Fig. 5, we illustrate the track in the n_a–T_a plane. This cyclic phase change is essentially the same as the limit cycle found in chemically reacting systems and biological systems (Nicolis & Prigogine 1977). Even if the starting point is set far from the track of a limit cycle, the actual track in the n_a–T_a plane rapidly converges to a cyclic track, the amplitude and frequency of which depend upon the supernova rate S and the gas density $\bar{n}$ (Ikeuchi & Tomita 1983).

(d) Runaway type: when the supernova rate is so high ($S_C < S$) that the radiative cooling is ignorable in region C, the ambient medium is heated further and all of the ISM is converted to a hot gas. Finally, this gas will escape from the galaxy. Such a possibility was discussed in the study of an early explosive era of a galaxy by Ikeuchi (1977).

These features of evolution or steady state of the ISM are well represented by simple model equations, and some types of non-linear behaviour have been discussed in another paper (Ikeuchi & Tomita 1983).

The dependence of the critical supernova rates S_A, S_B and S_C upon the gas density $\bar{n}$ can be roughly estimated by comparing the respective time-scales of interchange processes. Simple expressions for these are

sweeping time $\quad \tau_s = (R_f^3 S)^{-1}, \qquad (3.1)$

ionization time $\quad \tau_{\mathrm{ion}} = \zeta^{-1}, \qquad (3.2)$

evaporation time $\quad \tau_{\mathrm{evp}} = (R_{\mathrm{evp}}^3 S)^{-1}, \qquad (3.3)$

cooling time $\quad \tau_{\mathrm{cool}} = 3\bar{n}kTL^{-1}(\bar{n}, T), \qquad (3.4)$

heating time $\quad \tau_{\mathrm{heat}} = 3\bar{n}kTH_{\mathrm{SN}}^{-1}, \qquad (3.5)$

where equations (3.4) and (3.5) are defined at region C. Here, R_f and R_{evp} are, respectively, the final radii at the snowplough stage and at the evaporation-dominated one. Together with the cooling function L and the heating function H_{SN}, they are summarized in Appendices A and B. Each critical supernova rate is estimated as

$$\tau_{\mathrm{s}} \simeq \tau_{\mathrm{ion}} \rightarrow S_{\mathrm{A}} \simeq \zeta/R_{\mathrm{f}}^{3} \propto \bar{n} f_{\mathrm{A}}(T_{\mathrm{a}}), \tag{3.6}$$

$$\tau_{\mathrm{s}} \simeq \tau_{\mathrm{evp}} \simeq \tau_{\mathrm{heat}} \rightarrow S_{\mathrm{B}} \simeq (3\bar{n}kT/H_{\mathrm{SN}})^{-1}/R_{\mathrm{evp}}^{3} \propto \bar{n}^{2}, \tag{3.7}$$

$$\tau_{\mathrm{heat}} \simeq \tau_{\mathrm{cool}} \rightarrow S_{\mathrm{C}} \propto \bar{n}^{2} f_{\mathrm{c}}(T_{\mathrm{a}}). \tag{3.8}$$

In equation (3.7) the relation between n_{a} and $\bar{n}\,(\simeq N_{\mathrm{sc}} m_{\mathrm{c}})$ is determined from the condition $\tau_{\mathrm{s}} \simeq \tau_{\mathrm{evp}}$ and then the condition $\tau_{\mathrm{evp}} \simeq \tau_{\mathrm{heat}}$ is used to obtain the $\bar{n}^{2}$ dependence. The numerical results in Fig. 4 correspond to $T_{\mathrm{a}} = 8000$ K in (3.6) and 10^{6} K in (3.8).

This change of a steady state and/or an oscillating system due to the external parameters S and $\bar{n}$ is analogous to the bifurcation theory of a non-linear, non-equilibrium system. The transition from the E_{A} to E_{B} steady state suggests the existence of two separated, stable states, and the bifurcation of E_{B} to a limit-cycle state means the destabilization of the E_{B} state. In other words, this ISM system is similar to an excitable chemically reacting system. It is therefore important to analyse the ISM system from a viewpoint such as the bifurcation theory.

4 A six-component model of the ISM

4.1 INTRODUCTION

By including the formation of molecular clouds and the birth of stars from them, we can calculate the time variation of the total gas density $\bar{n}$ and supernova rate S. As a result we can follow the evolution of the ISM over a time-scale $\sim 10^{9}$ yr. If we also consider the change of the mass spectrum of stars, we can calculate the evolution of galaxies over a time-scale $\sim 10^{10}$ yr (Tanaka, Habe & Ikeuchi, in preparation). Since we want to make clear the role of molecular clouds in the star-formation process, the interchange processes with time-scales $\lesssim 10^{9}$ yr are examined in the present paper.

The rate of supernova explosions S is described as

$$S = \int_{m_1}^{m_2} \alpha_{\mathrm{SF}}(t - \tau_{\mathrm{m}})\, B(m)\, dm + S_0, \tag{4.1}$$

where τ_{m} is the lifetime of a star with mass m, and $B(m)$ is the initial mass function. The first term corresponds to the supernova rate of newly born stars from giant molecular clouds, and the second term S_0 to the supernova rate of old stars, which is assumed to be $S_0 = 3.5 \times 10^{-13} \exp(-R/4\,\mathrm{kpc})$ $SN\,\mathrm{pc}^{-3}\,\mathrm{yr}^{-1}$, R being the radial distance from the centre. The uppper and lower mass-limits of newly born stars are taken as $m_{\mathrm{u}} = 60\,M_{\odot}$ and $m_{\mathrm{l}} = 0.1\,M_{\odot}$, while those of exploding stars are taken as $m_2 = 30\,M_{\odot}$ and $m_1 = 5\,M_{\odot}$, respectively. Salpeter's mass function $B(m) \propto m^{-2.35}$ is always assumed and the lifetime of each star is estimated from Larson (1974) as

$$\log \tau(m) = 10.02 - 3.57 \log m + 0.90 (\log m)^{2}, \tag{4.2}$$

where the stellar mass m is expressed in terms of the solar mass and τ_{m} is in yr.

