On the renormalization structure in quantum conformal gravity

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平成２８（２０１６）年度

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December 9, 2016

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Chapter 1

Introduction

Einstein's general relativity and quantum mechanics are the two basic theory which support a present physics but it is known that they are incompatible. There are a lot of difficulties to construct the quantum gravity theory. First of all, the quantization of the gravitational field based on the Einstein gravity theory leads to the unrenormalizability problem. Moreover, the theory also becomes unstable even in a non-perturbative method since the Einstein-Hilbert action can takes indefinite value. Besides, we can not physically eliminate a spacetime solution with singularity since the action becomes finite for such a configuration.

In order to resolve these problems, the approach that we introduce higher derivative actions bounded from below involving the square of Riemann curvature tensor has been proposed [1, 2, 3, 5, 6, 7, 8, 9, 10]. In this approach, the action for singular spacetime configurations such as Schwarzschild solution diverges and thus such solutions are eliminated quantum mechanically. On the other hand, however, one could not avoid the problem that physical ghost mode, what is called massive graviton, emerges when one dealt all gravitational fields perturbatively. So, it is appropriate to suppose that we need some kind of non-perturbative method to quantize such higher derivative fields.

In our study, we consider the renormalizable quantum conformal gravity proposed in recent years [11, 12, 13, 14]. We then apply a non-perturbative method [15, 16, 17, 18, 19, 20, 21, 22, 23] learned from the development of two-dimensional quantum gravity [24, 25, 26, 27, 28, 29]. That is to quantize gravity by taking into account contributions induced from the path integral measure, what is called the Wess-Zumino action [30, 31] with respect to the conformal anomaly [32, 33, 34, 35, 36, 37, 38, 39, 40]. In order to carry out it, we manage the conformal mode of gravitational fields non-perturbatively without introducing its coupling constant. On the other hand, we dealt the traceless tensor mode perturbatively.

In the above quantization method, the following important natures emerges. The first is that the general coordinate invariance in such a non-perturbative treatment leads to the background-metric free nature [15, 16, 17, 18, 19, 20, 21, 22, 23]. It is realized as a gauge equivalency under the conformal transformation, which is called BRST conformal symmetry [22, 23]. Owing to this symmetry, we can choose the flat background without changing physics and can calculate quantum correction in the same way to usual quantum field theory. Besides, the BRST conformal symmetry makes the gravitational ghost mode unphysical. The second is that there is a new dynamical scale of quantum gravity [11, 12, 13, 14]. It is indicated from that the beta function of the coupling constant for the traceless tensor mode becomes negative. The background free spacetime and the classical spacetime are separated at this scale.

The conformal invariance has a crucial role in our non-perturbative formulation of quantum gravity. Its importance in physics is also suggested by the recent experiments on cosmic microwave background (CMB) [41, 42] which indicates the fluctuation in early universe to be scale invariant. From this, it is a good guess that conformal invariance is fundamental in quantum gravity.

The purpose of this study is to examine the renormalization structure in our quantum conformal gravity theory with using dimensional regularization [11, 12, 13, 14]. The advantages of using this regularization are that it preserves the diffeomorphism invariance at all orders in perturbation and that the path integral becomes independent of how to choose the functional measure. Then, the information of the conformal anomaly hides between the D and 4 dimensions, in contrast with the case using just four dimensional quantization method. So, it is quite important to determine the D dependence of the gravitational action. However, there are some ambiguities when we generalize the four-dimensional gravitational action into D-dimensional one.

Recently, such ambiguities was resolved by analyzing Hathrell's renormalization equation in the case of quantum field theory in curved space with dimensionless conformal coupling [36, 37, 38, 39, 40, 14]. It was then found that the D dimensional gravitational actions can be determined at all orders of perturbation, which are classified into two forms only that reduce to conformally invariant ones at four dimensions, namely, the square of the Weyl tensor and the Euler density [40, 14]. The renormalizable quantum conformal gravity we consider here is formulated using these actions [13, 14].

As mentioned above, a significant feature on this theory is that the dynamics of the conformal mode is induced quantum mechanically. In dimensional regularization, such dynamics emerges from Laurent expansion of the D-dimensional gravitational bare action around four dimensions. The pole terms are ordinary counterterms. On the other hand, the finite terms with non-negative powers of $D - 4$ produce interaction terms of the conformal mode, which are nothing but the Wess-Zumino action for conformal anomaly induced quantum mechanically. Especially, the lowest term is called the Riegert action [15] that gives the kinetic term of the conformal mode. The renormalization of quantum gravity is carried out with incorporating these induced terms. Then, the conformal mode does not receive renormalization, namely, its renormalization factor becomes unity. This property comes from the thing that the conformal mode is treated exactly without introducing the independent coupling for this mode.

