EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-LEP-TH/89-57

A Program for Computing Beam-Beam Modes

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6 October 1989

Abstract

A computer program is described which calculates the complex eigenvalues of the coherent dipole beam-beam modes. The program is more general than earlier programs. There may be up to 10 bunches in each beam, colliding in up to 20 interaction points. The β function may be different in each beam and each interaction point. The phase advances may be different in each arc and each beam. Each bunch may have a different number of particles. The energies of all bunches may differ at each interaction point. The two beams may have different emittances. The two beams may be vertically separated at each interaction point.

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Contents

1 Introduction

The coherent beam-beam oscillation is a good probe for the beam-beam interaction in storage rings[1]. It can be excited and measured by the usual tune measurement system. The tune shifts are related to horizontal and vertical emittances and luminosities. To use this phenomena in an actual storage ring, however, we should consider all possibly important factors that break the symmetry between beams and bunches. The asymmetry may be present spontaneously[2,3]. We propose a new Fortran program BBMODE to calculate observable tune shifts. It is a generalization of the BBMTRX[4] code. In the BBMODE code,

1. there may be up to 10 bunches in each beam.

2. the β function may be different in each beam and each interaction point IP.

3. the phase advances may be different in each arc and each beam.

4. each bunch may have a different number of particles.

5. the energies of all bunches may differ at each IP.

6. the two beams may have different emittances.

7. the two beams may be vertically separated at each IP.

In the next section, we give the basic theory. In Sect. 3, we show the details of the computation. Appendix A is a user's guide for the BBMODE program.

2 Theory

2.1 Beam-Beam Force

When two beams are in collision, a particle in a bunch receives a kick

$$\delta(x', y') = -\frac{N_* r_e}{\gamma} \vec{f}(x - \bar{x}_*, y - \bar{y}_*; \sigma_x^*, \sigma_y^*) \tag{2.1}$$

where x (y) and x' (y') are the coordinate and its slope of horizontal (vertical) transverse motion and $\bar{x}_*$ and $\bar{y}_*$ refer to the barycentres of the counter-rotating bunch. Here r_e is the classical electron radius, N_* is the number of particle of the counter-rotating bunch, γ is the relativistic Lorentz factor, and σ is the r.m.s. beam size at the interaction point (IP). A quantity with $*$ belongs to the counter-rotating bunch. Hereafter we use z to denote either x or y.

The vector $\vec{f}$ is determined by the density distribution of the counter-rotating bunch. For a Gaussian distribution, it can be written as[5]

$$\begin{aligned} & f_y(x, y; \sigma_x, \sigma_y) + i f_x(x, y; \sigma_x, \sigma_y) \\ = & \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}} \left\{ w\left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\frac{\sigma_y}{\sigma_x} x - i\frac{\sigma_x}{\sigma_y} y}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right) \right\}, \end{aligned} \tag{2.2}$$

where w is the complex error function. When, further, $z - \bar{z}_* \ll \sigma_z$, we have

$$\delta z' = \frac{-2N_* r_e}{\gamma \sigma_z^* (\sigma_x^* + \sigma_y^*)} (z - \bar{z}_*) \equiv -\frac{4\pi\xi_z}{\beta} (z - \bar{z}_*) \tag{2.3}$$

The beam-beam strength parameter ξ is defined by

$$\xi_z = \frac{N_* r_e}{\gamma} \frac{\beta}{2\pi \sigma_z^* (\sigma_x^* + \sigma_y^*)}.$$

In the operation of colliding-beam storage rings, it is not easy to observe the deflection of individual particles, but it is possible to observe deflections of bunches. In Ref.[6], it was shown that the kick $\delta \bar{z}'$ is, instead of Eq.(2.1),

$$\delta(\bar{x}', \bar{y}') = -\frac{N_* r_e}{\gamma} \vec{f}(\bar{x} - \bar{x}_*, \bar{y} - \bar{y}_*; \Sigma_x, \Sigma_y) \tag{2.4}$$

where Σ's are the effective beam sizes

$$\Sigma_z = \sqrt{(\sigma_z)^2 + (\sigma_z^*)^2},$$

under the assumptions (rigid Gaussian model) that

1. two bunches have Gaussian distributions in coordinate space.