For the initial conditions we take $n_{\mathrm{a}} = \bar{n}/4$, $n_{\mathrm{sc}} = M_{\mathrm{sc}}/V = \bar{n}/4$, $\bar{n}_{\mathrm{gmc}} = M_{\mathrm{gmc}}/V = \bar{n}/2$ and $n_{\mathrm{w}} = n_{\mathrm{h}} = n_{\mathrm{mc}} = 0$. It is seen later that the results are independent of the initial conditions. For the average total gas density $\bar{n}$ at each radius in our Galaxy, we take values estimated by Solomon, Sanders & Scoville (1979). Other parameters such as the epicyclic frequency κ and the average scale-height of the gas distribution H_{mc} are summarized in Table 1 (Cowie 1981).

Table 1. Adopted parameters for our Galaxy.

R (kpc)	(km s^{-1} kpc^{-1})	H_{mc} (pc)	$\bar{n}$ (cm^{-3})	Σ_{crit} (10^{20} g cm^{-2})
2	128.2	50	1.87–2.5	20.5
3	85	50	1.56–1.87	13.6
5	73	50	3.7 –10.1	11.3
7	49	50	2.5 –6.5	7.6
10	26	100	0.9 –1.8	4.04
12	15	150	0.6 –0.7	2.33

4.2 MODEL FOR THE SOLAR NEIGHBOURHOOD

The time variations of the masses of the six components are illustrated in Fig. 6. A noticeable feature is that the masses of molecular clouds, giant molecular clouds and the warm gas show a periodic change with period $\sim 1.3 \times 10^8$ yr. This comes about because gravitational instability of a cloud ensemble occurs when the condition (B.23) is satisfied. The giant molecular clouds thus formed are then destroyed by star formation and expanding H II regions in a time $\sim 10 R_{gmc}/C_I \sim 3 \times 10^7$ yr. Before long, supernova explosions of newly born stars begin to occur and the supernova rate increases as also shown in Fig. 6. This active phase continues for $\sim 10^8$ yr, which corresponds to the lifetime of a $5 M_\odot$ star, the lower mass limit for a supernova. During this stage, the warm gas supplied from the giant molecular clouds is converted to small clouds. With a time-scale $\alpha_{cc}^{-1} \sim 1.3 \times 10^8$ yr, molecular clouds are formed by mutual collisions of small clouds and the next instability occurs. In this way, the ISM changes periodically, and the active phase ($\sim 10^8$ yr) and quiet phase ($\sim 3 \times 10^7$yr) arise in turn. These intermittent changes of the ISM continue until the total gas mass is less than a critical one below which the gravitational instability of a cloud ensemble does not occur. In the present model, we assume that about 4 per cent of the mass of giant molecular clouds is converted to stars ($f_s = 0.04$). Then, within about (2–3) $\times 10^9$ yr the total gas density decreases to $\bar{n}_{crit} \sim 0.7\,\text{cm}^{-3}$ and the ISM evolution becomes monotonic, as described later.

It may be noted that the mass of small clouds does not show such a distinct change. This is because the total mass of small clouds is sufficiently higher than Σ_{crit}, above which

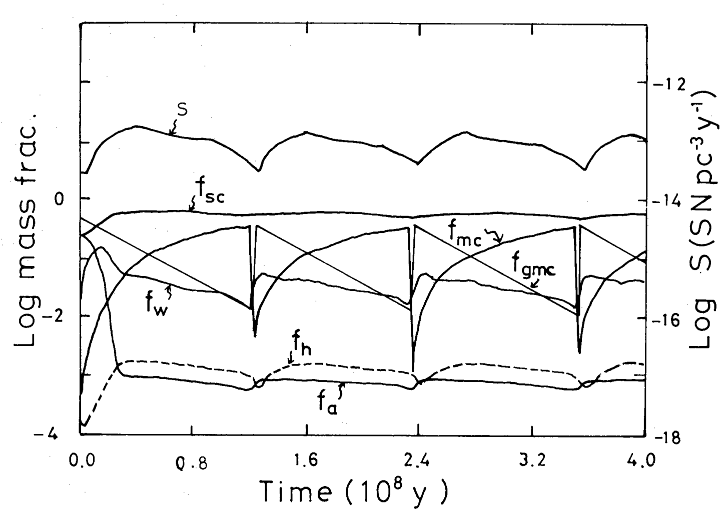

Figure 6. Time variations of supernova rate S and mass fraction of each component $f_i \equiv M_i/\Sigma M_i$ (i = sc, mc, gmc, h, w, a) for a model at the solar neighbourhood ($\bar{n} = 1.7$ cm^{-3}). A cyclic change with the period $\sim 1.2 \times 10^8$ yr is clearly seen.