For the traceless tensor mode, its dynamics is described by the Weyl action with introducing a dimensionless coupling constant t. The perturbation theory is then defined around the conformally flat spacetime in which the Weyl tensor vanishes. Since the beta function becomes negative, its coupling constant indicates what is called the asymptotic freedom. However, we here emphasize that it does not show the free graviton picture propagating on flat spacetime at high energy limit since the conformal mode still non-perturbatively fluctuates and thus the spacetime is in completely quantum mechanical phase.

In this thesis, we consider the system with adding the cosmological term and the Einstein term. The anomalous dimensions of the cosmological constant and the Planck mass can be calculated properly by taking the Wess-Zumino interactions into account, and it has been confirmed that the result at the zeroth order of t agrees with the exact solution derived from the BRST conformal symmetry [13]. One of the main achievements done in this thesis is that we calculate the loop corrections to the cosmological constant and the Planck mass at the order of $\alpha_t = t^2/4\pi$ [14]. Especially, for the cosmological constant, it is given at the two-loop level in this formulation. The result of the cosmological constant shows that its t-dependent part is negative while that of the Planck mass is positive. Also, the demonstration of two-loop renormalizability itself is one of the important purposes of this work.

Furthermore, we consider the effective action related to the cosmological term that depends on the conformal mode only and examine its renormalization group equation. Owing to the non-renormalization property of the conformal mode, we then find that it does not depend on the renormalization group parameter. Since the physical quantity should be invariant under the renormalization group flow, this implies that the effective potential for the cosmological term is the physical cosmological constant in this formulation. We then calculate the physical cosmological constant at the one-loop level explicitly and show that it is given by a function of the renormalized cosmological constant and the renormalized Planck mass. So, we consider its value to be the observed value.

This thesis is organized as follows. In chapter 2, we first mention the basic structure of quantum conformal gravity, emphasizing the importance of the Wess-Zumino action for conformal anomaly in order to preserve diffeomorphism invariance at the quantum level. And also, how to remove the spacetime singularity and how to overcome the negative-metric mode problem are briefly summarized here. In chapter 3, we present the formulation using dimensional regularization. After mentioned how to determine the D-dimensional gravitational action without ambiguities, we provide the propagators and interactions of the gravitational fields, including the Wess-Zumino actions. We then explain the renormalization techniques to systematically incorporate the induced dynamics of the conformal mode, in which it is emphasized that the information of the Wess-Zumino action hides between D and 4 dimensions in this regularization. Here, we present the beta function of the coupling which becomes negative, we also demonstrate the non-renormalization property of the conformal mode at the order of α_t. In chapter 4, we consider the system with adding the cosmological term and the Einstein term and calculate the anomalous dimensions at higher loops, including the corrections of α_t. For these calculations, we choose Landau gauge in order to reduce the numbers of the Feynman diagrams considerably. In chapter 5, we study the effective action with respect to the conformal mode. We first examine the renormalization group equation and show that such effective action becomes invariant under renormalization group flow. And then we calculate such a effective potential as the physical cosmological constant explicitly at the one-loop level. In chapter 6, we summarize our study and discuss its physical meanings. In appendix B, we demonstrate that the D-dependence of the gravitational action can be determined uniquely at all orders in the case of QCD in curved space. In other appendices, we present various gravitational formulae, details of calculations and integral formulae for loop calculations.

Chapter 2

Basic structure

First of all, we overview the basic structure of four-dimensional quantum gravity we discuss in this thesis.

In order to construct the quantum gravity theory, we here impose basic three conditions: the diffeomorphism invariance, the finiteness of the theory and 4-dimensional spacetime. Diffeomorphism invariance is one of the basic principles of Einstein gravity, and we assume that this symmetry remains also in quantum theory. The second condition means that physical quantities should be finite. That is to say, the theory is renormalizable as well as there is no singular point. Third, we consider the 4-dimensional spacetime since extra dimensions are not observed yet and there is no reason to consider them at the present stage.

2.1 Quantum conformal gravity action

It is known that quantum gravity theories based the Einstein action can not be renormalizable since the gravitational coupling has dimension. In order to resolve this problem and also ensure the positivity of the action, we introduce the fourth derivative action. Besides, we think that the conformal symmetry is important in a high energy region. So, we consider four-dimensional quantum gravity theory that has conformally invariant actions for fourth order gravitational terms and matter fields, which is given as

$$I = \int d^4x \sqrt{g} \left\{ \frac{1}{t^2} C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} + bG_4 - \frac{M_P^2}{2} R + \Lambda + \mathcal{L}_{\text{conf.matter}} \right\}, \tag{2.1}$$

where t is a dimensionless coupling governing the dynamics of quantum gravity. The coefficient b is introduced to eliminate divergences proportional to G_4. We should note that this constant is not the independent coupling since the Euler term does not include the kinetic term. Here, we also add the Einstein term and the cosmological term, which do not prevent the renormalizability since we consider the energy scale beyond the Planck scale.