2. during the collision, only barycentre can change but r.m.s. sizes do not change.

2.2 Canonical Variables

Since N may differ from bunch to bunch, and γ may vary from IP to IP, the canonical variables should be chosen carefully. We choose

$$\begin{pmatrix} Z \\ P_z \end{pmatrix} = \sqrt{N\gamma} \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} \bar{z} \\ \bar{z}' \end{pmatrix}. \tag{2.5}$$

where $N\gamma$ plays the role of the bunch mass[6]. By this choice, the kick, Eq.(2.4), is written in an explicitly symplectic form. When $Z - Z_* \ll \Sigma_z$,

$$\delta P_z = -4\pi\sqrt{\Xi}\left(\sqrt{\Xi}Z - \sqrt{\Xi_*}Z_*\right), \tag{2.6}$$

where Ξ's are the effective beam-beam strength parameter

$$\Xi_z = \frac{N_* r_e}{\gamma} \frac{\beta_z}{2\pi\Sigma_z(\Sigma_x + \Sigma_y)}.$$

That is, we replace σ's in ξ by Σ's to get Ξ's. In the arcs, the canonical variables transform simply as

$$\begin{pmatrix} Z \\ P_z \end{pmatrix}' = U(\mu)\begin{pmatrix} Z \\ P_z \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} Z \\ P_z \end{pmatrix}'_* = U(\mu_*)\begin{pmatrix} Z \\ P_z \end{pmatrix}_*,$$

where

$$U(\mu) = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix},$$

and μ and μ_* are the phase advances for each bunch.

2.3 Closed Orbit Difference

There may be some difference between the closed orbits of the two beams, artificially (by separators) or unintentionally. When the centres of the two bunches are vertically separated at the collision point by D_y, the linearized kick for a single particle close to the centre of the bunch, Eq.(2.3), is weakened by a multiplying factor[7]:

$$\xi_z \to F_z \xi_z$$

where

$$\left.\begin{aligned} F_x = F_x(\kappa, d) &= \frac{\kappa}{\kappa - 1}\left[1 - \frac{\exp(-d^2/2)}{\kappa} - \Phi(\kappa, d)\right], \\ F_y = F_y(\kappa, d) &= -\frac{1}{\kappa - 1}\left[1 - \kappa\exp(-d^2/2) - \Phi(\kappa, d)\right], \end{aligned}\right\} \tag{2.7}$$

$$\kappa = \frac{\sigma_x^*}{\sigma_y^*}, \quad d = \frac{D_y}{\sigma_y^*},$$

and

$$\Phi(\kappa, d) = \frac{d\sqrt{\pi}}{\sqrt{2(\kappa^2 - 1)}}\left[w\left(\frac{id}{\sqrt{2(\kappa^2 - 1)}}\right) - \exp(-d^2/2)w\left(\frac{i\kappa d}{\sqrt{2(\kappa^2 - 1)}}\right)\right] \tag{2.8}$$

These are obtained from Eq.(2.3) by performing a Taylor expansion around D_y and setting $x = 0$.

For the deflection of the bunches, we use the same factor F_z,

$$\delta P_z \rightarrow F_z \delta P_z,$$

but with the replacement of σ by Σ, according to Eq.(2.4). It follows that, in Eq.(2.7), we replace

$$\kappa \longrightarrow \frac{\Sigma_x}{\Sigma_y}, \quad \text{and} \quad d \longrightarrow \frac{D_y}{\Sigma_y}.$$

Note that, in applying this formula, D_y should be the real separation, not the nominal one. The nominal separation can be calculated from the electrostatic separator settings and errors in the machine[2], assuming that the trajectories of the bunch centres are straight lines through the interaction region. The real separation is determined by the nominal one and the deflection in the trajectory due to the beam-beam force itself[8]. In some cases, there can be a large difference between the two[9]. For the present, BBMODE **does not** calculate the real separation from the nominal one: this function will be added in future.