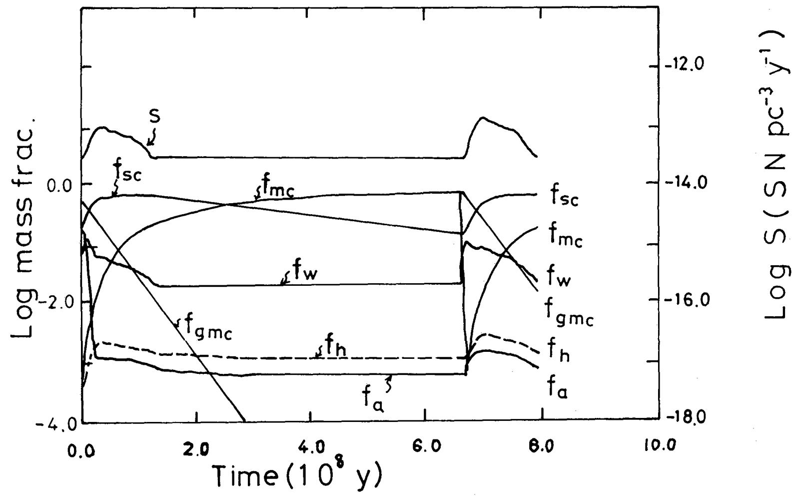

Figure 7. The same as in Fig. 6, but for the total gas density $\bar{n}$ = 0.9 cm^{-3} at the solar neighbourhood (Burton & Gordon 1978). A cyclic change with the period 6.7 × 10^7 yr is seen, but five-sixths of each cycle are nearly in a stationary state.

giant molecular clouds are formed. Therefore, it can be said that the abundance of H I clouds does not correspond directly to the star-formation rate.

This periodic change of the ISM suggests two interesting possibilities for the evolution and structure of a galaxy. One is that the star-formation process is not monotonic, but burst-like. Some observations indicate that the rate of star formation can be considerably different in galaxies with similar gas abundances, and we may suppose that intermittent star-formation is occurring in these galaxies. The other is that the spiral structures of young stars, H II regions and giant molecular clouds are connected with the periodic change of the ISM, i.e., the propagation of star-forming regions corresponds to spiral structures (Seiden & Gelora 1979; Cowie & Rybicki 1982). These possibilities should be explored further.

The criterion (B.23) of gravitational instability depends sensitively upon the total gas density. On the other hand, there is a controversy concerning the radial distribution of interstellar gas. Burton & Gordon (1978) have proposed a more gently sloping distribution than have Solomon *et al.* (1979), in which the total gas density at $R = 10$ kpc is estimated as $\bar{n} = 0.9\,\mathrm{cm}^{-3}$. In this case, the time-scale for the gravitational instability of a cloud ensemble to occur is too large, because $n_{\mathrm{crit}} = 0.7\,\mathrm{cm}^{-3}$ ($\lesssim \bar{n}$) and the ISM stays in the stationary state for a long time ($\sim 6.7 \times 10^8$ yr), as shown in Fig. 7. Since the giant molecular clouds are exhausted at an early stage ($\sim 10^7$ yr) of this stationary state, the supernova explosion rate becomes constant. This stationary state is therefore similar to the E_{B}-type in Section 3. In comparison with the abundance of giant molecular clouds detected in the solar neighbourhood, this model seems to be insufficient and we must seek other mechanisms for the formation of giant molecular clouds.

4.3 RADIAL CHANGE OF THE ISM IN OUR GALAXY

In our Galaxy, the epicyclic frequency κ increases towards the centre, but the total gas density $\bar{n}$ does not increase monotonically. The evolutionary characteristics of the ISM therefore vary with the radial distance R. We have examined several models as described in Table 1 and the numerical results are summarized in Table 2.

Table 2. Numerical results of six-component model.

R (kpc)	$\bar{n}$ (cm^{-3})	$\langle S \rangle$ (10^{-13} SN pc^{-3} yr^{-1})	τ_{cyc}† (10^8 yr)	$\langle V_h \rangle$★	HG $\langle T_h \rangle$ (10^5 K)	$\langle n_h \rangle$ (cm^{-3})	$\langle V_a \rangle$★	AM $\langle T_a \rangle$ (10^4 K)	$\langle n_a \rangle$ (cm^{-3})	SC $\langle f_{sc} \rangle$	MC $\langle f_{mc} \rangle$	GMC $\langle f_{gmc} \rangle$
5	10.1	11.0	0.52	0.77	6.0	9.5E-3	0.21	1.4	1.0	0.36	0.17	0.27
7	6.5	5.6	0.65	0.78	4.9	6.8E-3	0.20	1.4	0.5	0.41	0.18	0.23
10	1.8	0.85	1.3	0.64	2.3	2.3E-3	0.34	0.96	0.12	0.53	0.18	0.10
12	0.7	0.29	2.8	0.79	2.6	1.1E-3	0.18	1.1	0.09	0.58	0.14	0.07
10	0.9‡	0.35	6.6	0.61	2.8	1.7E-3	0.37	0.95	0.04	0.40	0.48	0.03
12	0.6‡	0.22	4.0	0.78	2.8	9.6E-4	0.20	1.1	0.09	0.56	0.21	0.04

★ V_h and V_a are the fractions of the volumes occupied by HG and AM. The bracket means they are time averaged over a long time.
† τ_{cyc} is the time between the bursts of star formation.
‡ These are the values estimated by Burton & Gordon (1978).

