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論 文 内 容 の 要 旨

Ideal magnetohydrodynamics (MHD) equilibria are subjected to two kind of instabilities, i.e., current-driven instabilities and pressure-driven instabilities. In three-dimensional (3-D) configurations with vacuum magnetic flux surfaces, the equilibria can be obtained without net toroidal current, where the current-driven instabilities become unimportant and only the pressure-driven instabilities need to be intensively studied. The pressure-driven modes consists of interchange modes and ballooning modes, and impose MHD stability β limits. Interchange modes are basically driven by average unfavorable magnetic curvature. Thus these modes localize on mode rational magnetic field lines and are almost constant along these lines. On the other hand, ballooning modes are basically driven by local unfavorable magnetic curvature, so that they localize on unfavorable magnetic curvature region and change along the magnetic field line. Ballooning modes are considered to be more stringent than interchange modes, whose properties have not been clarified in 3-D configurations. To study the properties of ballooning modes, one can proceed in two different ways, namely, local mode analysis and global mode analysis. In axisymmetric systems, the global modes can be constructed easily from the results of the local modes analysis. But this is not the case in non-axisymmetric systems, namely, 3-D systems. In fully 3-D systems, we can only make some conjectures for global modes from the properties of the local modes.

Through the local mode analysis of ballooning modes in an $L = 2/M = 10$ planar axis heliotron system with an inherently large Shafranov shift(where L and M are the polarity and toroidal field period of the helical coils, respectively), it has been demonstrated that [N. Nakajima, Phys. Plasmas **3**, 4545 and 4556(1996)]:

- The local magnetic shear (which is a stabilizing term for high-mode-number ballooning modes) is related to <u>helicity</u> of the helical coils in the considered vacuum configuration. Its change due to a large Shafranov shift is essentially axisymmetric, i.e., related to <u>toroidicity</u>. This change leads to the disappearance of the (integrated) local magnetic shear on the outer side of torus, even in the region with a stellarator-like global magnetic shear, leading to the destabilization of the high-mode-number ballooning modes.

- The local magnetic curvature (which constructs a potentially destabilizing term for high-mode-number ballooning modes together with the pressure gradient) consists of parts due to both <u>toroidicity</u> and <u>helicity</u> of the helical coils, which determines the 3-D properties of the high-mode-number ballooning modes.

In general 3-D MHD equilibria, the eigenvalues ω^2 for high-mode-number ballooning modes are functions of the labels of the flux surface ψ, the magnetic field line α, and the radial wave number θ_k: $\omega^2 = \omega^2(\psi, \theta_k, \alpha)$. Since ω^2 has no α-dependence in axisymmetric systems, the stronger the α-dependence of ω^2 is (mainly coming from the helicity part of the local magnetic curvature), the more significant the 3-D properties of ω^2 are. The topological properties of the unstable eigenvalues $\omega^2(< 0)$ in (ψ, θ_k, α) space for the $L = 2/M = 10$ planar axis heliotron system are shown that [N. Nakajima, Phys. Plasmas **3**, 4556 (1996)]:

- In Mercier unstable equilibria, there coexist two types of topological level surfaces for ω^2 in (ψ, θ_k, α) space. One is a tokamak-like topologically cylindrical level surface with the axis in α direction, The other is a topologically spheroidal level surface inherent to 3-D systems. The topologically spheroidal level surfaces are surrounded by the topologically cylindrical level surfaces. From their relative positional relation, it is clear that modes with spheroidal level surfaces have larger growth rates than those with cylindrical level surfaces.

- In Mercier stable equilibria, only a topologically spheroidal level surface exists. In contrast to Mercier unstable equilibria, this spheroidal level surfaces are surrounded by the level surfaces of stable Toroidicity-induced Alfvén Eigenmodes (TAE).

From these results it is conjectured that the global structure of pressure-driven modes has the following properties [N. Nakajima, Phys. Plasmas **3**, 4556 (1996)]:

- Global modes that correspond to modes in the local mode analysis with a topologically cylindrical level surface will be poloidal localized tokamak-like ballooning modes or interchange modes. Effects of the toroidal mode coupling on these modes are weak.

- Global modes that correspond to modes in the local mode analysis with a topologically spheroidal level surface will be ballooning modes inherent to 3-D systems, with quite high poloidal and toroidal mode numbers and localized in both the poloidal and toroidal directions. These modes become to be localized within each toroidal field period of the helical coils, as their typical toroidal mode numbers become higher.