2.4 Factor of Yokoya

Recently, multiplicative factors for δP_z representing the deviation from rigid-Gaussian approximation were proposed by Yokoya et.al[10]. Under assumptions that

1. the two beams are symmetric,
2. the currents are infinitely small (to apply the perturbation technique),
3. the separation D_y is zero,

they claimed that δP_z should be multiplied by a factor Y,

$$\begin{array}{lllll} Y_x & = & \Lambda(r) & = & 1.330 - 0.370r + 0.279r^2, \quad \text{(horizontal)} \\ Y_y & = & \Lambda(1-r) & = & 1.239 - 0.188r + 0.279r^2, \quad \text{(vertical)} \end{array}, \qquad (2.9)$$

where

$$r = \frac{\sigma_y}{\sigma_x + \sigma_y}.$$

We use it with the replacement of

$$\sigma_{x,y} \rightarrow \Sigma_{x,y}.$$

Note that the factor Y, as well as F is common to both the beam so that we replace Eq.(2.6) by

$$\delta P_z^{\pm} = -4\pi F_z Y_z \sqrt{\Xi_+} \left(\sqrt{\Xi_+} Z_+ - \sqrt{\Xi_-} Z_- \right), \qquad (2.10)$$

Note that, for the moment, it is not known what happens when one of the assumptions above does not hold. Our estimate may not be quite accurate, when

1. the current is large. Yokoya et.al. assert that the factor should be multiplied to the kick. In deriving their factor, however, they ignored the localized nature of the beam-beam kick so that there seems no justification on the assertion. In fact, according to an experiment on a linear collider[11], the deflection due to the kick is described by Eq.(2.4) quite well. The data in Ref.[10] also show a systematic deviation from this assertion. (It seems as if the factor should be multiplied to the tune shift.) This point should be studied theoretically and experimentally.

2. the $D_y \simeq \Sigma_y$. Presumably, the factor Y is smaller when the separation is larger. When it is large enough, the kick is much suppressed by the factor F so that this ambiguity is not important. When the separation exists and is small, on the other hand, the F is not small and this ambiguity becomes important.

3 Eigentune

We have analyzed the beam-beam force under a collision. The tune measurement system deflects one or some of bunches and observe the excited harmonic oscillations of bunches. When the beam-beam force and the excitation are not extremely large, this response can be treated in terms of matrix calculation. The response is large at the tunes corresponding to the eigenvalues of the matrix.

In the next section, we will show how BBMODE calculates the eigentunes. It seems convenient here to define an effective beam-beam parameter Ξ^{eff} as

$$\Xi_z^{eff} = (\Xi_z + \Xi_z^*)F_z \tag{3.1}$$

This is closely related to the tune shifts. When Ξ's are small, the contribution of each collision to the largest tune shift is roughly

$$\text{largest tuneshift} \simeq \Xi_{eff} \times Y_z.$$

4 Program

Here we explain some fundamental computation processes.

Numbering

We consider K e^+ bunches and K e^- bunches and $2K$ interaction points. We define

the position of an IP, denoted by I_s, $I_s = 1, 2, \cdots 2K$,

the time, denoted by I_t, $I_t = 1, 2, \cdots 2K$,

the index of the M-th bunch of i-th beam, denoted by (M, i), $M = 1, 2, \cdots K$, $i = 1$ or 2.

Fig.1 shows how the position I_s of the bunch (M, i) moves with I_t. For a given I_t, the