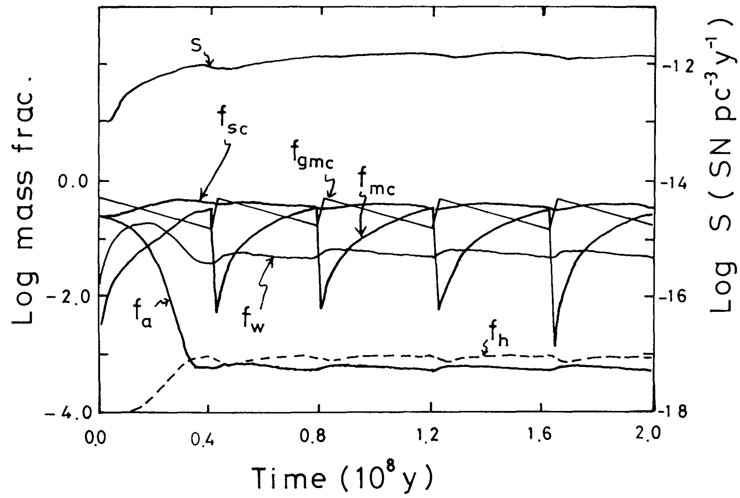

Figure 8. The same as in Fig. 6, but for a model at $R = 5$ kpc ($\bar{n} = 10.1\,\mathrm{cm}^{-3}$). A short-period ($4 \times 10^7$ yr) and small-amplitude cycle is seen. The supernova rate hardly depends upon the time.

In Fig. 8, we illustrate the time variations of the masses of the various components at $R = 5$ kpc ($\bar{n} = 10.1\,\mathrm{cm}^{-3}$), a region corresponding to the molecular ring. As above, the ISM changes periodically with a relatively small amplitude and short period. The gas density is so high that the molecular clouds are rapidly ($\tau_{cc} \sim 5 \times 10^7$ yr) formed from small clouds and their gravitational instability follows. On the other hand, a fraction $\exp(-\tau_{cc}/\tau_e) \sim 0.19$ of giant molecular clouds are not exhausted in this time τ_{cc}, and they are formed later. As a result, a cyclic change of small amplitude and short period occurs. The supernova explosion rate is high and almost constant in time, because the lifetime of exploding stars ($\lesssim 10^8$ yr) is longer than the period τ_{cc}. In Table 2, the values averaged over one period are summarized. As can be seen, the interstellar space is almost occupied by supernova remnants. In this region, the condition of gravitational instability cannot be satisfied if the total gas density decreases below $\bar{n}_{crit} \simeq 3.6\,\mathrm{cm}^{-3}$. This periodic change of the ISM therefore continues for $\sim 1.3 \times 10^9$ yr, unless the gas supply from stars is considered.

Within a radius ~ 4 kpc from the galactic centre, the gas density is very small (Burton & Gordon 1978; Solomon *et al.* 1979). Since the epicyclic frequency increases there, gravitational instability of a cloud system does not occur and the ISM rapidly reaches a stationary state similar to the low-density case in the solar neighbourhood.

On the other hand, in the outer part ($R > 10$ kpc) the total gas density is small ($\bar{n} \lesssim 1\,\mathrm{cm}^{-3}$) but the epicyclic frequency also decreases rapidly. Thus a long period and large-amplitude change of the ISM is expected, as shown in Fig. 9 for $R = 12$ kpc. After almost all giant molecular clouds are exhausted, they are produced from molecular clouds by means of gravitational instabilities. In this regime, the mean growth time of giant molecular clouds is $\sim 2.8 \times 10^8$ yr, which corresponds to the period of a cycle.

In the present paper, we make no attempt to explain why such a gas distribution occurs in the Galaxy, although this seems to be closely connected with the features of evolution of the ISM which have been discussed.

5 Conclusions

In Paper I, we have shown that the ISM attains a steady state within the time-scale $\sim 10^7$ yr. This result confirms the treatment of Brand & Heathcote (1982) on the relation between the spiral shock wave and the state of the ISM. In the present paper, two interesting results are found.

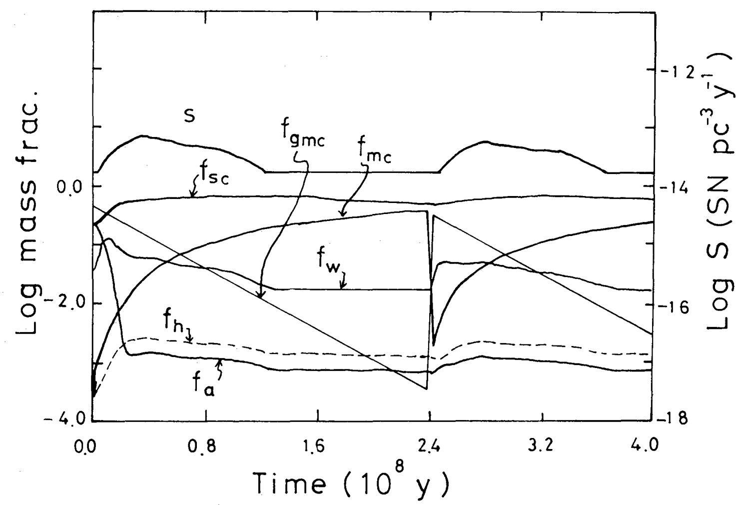

Figure 9. The same as in Fig. 7, but for a model at $R = 12$ kpc ($\bar{n} = 0.7$ cm^{-3}). A long-period (2.4×10^8 yr) and large-amplitude cycle is seen.