Quantum gravity is defined by path integral over the gravitational fields with the weight e^{-I}. The fourth order part of the action gives the kinetic term of this theory and here we consider this part. The dynamics of the traceless tensor mode is governed by the Weyl action. The coupling constant t introduced in front of this action defines the perturbation theory expanded about the conformally flat spacetime satisfying $C_{\alpha\beta\gamma\delta} = 0$. On the other hand, since the forth order action is conformally invariant, it does not depend on the conformal mode. Therefore, since the conformal mode is not restricted by this action, it must be treated exactly. From this, the gravitational field is expanded as

$$g_{\alpha\beta} = e^{2\phi}\bar{g}_{\alpha\beta}, \quad \bar{g}_{\alpha\beta} = (\hat{g}e^{th})_{\alpha\beta} = \hat{g}_{\alpha\beta} + th_{\alpha\beta} + \frac{t^2}{2}h_{\alpha\gamma}h^{\gamma}_{\ \beta} + \cdots , \tag{2.2}$$

where $h_{\alpha\beta}$ is a traceless tensor mode satisfying $h^{\alpha}_{\ \alpha} = \hat{g}^{\alpha\beta}h_{\alpha\beta} = 0$. We should note that the conformal factor is written in the exponential of the conformal mode ϕ in order to ensure its positivity and we do not introduce the coupling constant for this mode, unlike the case of the traceless tensor mode.

One of the most important features of this quantum gravity theory is that although there is no fourth order dynamics of the conformal mode in this action, it is induced from the path integral measure, what is called Wess-Zumino action. It is Jacobian necessary to preserve the diffeomorphism invariance when we translate the diffeomorphism invariant measure on $g_{\alpha\beta}$ into the practical measure defined on the background metric $\hat{g}_{\alpha\beta}$. From this, we can write the path integral as

$$e^{-\Gamma} = \int [dgdf]_g e^{-I(f,g)} = \int [d\phi dhdf]_{\hat{g}} e^{-S(\phi,\bar{g})-I(f,g)}, \tag{2.3}$$

where S is the Wess-Zumino action for conformal anomaly. The field f expresses a conformally coupled matter field. The Wess-Zumino action emerges from the zeroth order of the expansion with respect to coupling constant t, which is called Riegert action [15] defined by

$$S_R(\phi, \bar{g}) = \frac{b_c}{(4\pi)^2}\int d^4x\sqrt{\bar{g}}\left[2\phi\bar{\Delta}_4\phi + \left(\bar{G}_4 - \frac{2}{3}\bar{\nabla}^2\bar{R}\right)\phi\right], \tag{2.4}$$

where Δ_4 is the fourth-order differential operator defined as

$$\Delta_4 = \nabla^4 + 2R^{\alpha\beta}\nabla_\alpha\nabla_\beta - \frac{2}{3}R\nabla^2 + \frac{1}{3}\nabla^\alpha R\nabla_\alpha, \tag{2.5}$$

which becomes conformally invariant at four dimensions for scalar quantities and it is self-adjoint. The coefficient b_c is obtained from the one-loop calculation of the conformal anomaly as

$$b_c = \frac{1}{360}(N_S + 11N_F + 62N_A) + \frac{769}{180}. \tag{2.6}$$

The first three terms are the contribution from matter fields in which N_S, N_F, N_A are respectively the number of scalar, fermion and gauge fields [32, 33, 34]. The last term is the sum of 87/20 [9] and −7/90 [17] respectively coming from the gravitational fields $h_{\alpha\beta}$ and ϕ. Thus, since b_c is positive, the Riegert action becomes positive definite.

The dynamics of the traceless tensor mode is governed by the Weyl action with the dimensionless coupling t. The beta function of the coupling $\alpha_t = t^2/4\pi$ is calculated at one-loop level as

$$\beta_t \equiv \frac{\mu}{\alpha_t}\frac{d\alpha_t}{d\alpha} = -\left[\frac{1}{120}(N_S + 6N_F + 12N_A) + \frac{197}{30}\right]\frac{\alpha_t}{4\pi}, \tag{2.7}$$

which becomes negative. So, we find that the traceless tensor mode indicates the asymptotically free nature. This justifies that we consider the perturbation theory about conformally flat spacetime.

We should note that this asymptotic freedom does not mean the realization of the picture that free gravitons are propagating on the flat background at high energy regions. That is because the conformal mode still fluctuates non-perturbatively and thus there is no classical spacetime as the base of such picture.

Here, we also mention that this theory does not have spacetime singularity. It is because the Weyl action is bounded from below and includes the square of the Riemann curvature tensor, and thus the singular configurations that this action diverges are excluded quantum mechanically.