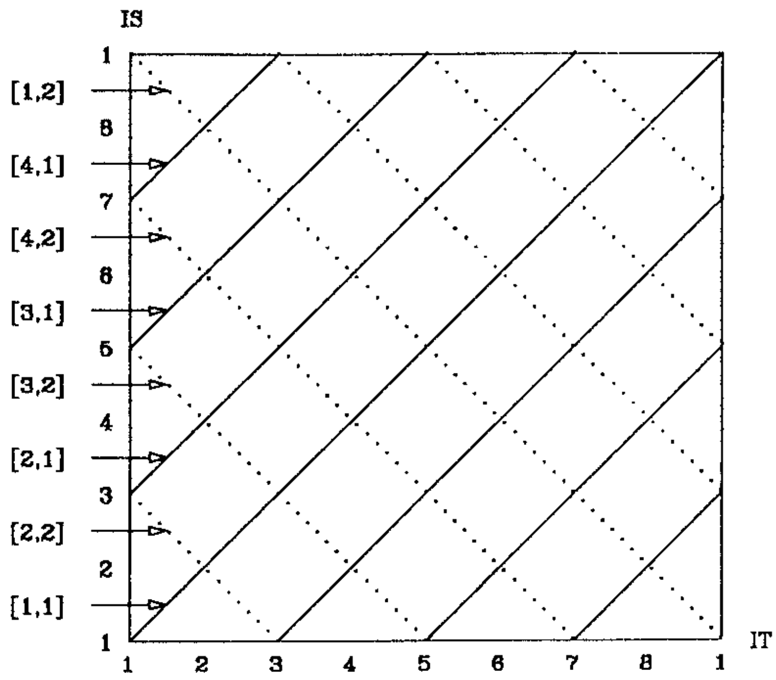

Figure 1: The definition of I_s, I_t and (M,k).

bunch (M,i) is at

$$I_s = I_s(I_t, M, i) = \begin{cases} \mathrm{mod}(2(M-1)+I_t-1, 2K)+1, & (i=1) \\ \mathrm{mod}(2K+2M-I_t-1, 2K)+1, & (i=2) \end{cases} .$$

in Fortran convention of *mod*. At $I_t = 1$, the bunch (M,i) is at $I_s = 2M-1$. Thus at I_t, $(M,1)$ collides with $(M',2)$, such that

$$M' = \mathrm{mod}(M-1+I_t-1, K)+1, \tag{4.1}$$

at the IP of

$$I_s = \mathrm{mod}(2(M-1)+I_t-1, 2K)+1. \tag{4.2}$$

Fundamental Vector

We construct $4K \times 4K$ matrices C_x and C_y, describing the horizontal and vertical motions for one turn, respectively, and find their eigenvalues. Since the descriptions of C_x and C_y go parallel, we use C_x for the explanation. C_x is the product of

$$C_x = \prod_{j=1}^{2K} O_j(R_j + I),$$

where R_j represents the beam-beam kick at a $I_t = j$, O_j the betatron oscillation between $I_t = j$ to $j+1$ and I is the unit matrix.

We define a $4K$ vector $\vec{X}$ as

$$\vec{X} = \begin{pmatrix} X(1) \\ X(2) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ X(4K) \end{pmatrix} \equiv \begin{pmatrix} \vec{Z}(1,1) \\ \vec{Z}(2,1) \\ \vdots \\ \vec{Z}(K,1) \\ \vec{Z}(1,2) \\ \vdots \\ \vec{Z}(K,2) \end{pmatrix},$$

where each $\vec{Z}$ is defined by Eq.(2.5). That is, for the M-th bunch of the i-th beam,

$$\begin{array}{lll} & \text{Coordinate} & \text{Momenta} \\ i=1: & X(2M-1) & X(2M) \\ i=2: & X(2K+2M-1) & X(2K+2M) \end{array} .$$

Beam-Beam Kick

At I_t, K pairs of bunches collide, such that the bunch labelled by $(M,1)$ collides with the bunch labelled by $(M',2)$, Eq(4.1), at I_s, Eq.(4.2). Thus the contribution to matrix R of this collision is

$$\begin{aligned} R(2M,2M-1) &= -4\pi\Xi_+(M,M',I_s)Y_z(I_s), \\ R(2K+2M',2K+2M'-1) &= -4\pi\Xi_-(M,M',I_s)Y_z(I_s), \\ R(2M,2K+2M'-1) & \\ = R(2K+2M',2M-1) &= -4\pi\sqrt{\Xi_+(M,M',I_s)\Xi_-(M,M',I_s)}Y_z(I_s) \end{aligned}$$

where $\Xi_\pm(M,M',I_s)$ is Ξ in this collision.