One is that the ISM does not always attain a steady state, but a cyclically changing state or a runaway state may be expected, depending upon the total gas density and the supernova explosion rate. It seems that the cyclic model may be applicable to dwarf galaxies, which frequently show peculiarities in their star-formation history. Seiden, Shulman & Feitzinger (1982) proposed a model for dwarf galaxies in which burst-like star formation occurs intermittently. Although their fundamental ideas are different from ours, some similarity is suggested in the sense that a new time-scale characterizing the evolution of the ISM appears, different from the dynamical one, and it plays an important role in the star-formation process. In the model by Seiden *et al.* (1982), it is the time in which the star-formation front propagates over a galaxy. In the present model, the period of the limit cycle characterizes the physical properties of a galaxy. As shown in another paper (Ikeuchi & Tomita 1983), the period and amplitude of a limit cycle of phase change are prolonged and enlarged for a slowly evolving galaxy in which the supernova explosion rate is small and the average gas density is low.

The other interesting result is that the formation of giant molecular clouds and stars may possibly be intermittent. This result comes from the assumption that giant molecular clouds are formed by the gravitational instability of a cloud ensemble (Cowie 1981). If

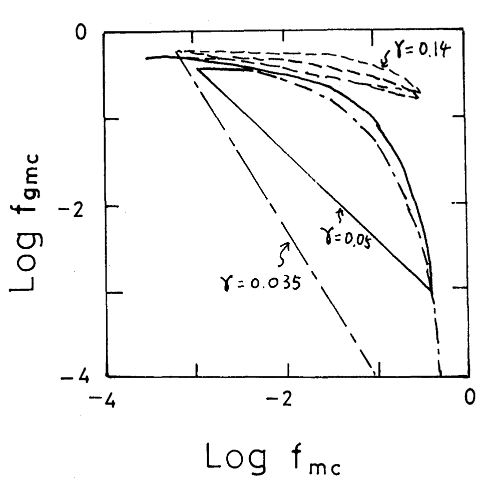

Figure 10. The cyclic orbits in the $f_{\rm gmc}$–$f_{\rm mc}$ plane for two cases of $\gamma = \bar{n}/\kappa = 0.035$, 0.05 and 0.14. With a decrease of the parameter γ, the ampltiude of a cycle increases.

there are other mechanisms more efficient than gravitational instability, this result must be modified. Also in this case, a cyclic change of cloud abundances occurs, and its amplitude and period depend upon the ratio $\gamma = \bar{n}/\kappa$. In Fig. 10, we illustrate the masses M_{mc} and M_{gmc} for $\gamma = 0.14$, 0.05 and $0.035\,\mathrm{cm}^{-3}\,\mathrm{km}^{-1}\,\mathrm{s\,kpc}$. With increasing γ, the amplitude and period of a cycle become smaller and shorter, so that giant molecular clouds are present every time and star formation is active. We may suppose that this case corresponds to the molecular ring. With decreasing γ, the amplitude and period of a cycle are enlarged and prolonged, respectively. There appear an active phase and a quiet phase of star formation in turn. This case is considered to be one of burst-like star formation. The period of a cycle is just the growth time of a cloud mass through mutual collisions, $\tau_{\mathrm{cc}} \simeq \alpha_{\mathrm{cc}}^{-1} \propto \bar{n}$. With a further decrease of γ, gravitational instability of a cloud ensemble does not occur, and the ISM rapidly reduces to a steady state.

At present, this picture of the ISM cannot directly be applied to a real galaxy. By calculating observable quantities such as the mean electron number $\langle n_{\mathrm{e}} \rangle$, the diffuse X-ray flux $\langle I_{\mathrm{X}} \rangle$, the expected absorption lines, etc, we may check the model. Such calculations will be described elsewhere.

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References

Brand, P. W. J. L. & Heathcote, S. R., 1982. *Mon. Not. R. astr. Soc.*, **198**, 545.
Burton, W. B. & Gordon, M. A., 1978. *Astr. Astrophys.*, **63**, 7.
Chevalier, R. A., 1974. *Astrophys. J.*, **188**, 501.
Cowie, L. L., 1980. *Astrophys. J.*, **236**, 863.
Cowie, L. L., 1981. *Astrophys. J.*, **245**, 66.
Cowie, L. L. & Rybicki, G. B., 1982. *Astrophys. J.*, **260**, 504.
Elmegreen, B. G. & Lada, C., 1977. *Astrophys. J.*, **214**, 725.
Field, G. B., 1965. *Astrophys. J.*, **142**, 531.
Habe, A., Ikeuchi, S. & Tanaka, Y. D., 1981. *Publ. astr. Soc. Japan*, **33**, 23.
Ikeuchi, S., 1977. *Prog. Theor. Physics*, **58**, 1742.
Ikeuchi, S. & Tomita, H., 1983. *Publ. astr. Soc. Japan*, **35**, 56.
Larson, R. B., 1974. *Mon. Not. R. astr. Soc.*, **166**, 585.
McKee, C. F. & Cowie, L. L., 1977. *Astrophys. J.*, **215**, 213.
McKee, C. F. & Ostriker, J. P., 1977. *Astrophys. J.*, **218**, 148.
Nicolis, G. & Prigogine, I., 1977. In *Self-Organization in Non-Equilibrium Systems*, p. 110, John Wiley, New York.
Raymond, J. C., Cox, D. P. & Smith, B. W., 1976. *Astrophys. J.*, **204**, 290.
Sedov, L. I., 1959. In *Similarity and Dimensional Method in Mechanics*, p. 210, Academic Press, New York.
Seiden, P. E. & Gerola, H., 1979. *Astrophys. J.*, **233**, 56.
Seiden, P. E., Shulman, L. S. & Feitzinger, J. V., 1982. *Astrophys. J.*, **253**, 91.
Solomon, P. M., Sanders, D. B. & Scoville, M. Z., 1979. In *Large Scale Characteristics of the Galaxy, IAU Symp. 84*, p. 35, ed. Burton, W. B., Reidel, Dordrecht, Holland.
Spitzer, L., 1978. In *Physical Processes in Interstellar Medium*, John Wiley & Sons, New York.
Whitworth, A., 1979. *Mon. Not. R. astr. Soc.*, **186**, 59.
Woltjer, L., 1972. *A. Rev. astr. Astrophys.*, **10**, 129.