2.2 Diffeomorphism invariance

Under the metric decomposition (2.2), the general coordinate transformation defined as

$$\delta_\xi g_{\alpha\beta} = g_{\alpha\gamma}\nabla_\beta \xi^\gamma + g_{\gamma\beta}\nabla_\alpha \xi^\gamma \tag{2.8}$$

is completely separated into the transformations of the conformal mode and the traceless tensor mode as

$$\begin{aligned} \delta_\xi \phi &= \xi^\gamma \hat{\nabla}_\gamma \phi + \frac{1}{4}\hat{\nabla}_\gamma \xi^\gamma, \\ \delta_\xi \bar{g}_{\alpha\beta} &= \bar{g}_{\alpha\gamma}\bar{\nabla}_\beta \xi^\gamma + \bar{g}_{\beta\gamma}\bar{\nabla}_\alpha \xi^\gamma - \frac{1}{2}\hat{g}_{\alpha\beta}\hat{\nabla}_\gamma \xi^\gamma, \end{aligned} \tag{2.9}$$

where $\bar{\nabla}_\alpha \xi^\alpha = \hat{\nabla}_\alpha \xi^\alpha$ is used. Moreover, we can rewrite the second equation in terms of $h_{\alpha\beta}$ as

$$\begin{aligned} \delta_\xi h_{\alpha\beta} = &\frac{1}{t}\left(\hat{\nabla}_\alpha \xi_\beta + \hat{\nabla}_\beta \xi_\alpha - \frac{1}{2}\hat{g}_{\alpha\beta}\hat{\nabla}_\gamma \xi^\gamma\right) \\ &+ \hat{\nabla}_\gamma h_{\alpha\beta}\xi^\gamma + \frac{1}{2}h_{\alpha\gamma}(\hat{\nabla}_\beta \xi^\gamma - \hat{\nabla}^\gamma \xi_\beta) + \frac{1}{2}h_{\beta\gamma}(\hat{\nabla}_\alpha \xi^\gamma - \hat{\nabla}^\gamma \xi_\alpha) \\ &+ \mathcal{O}(h^2), \end{aligned} \tag{2.10}$$

where $\xi_\alpha = \hat{g}_{\alpha\beta}\xi^\beta$.

BRST conformal symmetry Because of the non-perturbative treatment of the conformal mode, our quantum conformal gravity has a very important property at $t = 0$, called the BRST conformal symmetry [22, 23]. It arises as a part of diffeomorphism symmetry in which the gauge parameter ξ^α is given as the conformal Killing vectors satisfying the equation

$$\hat{\nabla}_\alpha \zeta_\beta + \hat{\nabla}_\beta \zeta_\alpha - \frac{1}{2}\hat{g}_{\alpha\beta}\hat{\nabla}_\gamma \zeta^\gamma = 0. \tag{2.11}$$

In this case, the first term of the transformation (2.10) vanishes and the second term becomes effective. We then obtain the conformal transformations

$$\delta_\zeta \phi = \zeta^\gamma \partial_\gamma \phi + \frac{1}{4}\hat{\nabla}_\gamma \zeta^\gamma, \tag{2.12}$$

$$\delta_\zeta h_{\alpha\beta} = \zeta^\gamma \hat{\nabla}_\gamma h_{\alpha\beta} + \frac{1}{2}h_{\alpha\gamma}\left(\hat{\nabla}_\beta \zeta^\gamma - \hat{\nabla}\zeta_\beta\right) + \frac{1}{2}h_{\beta\gamma}\left(\hat{\nabla}_\alpha \zeta^\gamma - \hat{\nabla}^\gamma \zeta_\alpha\right) \tag{2.13}$$

as a gauge transformation. The BRST conformal transformation is defined by replacing the gauge parameter ζ^α with the corresponding ghost field. This symmetry means that all theories connecting one another by conformal transformations becomes gauge-equivalent.

Thus, at the vanishing coupling limit of $t = 0$, the quantum gravity system can be described as a conformal field theory with BRST conformal symmetry. This system is governed by the induced Riegert action (2.4) and the kinetic term of the Weyl action. The physical states of quantum gravity are defined at the $t = 0$ limit. We then have found that the BRST conformal symmetry makes all negative-metric modes unphysical and physical fields are given by a real primary scalar with a definite conformal dimension in terms of conformal field theory. The anomalous dimensions of physical fields at $t = 0$ can be determined exactly using this symmetry.

The BRST conformal symmetry is nothing but a representation of the background change of the background metric

$$\hat{g}_{\alpha\beta} \to e^{2\sigma} \hat{g}_{\alpha\beta}. \tag{2.14}$$

This invariance originally comes from that the conformal mode is treated exactly so that the shift change of the conformal mode $\phi \to \phi + \sigma$ is equivalent to the conformal change of the background metric (2.14). Since the conformal mode is now the integration variable and the path integral measure is invariant under the local shift, the theory becomes invariant under the conformal change of the background metric. As a consequence, we can choose any background as far as it is conformally flat. In what follows, when we calculate loop corrections, we take the flat background $\hat{g}_{\alpha\beta} = \delta_{\alpha\beta}$.