Transfer Matrix

The phase advance μ from an IP to another may be different in each arc and for each beam. Our convention of the μ is as follows:

$$\begin{array}{l} i=1: \ 1 \xrightarrow{\mu(1,1)} 2 \xrightarrow{\mu(2,1)} 3 \cdots 2K-1 \xrightarrow{\mu(2K-1,1)} 2K \xrightarrow{\mu(2K,1)} 1, \\ i=2: \ 1 \xleftarrow{\mu(2,2)} 2 \xleftarrow{\mu(3,2)} 3 \cdots 2K-1 \xleftarrow{\mu(2K,2)} 2K \xleftarrow{\mu(1,2)} 1. \end{array} \quad (4.3)$$

Just after the beam-beam kick at time I_t, that is, from I_t to I_t+1, a matrix O applies, which is a block-wise diagonal matrix composed of $U(\mu)$'s: in the subspace of $\vec{Z}(M,i)$, O is $U(\mu)$ with

$$\mu = \mu[I_s(I_t,M,i),i],$$

where $\mu(I_s,i)$ is defined by Eq.(4.3).

Symplecticity check

Because of numerical errors, the matrix C_x thus obtained may be slightly non-symplectic. We calculate

$$C_x^t J C_x - J,$$

where J is the $4K \times 4K$ symplectic metric, and print a warning when any of its elements differs from 0 by more than 10^{-4} in absolute value.

Eigenvalues

The eigenvalues are calculated numerically. For our purpose, however, it is not enough to know all eigenvalues. In order to select relevant tunes only, we need eigen vectors also.

The observable tunes are listed in descending order. The highest mode (π mode) and the lowest mode (σ mode) can be easily observed by the tune measurement system. (Intermediate modes are a little difficult).

When two beams have the same nominal tune, the tune of the σ mode does not depend on the strength of the beam-beam interaction and is the same as the nominal tune. When the nominal tunes are different between two beams, the σ mode is affected also by the beam-beam interaction. In applying BBMODE, note that σ mode tune can also be affected by impedance.

Luminosity

The luminosity L of each IP,

$$L = \frac{f_{rev}}{2\pi} \sum_{col} \frac{N N_*}{\Sigma_x \Sigma_y} \exp[-\frac{1}{2}(\frac{D_y}{\Sigma_y})^2]$$

can be estimated easily and is listed. Here the sum extends over all collision at the IP and f_{rev} is the repetition rate of the same kind of collision (revolution frequency). This can be compared with the luminosity monitors. It seems more reliable if we compare the integrated luminosity. The latter can also be calculated if the bunch currents are provided almost continuously.

A User's guide to BBMODE

In the first version of the program, the following keywords are used. Data should be written in the line following each keyword (see example below), in the fields marked by asterisks. The order of presenting data is irrelevant, except for NUB (see below).

COM Comment. A line with COM is always ignored.

TIT Title which will be written in the output.

NUB Number of bunches in each beam. Only COM and TIT may precede it.

EMT Emittances, ϵ_x and ϵ_y for both beams. (metre rad).

BET The β_x and β_y at each IP for both beams. [metre]

TUN Tunes (phase advance/2π), ν_x and ν_y for each arcs for both beams.

CUR The current of each bunch and both beams. [A]

ENG The energy of both beams at each IP. [eV]

COD The (real) vertical separation between both beams at each IP. [metre]

CIR Circumference. [metre] Default is the value for LEP.

CAL By this, BBMODE starts calculation with data thus given. One may repeat the calculation by changing some of data using the keywords and saying CAL again. Note that NUB cannot be changed by this.

END Finish the job. This is necessary.

In case the same number repeats for EMT, BET, TUN, CUR, ENG or COD, one can simplify inputs by writing some negative number in the second line (see example below): it implies the same numbers in the first line repeat itself.