- In Mercier unstable equilibria, where both topologically cylindrical and spheroidal level surface coexist, tokamak-like ballooning modes or interchange modes appear when their typical toroidal mode numbers are relatively small. As the typical toroidal mode numbers become larger, ballooning modes inherent to 3-D systems appear with larger growth rates.

- In Mercier stable equilibria, where only a topologically spheroidal level surface exists, only ballooning modes inherent to 3-D systems appear.

The purposes of the work are to prove the above conjecture from local mode analysis and to clarify the inherent properties of pressure-driven modes through a global mode analysis in the $L = 2/M = 10$ planar axis heliotron system with an inherently large Shafranov shift [J. Chen, N. Nakajima, and M. Okamoto, Global mode analysis of ideal MHD modes in a heliotron/torsatron system: I. Mercier-unstable equilibria].

First the Mercier-unstable equilibria are categorized into two types, namely, toroidicity-dominant Mercier-unstable equilibria and helicity-dominant Mercier-unstable equilibria. This categorization is motivated by the conjecture that tokamak-like ballooning modes or interchange modes exist for relatively small toroidal mode numbers, and is related to the local properties of Mercier-unstable equilibria brought by Shafranov shift. The properties of the vacuum configuration are understood as a straight helical configuration toroidally bended. Since the aspect ratio is relatively large: $R_0/a = 7 \sim 8$ [here R_0 and a are the major and minor radii, respectively], the global and local properties of the vacuum configuration are mainly determined by helicity of the helical coils. The properties of the finite-β equilibria are basically understood as a modification of the vacuum configuration by an essentially axisymmetric and inherently large Shafranov shift. As the Shafranov shift becomes larger, the stabilizing term due to the local magnetic shear is more reduced. The toroidicity-dominant Mercier-unstable equilibria are characterized by properties that it is easy for the local magnetic shear to vanish on the outer side of torus, which is brought by a relatively large Shafranov shift. In these equilibria, it is relatively easy for ballooning modes to be destabilized. The helicity-dominant Mercier-unstable equilibria are characterized by properties that it is hard for the local magnetic shear to vanish on the outer side of torus, which is brought by a relatively small Shafranov shift. In these equilibria, it is relatively hard for ballooning modes to be destabilized. Note that, in both types of equilibria, the Shafranov shift locally reduces (enhances) the unfavorable normal magnetic curvature on the outside (inside) of torus, which is another local property due to Shafranov shift.

On the basis of these considerations, the following two types of Mercier-unstable equilibria have been adopted. The toroidicity-dominant Mercier-unstable equilibrium is created with a peaked pressure profile $P = P_0(1-\psi_N)^2$ and $\beta_0 = 5.9\%$, under the flux conserving condition, i.e., with a specified profile for the rotational transform. The helicity-dominant Mercier-unstable equilibrium is created with a broad pressure profile $P = P_0(1-\psi_N^2)^2$ and $\beta_0 = 4.0\%$, under the currentless condition.

The global mode analysis are done by CAS3D2MN, a version of CAS3D: Code for Analysis of the MHD Stability of 3-D equilibrium [C. Schwab, Phys. Fluids B **5**, 3195 (1993)]. CAS3D have been designed to analyze the global ideal MHD modes of 3-D equilibria based on a formulation of the ideal MHD energy principle with incompressibility in Boozer coordinate system and the application of Ritz-Galerkin method. In CAS3D2MN, a phase-factor transformation was used in order to save memory and flops.

The inverse iteration with spectral shift is an essential concept in the solution of eigenproblems. It is very efficient if the spectral shift is given to be very close to the desired eigenvalue and the initial vector is chosen to be dominant along the corresponding eigenvector. It is demonstrated in our simulation that convergence will occur after only 3 or 4 steps if the spectral shift itself is a good approximation of the desired eigenvalue and the initial vector has dominant component along the corresponding eigenvector. The problem left to be done is how to guess the spectral shift and give a good initial vector. The spectral shift was calculated by matrix transformation in CAS3D2MN. Since the bandwidth will be destroyed by matrix transformation the resultant memory and flops will be $O(n^2)$ and $O(n^3)$, respectively. It is shown in our work that the use of matrix transformation is unsuitable, not only because it becomes very expensive in the sense of flops and storage if the matrix order extends beyond $10,000$ but also the problem size we can deal with is limited by the available computer resources. Here this problem is solved by using the Lanczos algorithm with no re-orthogonalization which keeps the matrix bandwidth from begin to end. The arithmetic operation mainly come from the matrix-vector multiplies and only 3 recently created Lanczos vectors need to be stored. The resultant memory and flops can be controlled to $O(n)$ and $O(n^2)$ order. This iteration process is accelerated by an shift-and-invert technique. In the new version CAS3D2MNv1, an efficient initial vector generation is also introduced [J. Chen, N. Nakajima, and M. Okamoto, Comput. Phys. Commun., **113**, 1 (1998)].