Chapter 3

Renormalizable quantum gravity

In the previous chapter, we overviewed the basic structure of four-dimensional quantum conformal gravity. In this chapter, we formulate this theory with using dimensional regularization. The advantages of using this regularization are that it is only the method we can calculate higher loop corrections with preserving the diffeomorphism invariance. And also, the theory becomes independent of the choice of the path integral measure owing to the property of $\int d^D p = \delta^{(D)}(0) = 0$. Moreover, there are no quadratic and quartic divergences, which are substantial in UV theories without Landau pole. On the other hand, since the contributions from the measure such as conformal anomalies are hidden between D and 4 dimensions, we have to determine the D dependence of the gravitational action exactly.

When we generalize the fourth derivative action in four dimensions into that in arbitrary dimensions, ambiguities emerge. We recently resolved this problem with using Hathrell's renormalization group equation and then we determined the form of the gravitational actions: the square of the D-dimensional Weyl tensor F_D and the modified Euler density G_D (Details are presented in appendix B).

Furthermore, as mentioned before, this theory has the background metric independence represented as BRST conformal symmetry. Owing to this symmetry, we can choose the flat metric as a background. Thus, we can formulate quantum gravity theory as a usual quantum field theory on the flat spacetime.

3.1 Quantum conformal gravity action in D dimensions

Here, we use the obtained counterterms as an quantum gravity action and consider the system with adding the Einstein-Hilbert action, the cosmological constant term and the conformally coupled matter field action [13, 14]. Then, our quantum gravity action is expressed as

$$S = \int d^D x \sqrt{g} \left[\frac{1}{t_0^2} F_D + b_0 G_D + \Lambda_0 - \frac{M_0^2}{2} R + \mathcal{L}_{matter} \right], \tag{3.1}$$

where t_0 is the bare dimensionless gravitational coupling constant. The b_0 is another dimensionless bare quantity, but it is not an independent coupling as mentioned below. The mass parameters M_0 and Λ_0 are the bare Planck mass and the bare cosmological constant.

Here again, we present the D-dimensional Weyl action

$$F_D = C^2_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - \frac{4}{D-2}R_{\alpha\beta}R^{\alpha\beta} + \frac{2}{(D-1)(D-2)}R^2 \tag{3.2}$$

and the D-dimensional Euler density, which is determined to be

$$G_D = G_4 + (D-4)\chi(D)H^2, \tag{3.3}$$

where G_4 is the usual Euler combination and H is a rescaled scalar curvature defined as $G_4 = R^2_{\alpha\beta\gamma\delta} - 4R^2_{\alpha\beta} + R^2$ and $H = R/(D-1)$, respectively. The function χ is expanded around four dimensions as

$$\chi(D) = \sum_{n=1}^{\infty} \chi_n (D-4)^{n-1}. \tag{3.4}$$

The coefficient χ_n can be determined order by order and the first two coefficients have been calculated (see eq. (B.86)) as

$$\chi_1 = \frac{1}{2}, \qquad \chi_2 = \frac{3}{4}. \tag{3.5}$$

We have then shown that these are universal values independent of gauge group and the contents of matter fields as far as they are conformally coupled. Further from the same analysis of QED, it has been found that $\chi_{1,2}$ are the same values and χ_3 is given by $1/3$. We use these values also in quantum gravity since it should reduce to the curved theory in the classical limit of gravity such as, for instance the large N limit of gauge group $SU(N)$.

3.2 Renormalization procedure

The perturbation in t_0 implies that the gravitational field is expanded around a conformally flat spacetime where the Weyl tensor vanishes. Thus, we quantize the gravitational field by separating into the conformal mode ϕ and the traceless tensor mode $h_{0\alpha\beta}$ as

$$g_{\alpha\beta} = e^{2\phi}\bar{g}_{\alpha\beta}, \quad \bar{g}_{\alpha\beta} = (e^{t_0 h_0})_{\alpha\beta} = \delta_{\alpha\beta} + t_0 h_{0\alpha\beta} + \frac{t_0^2}{2} h^{\gamma}_{0\alpha} h_{0\gamma\alpha} + \cdots , \tag{3.6}$$

where $h^{\alpha}_{0\alpha} = 0$ and the flat background is taken in what follows.

The renormalization factors of the traceless tensor mode and the coupling constant are defined as usual by

$$h_{0\alpha\beta} = Z_h^{\frac{1}{2}} h_{\alpha\beta}, \quad t_0 = \mu^{2-\frac{D}{2}} Z_t t, \tag{3.7}$$

where μ is a mass scale and the renormalized coupling t is dimensionless. On the other hand, since we do not introduce the coupling constant for the conformal mode, diffeomorphism invariance requires that it is not renormalized such that

$$Z_\phi = 1. \tag{3.8}$$

It can be easily understand from the fact that the gauge invariance results in the relationship between the renormalization factors of the coupling constant and the corresponding gauge field.[1] No coupling constant thus implies that there is no field-renormalization factor. For this reason, we write the conformal mode ϕ, not ϕ_0 in (3.6). This is one of the most important features in our renormalization calculations, which reflects the independence of the choice of the background metric as mentioned above.