The outputs are

- Factors F_x, F_y, Y_x and Y_y and the effective beam-beam parameters at each collision.
- All tunes of beam-beam modes in descending order.
- Luminosity estimates at each IP.
- Warning for non-symplecticity and linear instability, when necessary.

An example for Input

Here is the sample input form for LEP parameters. For the sake of brevity, we assumed that only one IP is alive and $K = 1$. In ENG, we use the convention for the minus sign. The corresponding output is shown afterwards.

```
COM This is comment.
TIT    The next line is a title written on output.
TEST DATA  (K=1)
NUB          *      Number of bunches in each beam
             1
TIT    This is the longest possible Title
LEP at 55 GeV (Optimal Coupling) in 1+1=1 operation with design current.
COM        **********   **********   **********   **********
COM       emittances (meter*rad)
EMT        EMITX(1)     EMITX(2)     EMITY(1)     EMITY(2)
           +5.270E-08   +5.270E-08    2.108E-09    2.108E-09
COM Beta functions at IP's in meter
BET  IP    BETAX(1)     BETAX(2)     BETAY(1)     BETAY(2)
      1     1.950E+01    1.950E+01    0.780E-00    0.780E-00
L3    2     1.750E+00    1.750E+00    0.070E-00    0.070E-00
COM Tunes for each arc (from No.X to next) **********
TUN  IP    NUX(1)       NUX(2)       NUY(1)       NUY(2)
      1     8.025E-01    8.025E-01    7.938E-01    7.938E-01
      2     8.025E-01    8.025E-01    7.938E-01    7.938E-01
COM  Vertical separation at IP's. Use real separation.
COD  IP    DY
      1     1.000E-03
      2     0.000E-00
CUR BUNCH CURRNT(1)     CURRNT(2)     in Ampere
      1     0.750E-03    0.750E-03
ENG  IP    ENERG(1)     ENERG(2)      in eV
      1     5.500E+10    5.500E+10
      2    -5.500E+20    0.000E+10
CIRCUMFERENCE      in meter      (this is not necessary for LEP)
            2.6658883376000E+04
CAL
TIT
When the separator does not work...
COD  IP    DY
      1     0.000E-03
      2     0.000E-00
CAL
END
```