Since the local magnetic curvature due to helicity has the same period M in the toroidal direction as the toroidal field period of the equilibria, the characteristics of the pressure-driven modes in such Mercier-unstable equilibria dramatically change according to how much the local magnetic shear is reduced (whether the equilibrium is toroidicity-dominant or helicity-dominant) and also according to the relative magnitude of the typical toroidal mode numbers n of the perturbations compared with the toroidal field period M of the equilibria.

In the toroidicity-dominant Mercier-unstable equilibria, the pressure-driven modes change from interchange modes with negligible toroidal mode coupling for low toroidal mode numbers $n < M$, to tokamak-like poloidally localized ballooning modes with weak toroidal mode coupling for moderate toroidal mode numbers $n \sim M$, and finally to both poloidally and toroidally localized ballooning modes purely inherent to three-dimensional systems with strong poloidal and toroidal mode couplings for fairly high toroidal mode numbers $n \gg M$. Strong toroidal mode coupling, in cooperation with the poloidal mode coupling, makes the perturbation localize to flux tubes.

In the helicity-dominant Mercier-unstable equilibria, the pressure-driven modes change from interchange modes, with negligible toroidal mode coupling for $n < M$ or with weak toroidal mode coupling for $n \sim M$, directly to poloidally and toroidally localized ballooning modes purely inherent to three-dimensional systems with strong poloidal and toroidal mode couplings for $n \gg M$.

In the Mercier-unstable equilibria, interchange modes with low toroidal mode numbers $n < M$, experiencing the unfavorable magnetic curvature with its local structure averaged out, occur for both toroidicity-dominant and helicity-dominant equilibria. For fairly high toroidal mode numbers $n \gg M$, the perturbations can feel the fine local structure of the magnetic curvature due to helicity and also the local magnetic shear is reduced more or less in both types of equilibria, and consequently poloidally and toroidally localized ballooning modes inherent to 3-D systems are destabilized for both toroidicity-dominant and helicity-dominant Mercier-unstable equilibria. The situation for moderate toroidal mode numbers $n \sim M$ is different. The local magnetic shear is more reduced in toroidicity-dominant Mercier-unstable equilibria than in helicity-dominant Mercier-unstable equilibria, and also the modes with moderate toroidal mode numbers $n \sim M$ can not feel the local structure of the normal magnetic curvature due to helicity effectively. Thus, tokamak-like poloidally localized ballooning modes with a weak toroidal mode coupling can be easily destabilized for toroidicity-dominant Mercier-unstable equilibria, and interchange modes, driven by the average unfavorable magnetic curvature and not experiencing the effect of toroidal mode coupling, can be destabilized for helicity-dominant Mercier-unstable equilibria. Since the normal magnetic curvature becomes more unfavorable on the inner side than on the outer side of the torus by the Shafranov shift, the interchange modes are localized on the inner side of the torus for both types of equilibria. This type of interchange mode is anti-ballooning with respect to the poloidal mode coupling.

In both types of Mercier-unstable equilibria, the pressure-driven modes, i.e., ballooning modes and interchange modes, become more unstable and more localized both on flux tubes and in the radial direction, and have stronger toroidal mode coupling through the normal magnetic curvature due to helicity, as the typical toroidal mode numbers increase. All of these properties of the pressure-driven modes in two types of Mercier-unstable equilibria are quite consistent with the conjecture from local mode analysis.