We here expand these renormalization factors as follows:

$$\ln Z_h = \sum_{n=1}^{\infty} \frac{f_n}{(D-4)^n}, \quad \ln Z_t^{-2} = \sum_{n=1}^{\infty} \frac{g_n}{(D-4)^n}. \tag{3.9}$$

Using these terms, we can renormalize UV divergences proportional to the F_D term. The beta function of the coupling constant is then defined as

$$\beta_t \equiv \frac{\mu}{\alpha_t} \frac{d\alpha_t}{d\mu} = D - 4 + \bar{\beta}_t, \tag{3.10}$$

where $\alpha_t = t^2/4\pi$ and $\bar{\beta}_t = \mu d(\ln Z_t^{-2})/d\mu$.

In comparison with the traceless tensor mode, the conformal mode has more complicated renormalization structure. Since the volume integral of G_D becomes topological at four dimension, its kinetic term emerges from the first order of $D-4$. This means that G_D does not contribute to classical gravitational dynamics. And so, we define the bare coupling constant b_0 in a pure pole series as follows:

$$b_0 = \frac{\mu^{D-4}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \frac{b_n}{(D-4)^n}. \tag{3.11}$$

Its residues b_n at $n \geq 2$ are the function of the coupling constant t only, while the residue of simple pole b_1 has both a constant part b and a coupling dependent part b' as

$$b_1 = b + b_1'(\alpha_t). \tag{3.12}$$

In order to work out various renormalization calculation with respect to the conformal mode, we need some kind of procedure which incorporates the dynamics induced quantum mechanically. Here, we propose that for the moment we temporarily treat b as another coupling constant for the conformal mode. In that case, the effective action becomes finite up to the topological term, which is expressed as follows:

$$\Gamma = \frac{\mu^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{b - b_c}{D-4} \int d^D x \sqrt{\hat{g}} \hat{G}_4 + \Gamma_R(\alpha_t, b), \tag{3.13}$$

[1] For instance, $Z_e Z_3^{1/2} = 1$ in QED. Precisely, the argument in general holds only for the background gauge field in the background field method [45]. For the conformal mode, however, it is true since this mode is not gauge-fixed unlike the traceless tensor mode so that the renormalization factor of the conformal mode is the same to that of the background and it becomes unity from diffeomorphism invariance. Later, the background field method is used in various loop calculations.

where Γ_R is a renormalized effective action. We should notice that the divergent term emerges only when we choose a curved background metric. The constant b_c comes from the direct one loop calculation given by eq. (2.6). After the renormalization procedure is carried out, we take $b = b_c$. In this way, we can obtain the finite effective action $\Gamma_R(t, b_c)$ whose dynamics is governed by a single gravitational coupling t.

From the renormalization group equation $\mu db_0/d\mu = 0$, we obtain the following expression:

$$\mu\frac{db}{d\mu} = (D-4)\bar{\beta}_b, \tag{3.14}$$

where $\bar{\beta}_b$ is a finite function given as

$$\bar{\beta}_g = -\left(\frac{\partial b_1}{\partial b}\right)^{-1}\left(b_1 + \alpha_t\frac{\partial b_1}{\partial \alpha_t}\right). \tag{3.15}$$

Here, in order to be able to replace the coupling b to the constant b_c at the end, the condition $\mu db/d\mu = 0$ should be satisfied at four dimensions. Therefore, (3.14) ensures the validity of the renormalization procedure proposed above.

From the renormalization group analysis of QED and QCD in curved space, we find that b'_1 in (3.12) arises at the fourth order of the gauge-coupling constant. From this fact and the similarity between the gauge field and the traceless tensor field ruled by the Weyl action, we can guess that the α_t dependence of b'_1 is also given as

$$b'_1 = \mathcal{O}(\alpha_t^2), \tag{3.16}$$

and then we obtain $\bar{\beta}_b = -b + \mathcal{O}(\alpha_t^2)$. This assumption should be verified through direct two-loop calculations of three-point functions of the traceless tensor mode or indirect calculations using the renormalization group equation, but this work is not complete yet.

3.3 Propagators and Wess-Zumino interactions

In this section, we will derive the conformal mode propagator and the traceless tensor mode propagator, and Wess-Zumino interactions to calculate some Feynman diagrams.

3.3.1 The F_D term

The Weyl term in D-dimensions is expanded as follows

$$\begin{aligned}\frac{1}{t_0^2}\int d^Dx\sqrt{g}F_D &= \frac{1}{t_0^2}\int d^Dx e^{(D-4)\phi}\bar{C}_{\alpha\beta\gamma\delta}\bar{C}^{\alpha\beta\gamma\delta} \\ &= \int d^Dx\left[\frac{1}{t_0^2}\bar{C}_{\alpha\beta\gamma\delta}\bar{C}^{\alpha\beta\gamma\delta} + \frac{D-4}{t_0^2}\phi\bar{C}_{\alpha\beta\gamma\delta}\bar{C}^{\alpha\beta\gamma\delta} + \cdots\right].\end{aligned} \tag{3.17}$$

The first term of R.H.S gives the propagator and self-interactions of the traceless tensor mode. The second and other terms are the induced Wess-Zumino actions associated with the Weyl-squared conformal anomaly, which give new interactions that involve the conformal mode.