Appendix A: Expansion law of a supernova remnant

As shown in Paper I, the structure and expansion law of a supernova remnant (SNR) are expressed in terms of the explosion energy E_0, the density of the ambient medium n_a, the space density of evaporating clouds N_{sc}, the cloud size a, and the age of an SNR τ. In Table 3, the radius R_s of the shock front, the temperature T_h and the density n_h of an SNR cavity are summarized, where we assume $E_0 = 10^{51}$ erg and the fraction $\alpha_{th} = 0.72$ of it to be retained in the cavity. The transition time is expressed as

$$\tau_1 = 3.93 \times 10^4 n_a^{-1} N_{sc}^{1/2} a^{1/2} \text{ yr}, \tag{A.1}$$

$$\tau_2 = 2.09 \times 10^4 n_a^{-0.56} \text{ yr}, \tag{A.2}$$

$$\tau_3 = 1.09 \times 10^9 T_a^{-0.70} n_a^{-0.37} \text{ yr}, \tag{A.3}$$

where N_{sc} and a are measured in units of pc^{-3} and pc. In these expressions as well as those of Table 3, the results by Sedov (1959), Woltjer (1972), Chevalier (1974), McKee & Cowie (1977) and McKee & Ostriker (1977) are adopted. The cooling function L in erg cm^{-3} s^{-1} and the classical evaporation rate Ξ in g s^{-1} are taken (Raymond, Cox & Smith 1976; McKee & Cowie 1977) as

$$L = 1.59 \times 10^{-24} n^2 T_6^{1/2}, \qquad T > 4 \times 10^7 \text{ K}, \tag{A.4}$$

$$L = 1.99 \times 10^{-22} n^2 T_6^{-1}, \qquad 2 \times 10^5 \text{ K} < T < 4 \times 10^7 \text{ K}, \tag{A.5}$$

$$L = 1.11 \times 10^{-20} n^2 T_6^{3/2}, \qquad 10^4 \text{ K} < T < 2 \times 10^5 \text{ K} \tag{A.6}$$

and

$$\Xi = 2.75 \times 10^{19} T_6^{5/2} a, \tag{A.7}$$

with $T_6 = T/10^6$ K.

As indicated in the text, not all of the SNRs follow the above expansion laws. One reason is that mutual overlapping of SNRs may occur before τ_i ($i = 1, 2, 3$). As a criterion of overlapping, we calculate the porosity P defined as

$$P = \int_0^{\tau_0} \frac{4\pi}{3} R_s^3(\tau')\, S(t - \tau')\, d\tau'. \tag{A.8}$$

If $P > 1$ at $\tau_0 < \tau_i$ ($i = 1, 2, 3$), SNRs will be destroyed by mutual collisons. The regions B, D and F in Fig. 2 are determined from the conditions

$$\tau_2 < \tau_0 < \tau_3 \qquad \text{region B}, \tag{A.9}$$

$$\tau_1 < \tau_0 < \tau_2 \qquad \text{region D}, \tag{A.10}$$

$$\tau_0 < \tau_1 \qquad \text{region F}. \tag{A.11}$$

Another case is that the gas pressure behind the shock front of an SNR becomes smaller than the pressure of the ambient medium before τ_i ($i = 1, 2, 3$), which may be expected when the ambient temperature is fairly high. Defining the pressure-equilibrium time as τ_p, the regions A, C and E in Fig. 2 are determined as

$$\tau_2 < \tau_p \leqslant \tau_3 \qquad \text{region A}, \tag{A.12}$$

$$\tau_1 < \tau_p < \tau_2 \qquad \text{region C}, \tag{A.13}$$

$$\tau_p < \tau_1 \qquad \text{region E}. \tag{A.14}$$

Table 3. Expansion laws of SNRs.

Stage	R_s (pc)	T_h (K)	n_h (cm^{-3})
I ($0<t<\tau_1$)	$0.025\,E_{51}^{0.1}(N_{sc}a)^{-0.1}\,t^{0.6}$	$8.25\times10^9\,E_{51}^{0.2}(N_{sc}a)^{-0.2}\,t^{-0.8}$	$1.05\times10^5\,E^{0.5}(N_{sc}a)^{0.5}\,t^{-1}$
II ($\tau_1<t<\tau_2$)	$0.32\,E_{51}^{0.2}\,n_a^{-0.2}\,t^{0.4}$	$4.11\,E_{51}^{0.4}\,n_a^{-0.4}\,t^{-1.2}$	n_a
III ($\tau_2<t<\tau_3$)	$1.13\,E_{51}^{115/511}\,n_a^{-135/511}\,t^{2/7}$	$2.82\times10^8\,E_{51}^{134/511}\,n_a^{-24/511}\,t^{-4/7}$	$2.2\times10^3\,E_{51}^{76/511}\,n_a^{265/511}\,t^{-6/7}$

E_{51} is the expansion energy in units of 10^{51} erg, and t and N_{sc} are expressed in yr and pc^{-3}.