論文の審査結果の要旨

磁場閉じ込め核融合プラズマ実験装置における理想ＭＨＤ安定性の解析は、装置の性能の指標の一つである最大到達ベータ値を規定するため、極めて重要な研究対象である。入れ子状の真空磁気面を持つヘリオトロン系においては、トロイダル方向に正味電流を持たない無電流運転が可能であるため、圧力勾配と悪い磁気曲率の積が主要な駆動力となる、圧力駆動型理想ＭＨＤモードが不安定となりやすい。圧力駆動型モードは、磁力線に沿って平均的に悪い磁気曲率によって駆動される交換型モードと、局所的な悪い磁気曲率によって駆動されるバルーニングモードとに大別される。圧力駆動型モードは、高波数モードほど不安定となりやすいため、高波数近似を用いた３次元局所モード解析がグローバルモード解析に先立って実施される。その結果、平衡を２次元化する平均化法等では取り扱えない３次元ＭＨＤ平衡に固有のバルーニングモードの存在が指摘された。しかしながら、３次元平衡の場合、局所モード解析の結果をグローバルモード解析に結びつける数学的手法が確立されていないため、局所モード解析の結果は推定にすぎず、３次元グローバルモード解析による検証が待たれていた。

本論文では、３次元局所モード解析による推定を３次元グローバルモード解析により検証するとともに、交換型モードに対して不安定なメルシェ不安定３次元ＭＨＤ平衡における圧力駆動型モードの性質をかなり系統的に明らかにしている。この種の計算としては世界で初めてのものであり、今後のこの分野の研究の基礎となり得るものである。更に、大型ヘリカル装置（ＬＨＤ）等の実験に対して、基本的なデータベースを与えるものでもある。計算においては、安定性の問題をエネルギー原理に基づいた３次元理想ＭＨＤ安定性解析コード（ＣＡＳ３Ｄ）により実対称帯行列の固有値問題に帰結させている。この固有値問題は極めて大規模で（次数は10^5～10^6程度、メモリーにして、最大16GB）、通常の固有値解析コードでは計算は困難であった。これを解決するため、無直交化ランチョス法を取り入れ、引き続く逆反復法の初期固有ベクトルに乱数を導入するなど、様々な創意工夫をこらしたコードを新しく開発し、現有資源での計算を可能とした。

３次元グローバルモード解析により、メルシェ不安定３次元ＭＨＤ平衡における最も不安定な圧力駆動型不安定モードは、その典型的なトロイダルモード数nと平衡のトロイダル周期数Mとの大小関係により、モードパターンを変えることが明らかにされた。$n < M$に対しては、交換型モードが不安定となり、$n \gg M$に対しては、３次元ＭＨＤ平衡に固有の、ポロイダル及びトロイダル両方向に局在化するバルーニングモードが不安定となる。$n \sim M$に対しては、局所磁気シアによるバルーニングモードの安定化効果がシャフラノフシフトにより消失する度合いに応じて、トカマク的なポロイダル方向にのみ局在化するバルーニングモードが不安定化される平衡と、交換型モードが不安定化される平衡とが存在することが指摘されている。本論文の研究は、交換型モードに対して安定なメルシェ安定３次元ＭＨＤ平衡に対する計算へとつながるものであり、ヘリオトロン型核融合プラズマ実験装置のみならず、環状系プラズマの理想及び抵抗性ＭＨＤ安定性に対する統一的理解に貢献す

る、独創的、且つ、発展性のあるものである。

申請者は、審査委員全員の前で、先ず、提出論文の内容を詳細、且つ、明快に説明した。論文説明の間及び説明後に、博士論文の背景、動機、問題設定とモデル、問題解決の数値的手法、得られた結果とその物理的解釈、独創性、今後の発展等について審査委員から質問がなされた。又、博士論文に関連する核融合・プラズマ物理、及び、数値計算上の技法に対しても口頭試問が行われた。

その結果、本論文は、３次元ヘリオトロン型ＭＨＤ平衡の３次元グローバルモード安定性解析として、独創的、且つ、発展性のある研究であると認められた。更に、申請者は、研究の動機と目的、問題設定と解決法、新しく得た知見とその物理的重要性、独創性、研究の今後の発展等を的確に把握していると判断された。核融合・プラズマ物理、及び、問題解決法として用いた数値計算上の技法に関する知見も十分であると認めら、又、英語力、及び、日本語力も十分であると判断された。

申請者の研究は、数値計算上の技法に関するものは、Comp.Phys.Commun.[Vol.113 (1998) pp.1-9]に掲載済みで、物理に関するものは、Phys.Plasmasに掲載が受諾され、更に、申請者は、両論文の第一著者でもある。

２月４日に開催された論文公開発表会においては、指定時間内に内容を要領よくまとめ、聴衆からの質問にも明確に答えていた。

以上の結果により、審査委員会は試験に合格と判定した。