The kinetic term of the traceless tensor mode is then given as

$$\int d^D x \left[\frac{D-3}{D-2} \left(h_{0\alpha\beta}\partial^4 h_0{}^{\alpha\beta} + 2\chi_{0\alpha}\partial^2\chi_{0\beta} \right) - \frac{D-3}{D-1}\chi_0^\alpha \partial_\alpha \partial_\beta \chi_0^\beta \right], \tag{3.18}$$

where $\chi_{0\alpha}$ is defined as

$$\chi_{0\alpha} = \partial^\beta h_{0\alpha\beta}. \tag{3.19}$$

Also, we introduce the gauge fixing term defined in Appendix B as

$$\int d^D x \frac{1}{\zeta_0} \chi_{0\alpha} N^{\alpha\beta} \chi_{0\alpha\beta} \tag{3.20}$$

where $N_{\alpha\beta}$ is defined as

$$N_{\alpha\beta} = \frac{2(D-3)}{D-2} \left(-2\eta_{\alpha\beta}\partial^2 + \frac{D-2}{D-1}\partial_\alpha \partial_\beta \right) \tag{3.21}$$

and the gauge parameter is renormalized using the renormalization factor of h as $\zeta_0 = Z_h \zeta$. The renormalization of the ghost sector defined in Appendix C is carried out as usual by introducing its own renormalization factor.

Now, we derive the propagator of the traceless tensor mode in arbitrary gauge. The full kinetic action of the traceless tensor mode in momentum space is

$$S_h^{kin} = \frac{1}{2} \int \frac{d^D k}{(2\pi)^D} h_{\alpha\beta}(k) K^{(\zeta)}_{\alpha\beta,\gamma\delta}(k) h_{\gamma\delta}(-k), \tag{3.22}$$

where taking into account of the traceless condition and the symmetry of indices, the complete form of the kernel $K^{(\zeta)}_{\alpha\beta,\gamma\delta}(k)$ is given as

$$\begin{aligned} K^{(\zeta)}_{\alpha\beta,\gamma\delta}(k) = \frac{2(D-3)}{D-2} \Bigg\{ & I^H_{\alpha\beta,\gamma\delta} k^4 + \frac{1-\zeta}{\zeta} \Bigg[\frac{1}{2} (\delta_{\alpha\gamma} k_\beta k_\delta + \delta_{\beta\gamma} k_\alpha k_\delta + \delta_{\alpha\delta} k_\beta k_\gamma + \delta_{\beta\delta} k_\alpha k_\gamma) k^2 \\ & - \frac{1}{D-1} (\delta_{\alpha\beta} k_\gamma k_\delta + \delta_{\gamma\delta} k_\alpha k_\beta) k^2 + \frac{1}{D(D-1)} \delta_{\alpha\beta}\delta_{\gamma\delta} k^4 - \frac{D-2}{D-1} k_\alpha k_\beta k_\gamma k_\delta \Bigg] \Bigg\}. \end{aligned} \tag{3.23}$$

Here, $I^H_{\alpha\beta,\gamma\delta}$ is an identity operator defined as

$$I^H_{\alpha\beta,\gamma\delta} = \frac{1}{2} (\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) - \frac{1}{D} \delta_{\alpha\beta}\delta_{\gamma\delta}, \tag{3.24}$$

which satisfies $(I^H)^2 = I^H$. Then, the equation of motion is expressed as $K^{(\zeta)}_{\alpha\beta,\gamma\delta}(k) h^{\gamma\delta}(k) = 0$. By solving the inverse of $K^{(\zeta)}_{\alpha\beta,\gamma\delta}(k)$, we can obtain the traceless tensor mode propagator as follows

$$\langle h_{\alpha\beta}(k) h_{\gamma\delta}(-k) \rangle = \frac{D-2}{2(D-3)} \frac{1}{k^4} I^{(\zeta)}_{\alpha\beta,\gamma\delta}(k), \tag{3.25}$$

where

$$I^{(\zeta)}_{\alpha\beta,\gamma\delta}(k) = \frac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) - \frac{1}{D}\delta_{\alpha\beta}\delta_{\gamma\delta}$$
$$+ (\zeta - 1)\left\{\frac{1}{2}\left(\delta_{\alpha\gamma}\frac{k_\beta k_\delta}{k^2} + \delta_{\alpha\delta}\frac{k_\beta k_\gamma}{k^2} + \delta_{\beta\gamma}\frac{k_\alpha k_\delta}{k^2} + \delta_{\beta\delta}\frac{k_\alpha k_\gamma}{k^2}\right)\right.$$
$$\left. - \frac{1}{D-1}\left(\delta_{\alpha\beta}\frac{k_\gamma k_\delta}{k^2} + \delta_{\gamma\delta}\frac{k_\alpha k_\beta}{k^2}\right) + \frac{1}{D(D-1)}\delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{D-2}{D-1}\frac{k_\alpha k_\beta k_\gamma k_\delta}{k^4}\right\}. \quad (3.26)$$