An Example for Output

```
TEST DATA  (K=1)
bbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbb
b                   BBMODE                    b
mmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmm
LEP at 55 GeV (Optimal Coupling) in 1+1=1 operation with design current.
Number of Bunches  =          1  in each beam
      Circumference=  26658.8828
Revolution frequency=  11245.4961
Emittances
       EMITX(1)     EMITX(2)     EMITY(1)     EMITY(2)     m rad
       5.270E-08    5.270E-08    2.108E-09    2.108E-09
Beta functions and vertical separations
 IP    BETAX(1)     BETAX(2)     BETAY(1)     BETAY(2)     DY        m
  1    1.950E+01    1.950E+01    7.800E-01    7.800E-01    1.000E-03
  2    1.750E+00    1.750E+00    7.000E-02    7.000E-02    0.000E+00
Beam sizes
 IP    SIGX(1)      SIGX(2)      SIGY(1)      SIGY(2)       m
  1    1.014E-03    1.014E-03    4.055E-05    4.055E-05
  2    3.037E-04    3.037E-04    1.215E-05    1.215E-05
Tunes
 IP    NUX(1)       NUX(2)       NUY(1)       NUY(2)      [phase advance/2pi]
  1    8.025E-01    8.025E-01    7.938E-01    7.938E-01
  2    8.025E-01    8.025E-01    7.938E-01    7.938E-01
total  1.605E+00    1.605E+00    1.588E+00    1.588E+00

Number of particles and Current
bunch  N(1)         N(2)      *  CURNT(1)     CURNT(2)     Ampere
  1    4.163E+11    4.163E+11 *  7.500E-04    7.500E-04
                         TOTAL  7.500E-04    7.500E-04
Lorentz factors and Energies
 IP    GAMMA(1)     GAMMA(2)  *  ENERG(1)     ENERG(2)     (GeV)
  1    1.076E+05    1.076E+05 *  5.500E+01    5.500E+01
  2    1.076E+05    1.076E+05 *  5.500E+01    5.500E+01
BBMTR starts **********************************
TIME BUNCH IP  FX          FY          Xi(eff)x   Xi(eff)y   Yokoya-x   Yokoya-y
   1   1   1  1  4.775E-01 -1.910E-02  7.217E-03  1.155E-05  1.316E+00  1.232E+00
   2   1   1  2  1.000E+00  1.000E+00  3.165E-02  3.165E-02  1.316E+00  1.232E+00
BBMTR finished ********************************
Horizontal tunes
  K      REAL        IMAG        TUNE
  1 -4.946E-01 -8.691E-01       0.66766
  2 -7.901E-01 -6.129E-01       0.60500
Vertical tunes
  K      REAL        IMAG        TUNE
  1 -7.287E-01 -6.848E-01       0.62006
  2 -8.523E-01 -5.230E-01       0.58760
```

```
Luminosity estimate
 IP    Luminosity (cm^{-2}s^{-1})
  1    0.000E+00
  2    4.203E+30
bbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbb
b                  BBMODE                        b
mmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmm
When the separator does not work...
Number of Bunches  =          1  in each beam
      Circumference=  26658.8828
Revolution frequency=  11245.4961
Emittances
       EMITX(1)      EMITX(2)      EMITY(1)      EMITY(2)      m rad
       5.270E-08     5.270E-08     2.108E-09     2.108E-09
Beta functions and vertical separations
 IP    BETAX(1)      BETAX(2)      BETAY(1)      BETAY(2)      DY        m
  1    1.950E+01     1.950E+01     7.800E-01     7.800E-01     0.000E+00
  2    1.750E+00     1.750E+00     7.000E-02     7.000E-02     0.000E+00
Beam sizes
 IP    SIGX(1)       SIGX(2)       SIGY(1)       SIGY(2)        m
  1    1.014E-03     1.014E-03     4.055E-05     4.055E-05
  2    3.037E-04     3.037E-04     1.215E-05     1.215E-05
Tunes
 IP    NUX(1)        NUX(2)        NUY(1)        NUY(2)       [phase advance/2pi]
  1    8.025E-01     8.025E-01     7.938E-01     7.938E-01
  2    8.025E-01     8.025E-01     7.938E-01     7.938E-01
total  1.605E+00     1.605E+00     1.588E+00     1.588E+00

Number of particles and Current
bunch  N(1)          N(2)       *  CURNT(1)      CURNT(2)      Ampere
  1    4.163E+11     4.163E+11  *  7.500E-04     7.500E-04
                            TOTAL  7.500E-04    7.500E-04
Lorentz factors and Energies
 IP    GAMMA(1)      GAMMA(2)   *  ENERG(1)      ENERG(2)      (GeV)
  1    1.076E+05     1.076E+05  *  5.500E+01     5.500E+01
  2    1.076E+05     1.076E+05  *  5.500E+01     5.500E+01
BBMTR starts *********************************
TIME BUNCH IP  FX          FY          Xi(eff)x    Xi(eff)y    Yokoya-x    Yokoya-y
   1  1  1  1  1.000E+00   1.000E+00   3.165E-02   3.165E-02   1.316E+00   1.232E+00
   2  1  1  2  1.000E+00   1.000E+00   3.165E-02   3.165E-02   1.316E+00   1.232E+00
BBMTR finished *******************************
Horizontal tunes
  K     REAL       IMAG         TUNE
  1 -3.467E-01 -9.380E-01       0.69365
  2 -7.901E-01 -6.129E-01       0.60500
Vertical tunes
  K     REAL       IMAG         TUNE
  1 -4.848E-01 -8.746E-01       0.66944
```

```
  2 -8.523E-01 -5.230E-01       0.58760
Luminosity estimate
 IP     Luminosity (cm^{-2}s^{-1})
  1     3.772E+29
  2     4.203E+30
bbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbb
b                  BBMODE                          b
mmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmm
TOTAL CPU TIME is 0.222619995E-01
```

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