Finally, we comment briefly on the expansion of an SNR exploded inside a pre-existing SNR. The expansion law of the shock front is obtained if we replace n_a and T_a by n_h and T_h in Table 1. The shock heating rate H_{SN} in equation (2.8) is simply assumed to be

$$H_{SN} = (V_h/V)\,SE_0, \tag{A.15}$$

where (V_h/V) means the fraction of the volume occupied by SNRs and is defined in Appendix B.

Appendix B: Interchange rates

In the first place, we divide the volume into three parts as follows:

$$V = V_w + V_a + V_h, \tag{B.1}$$

where V_w, V_a and V_h are the volume occupied by warm gas, ambient medium and SNRs, respectively. The volume V_w is defined as

$$V_w = M_w/\rho_w, \tag{B.2}$$

and the density ρ_w is determined by the pressure balance with the ambient medium

$$\rho_w = \rho_a T_a/T_w = \rho_a T_a/8000\text{ K}. \tag{B.3}$$

The density of the ambient medium is determined as

$$\rho_a = M_a/V_a = M_a/(V - V_w - V_h) \tag{B.4}$$

and its temperature is calculated from the energy equation (2.7). The volume V_h is calculated from

$$\frac{dV_h}{dt} = V_{aw}\int_0^{\tau_f} 4\pi R_s^2(\tau)\,S(t-\tau)\,v_s(\tau)\,d\tau - V_h/\tau_f. \tag{B.5}$$

Here, the volume V_{aw} is defined as

$$V_{aw} \equiv V_a + V_w = V - V_h, \tag{B.6}$$

and the mean lifetime of SNRs, τ_f, is taken as

$$\tau_f = \text{minimum}\,(\tau_3, \tau_0, \tau_p). \tag{B.7}$$

Each interchange rate is described as follows:

(i) α_s^A, α_s^W: sweeping-up rate of an ambient medium and a warm gas by SNRs, whose progenitors exploded in the volume not occupied by pre-existing SNRs. They are given as

$$\begin{pmatrix}\alpha_s^{AH}\\ \alpha_s^{AC}\end{pmatrix} = V_{aw}\int_0^{\tau_f}\dot{M}_s^A(t,\tau)\,S(t-\tau)\begin{pmatrix}\beta_H(\tau)\\ 1-\beta_H(\tau)\end{pmatrix}d\tau, \tag{B.8}$$

and

$$\begin{pmatrix}\alpha_s^{WH}\\ \alpha_s^{WC}\end{pmatrix} = V_{aw}\int_0^{\tau_f} \dot{M}_s^{W}(t,\tau)\, S(t-\tau)\begin{pmatrix}\beta_H(\tau)\\ 1-\beta_H(\tau)\end{pmatrix} d\tau. \tag{B.9}$$

The sweeping-up rate of an SNR with the age τ is written as

$$\dot{M}_s^{(i)}(t,\tau) = 4\pi R_s^2(\tau)\, v_s(\tau)\; V_i\rho_i(t)/V_{aw} \qquad (i = a, w), \tag{B.10}$$

with $v_s(\tau) = dR_s/d\tau$. The fraction of the gas mass converted to a hot gas is given as

$$\begin{aligned}\beta_H(\tau) &= 1 \qquad 0<\tau<\tau_2,\\ &= 0 \qquad \tau_2<\tau.\end{aligned} \tag{B.11}$$

(ii) α_s^H: the sweeping-up rate of hot gas in SNR cavities is given as

$$\alpha_s^H = V_h\int_0^{\tau_f} \dot{M}_s^H(t,\tau)\, S(t-\tau)[1-\beta_H(\tau)]\; d\tau, \tag{B.12}$$

and the sweeping-up rate in a cavity is written as

$$\dot{M}_s^H(t,\tau) = 4\pi R_s^2(\tau)\, v_s(\tau)\, \rho_h(t). \tag{B.13}$$

(iii) α_{evp}: the evaporation rate of cold clouds is given as

$$\alpha_{evp} = \alpha_{evp}^{(1)} + \alpha_{evp}^{(2)}, \tag{B.14}$$

with

$$\begin{pmatrix}\alpha_{evp}^{(1)}\\ \alpha_{evp}^{(2)}\end{pmatrix} = \begin{pmatrix}N_{sc}\, V_a\\ N_{sc}\, V_h\end{pmatrix}\int_0^{\tau_2} 4\pi R_s^3(\tau)\, S(t-\tau)\, \Xi(\tau)\, d\tau/3. \tag{B.15}$$

In this expression, we include the evaporation of clouds at the Sedov stage.

(iv) α_{cc}; α_{cc}^M: for the growth rate of the cloud mass, we take a convenient way as follows. We divide the mass range of clouds into three parts

$$\text{(A)} \qquad m_{sc} \leqslant m_c \leqslant m_1 = 500\, M_\odot, \tag{B.16}$$

$$\text{(B)} \qquad m_1 \leqslant m_c \leqslant m_{mc} = 10^4\, M_\odot, \tag{B.17}$$

$$\text{(C)} \qquad m_{mc} \leqslant m_c \leqslant m_{gmc} = 3\times 10^5\, M_\odot. \tag{B.18}$$

In the mass ranges (A) and (B), the cloud mass increases by mutual encounters. The mean collision time is estimated as

$$\tau_{cc} = (\pi R_c^2 N_c v_c)^{-1}, \tag{B.19}$$

where R_c, N_c and v_c are the radius, the number density and the random velocity in each mass range. The geometrical cross-section πR_c^2 is proportional to $m_c^{2/3}$. For the mass range (A), we assume the random velocity to be constant, $v_c = 10\ \text{km s}^{-1}$ (Spitzer 1978). Then we take

$$\tau_m^A = 1.5(m_1/m_c)^{1/3}\, \tau_{cc} \tag{B.20}$$

with $m_1 = 500\, M_\odot$. For the mass range (B), we assume equipartition of kinetic energy in each mass as $m_c v_c^2$ = constant. Then the mean collision time is written as

$$\tau_m^B = 0.6(m_{mc}/m_1)^{5/6}(m_1/m_c)^{1/3}\, \tau_{cc} \tag{B.21}$$

with $m_{mc} = 10^4 M_\odot$. By means of these time-scales, we define the effective growth rate of *MC* as

$$\alpha_{cc} = (\tau_m^A + \tau_m^B)^{-1}. \tag{B.22}$$

For the formation of giant molecular clouds, we consider the gravitational instability of a cloud ensemble composed of many $10^4 M_\odot$ clouds (Cowie 1980, 1981). The cloud ensemble becomes gravitationally unstable when the Toomre–Goldreich–Lynden Bell criterion is satisfied, i.e.,

$$\Sigma > \Sigma_{crit} \equiv v_{mc}\kappa/\pi G, \tag{B.23}$$

where Σ, v_{mc} and κ are, respectively, the surface mass density of clouds, the velocity dispersion of a cloud ensemble and the epicyclic frequency. The surface mass density of *MC* is calculated as

$$\Sigma = M_{mc}H_{mc}/V, \tag{B.24}$$

where H_{mc} and V are the average scale height and the volume. The velocity dispersion v_{mc} is estimated by the equipartition condition between clouds with masses $500\,M_\odot$ and $10^4\,M_\odot$ as

$$v_{mc} = (500\,M_\odot/10^4\,M_\odot)^{1/2}\, v_c(500\,M_\odot) \simeq 2.2\ \mathrm{km\,s^{-1}}, \tag{B.25}$$

where we take $v_c(500\,M_\odot) = 10\ \mathrm{km\ s^{-1}}$. The adopted values of κ and H_{mc} are summarized in Table 1 in the text.

From the above, the formation rate of giant molecular clouds is described as

$$\alpha_{cc}^M = M_{mc}\delta(t - t_c), \tag{B.26}$$

where t_c is the epoch when the condition (B.23) is satisfied.

(v) α_{ion}: the ionization rate of cloud surfaces irradiated by UV fluxes and cosmic rays is given as

$$\alpha_{ion} = M_{sc}\zeta. \tag{B.27}$$

At present, we assume the ionization rate ζ to be

$$\zeta = 1.2 \times 10^{-15}(S/10^{-13}\,SN\,\mathrm{pc^{-3}\,yr^{-1}})\mathrm{s^{-1}}. \tag{B.28}$$

(vi) α_{mix}: the gas mixing rate at the destruction stage of SNRs is given as

$$\alpha_{mix} = M_h(t)/\tau_f. \tag{B.29}$$

As indicated above, we assume the mean destruction time of an SNR to be equal to its lifetime τ_f defined in (B.7).

(vii) α_{SF}, α_e: the star-formation rate and gas erosion rate in giant molecular clouds are assumed to be

$$\alpha_{SF} = M_{gmc} f_s/\tau_e \qquad \text{and} \qquad \alpha_e = (1 - f_s)\, M_{gmc}/\tau_e. \tag{B.30}$$

The factor f_s denotes the mass fraction of molecular clouds converted to stars, and the remaining fraction $(1 - f_s)$ of the cloud mass returns to a warm gas. The mean lifetime of a giant molecular cloud τ_e is assumed to be equal to the mean erosion time τ_e' (Whitworth 1979) as

$$\tau_e' = 0.36(R_{gmc}/R_{st})^{3/4}(R_{gmc}/C_I), \tag{B.31}$$

where $R_{\rm gmc}$ is the radius of a giant molecular cloud, $R_{\rm st}$ is the Strömgren radius within the giant molecular cloud, and $C_{\rm I}$ is the isothermal sound speed of an H II region. Furthermore, we modify this erosion time so that

$$\begin{aligned} \tau_{\rm e} &= \tau'_{\rm e} && \text{for } N_{\rm gmc}/N_{\rm O} \leqslant 1 \\ &= \tau'_{\rm e}\{N_{\rm gmc}/N_{\rm O}\} && \text{for } N_{\rm gmc}/N_{\rm O} > 1, \end{aligned} \tag{B.32}$$

$$\{N_{\rm gmc}/N_{\rm O}\} = \begin{cases} 1 & \text{for } N_{\rm gmc}/N_{\rm O} \leqslant 1, \\ N_{\rm gmc}/N_{\rm O} & \text{for } N_{\rm gmc}/N_{\rm O} \geqslant 1, \end{cases} \tag{B.33}$$

where $N_{\rm gmc}$ and $N_{\rm O}$ are the space densities of giant molecular clouds and O-type stars. This modification means that the collapse of giant molecular clouds occurs in proportion to the number of O-type stars. Here, we take data for giant molecular clouds as $R_{\rm gmc} = 30$ pc and $m_{\rm gmc} = 3 \times 10^5 M_\odot$, and take the standard mass of O-type stars as $15 M_\odot$ and $N_{\rm gmc}/N_{\rm O}$ as 10.