When we act momentum k^α on (3.25),

$$k^\alpha \langle h_{\alpha\beta}(k) h_{\gamma\delta}(-k)\rangle = \zeta \frac{D-2}{2(D-3)}\frac{1}{k^4}\left(\frac{1}{2}k_\gamma\delta_{\beta\delta} + \frac{1}{2}k_\delta\delta_{\beta\gamma} - \frac{1}{D}k_\beta\delta_{\gamma\delta}\right). \quad (3.27)$$

From this, the traceless tensor mode propagator satisfies the transverse condition when we take $\zeta = 0$, which is called Landau gauge.

When we expand the second term in equation (3.17), we obtain the three point interaction, which is given as

$$S^{(D-4)}_{F[\phi hh]} = (D-4)\int d^Dx\phi\Big[\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\gamma\partial^\delta h^{\alpha\beta} - 2\partial_\gamma\partial_\beta h_{\alpha\delta}\partial^\gamma\partial^\alpha h^{\beta\delta} + \partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\alpha\partial^\beta h^{\gamma\delta}$$
$$- \frac{4}{D-2}\left\{\frac{1}{4}\partial^2 h_{\alpha\beta}\partial^2 h^{\alpha\beta} - \partial^2 h^{\alpha\beta}\partial_\alpha\chi_\beta + \frac{1}{2}\partial_\alpha\chi_\beta\partial^\alpha\chi^\beta + \frac{1}{2}\partial_\alpha\chi_\beta\partial^\beta\chi^\alpha\right\}$$
$$+ \frac{2}{(D-1)(D-2)}\partial_\alpha\chi^\alpha\partial_\beta\chi^\beta\Big], \quad (3.28)$$

where $\chi_\alpha = \partial^\beta h_{\alpha\beta}$. We use this interaction to calculate the two-loop quantum gravity correction of the cosmological constant. The interaction term in momentum space is written in Appendix D because their expressions are very long.

3.3.2 The G_D term

The kinetic term of the conformal mode and their interaction arise from the D-dimensional Euler term G_D. From the expression of the bare coefficient b_0 (3.11), we can expand it in terms of the conformal mode as follows:

$$b_0\int d^Dx\sqrt{g}G_D = \frac{\mu^{D-4}}{(4\pi)^{\frac{D}{2}}}\int d^Dx\sqrt{\hat{g}}\left\{\left(\frac{b_1}{D-4} + \frac{b_2}{(D-4)^2} + \cdots\right)\bar{G}_4\right.$$
$$+ \left(b_1 + \frac{b_2}{D-4} + \cdots\right)\left(2\phi\bar{\Delta}_4\phi + \bar{G}_4\phi - \frac{2}{3}\bar{R}\bar{\nabla}^2\phi + \frac{1}{18}\bar{R}^2\right)$$
$$+ \{(D-4)b_1 + b_2 + \cdots\}\Big(\phi^2\bar{\Delta}_4\phi + \frac{1}{2}\bar{G}_4\phi^2 + 3\phi\bar{\nabla}^4\phi$$
$$+ 4\phi\bar{R}^{\alpha\beta}\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi - \frac{14}{9}\phi\bar{R}\bar{\nabla}^2\phi + \frac{10}{9}\phi\bar{\nabla}^\alpha\bar{R}\bar{\nabla}_\alpha\phi$$
$$\left. - \frac{7}{9}\bar{R}\bar{\nabla}^2\phi + \frac{1}{18}\bar{R}^2\phi + \frac{5}{108}\bar{R}^2\Big) + \cdots\right\}, \quad (3.29)$$

The first term in R.H.S of (3.29) is the counterterm to suppress the UV divergences proportional to $\bar{G}_4$, which determine the residue b_n in eq. (3.11). We here emphasize that the finite part proportional to b_1 in the second term gives the Riegert action (2.4), which is the Wess-Zumino action related to the conformal anomaly E_4 (B.87) at four dimensions.

We find that the kinetic term of the conformal mode is derived from the Riegert action as

$$S_{\phi}^{\text{kin}} = 2b \frac{\mu^{D-4}}{(4\pi)^{\frac{D}{2}}} \int d^D x \phi \partial^4 \phi. \tag{3.30}$$

From this, we can obtain the conformal mode propagator as follows:

$$\langle \phi(k)\phi(-k) \rangle = \frac{\mu^{4-D}(4\pi)^{\frac{D}{2}}}{4b} \frac{1}{k^4}. \tag{3.31}$$

Therefore, quantum corrections from this mode are expanded in a power series with respect to $1/b$, which corresponds to considering the large-N expansion for the number of matter fields N_S, N_F and N_A in eq. (2.6).

Next, we present gravitational interactions to calculate various diagrams in what follows. First, the three-point self-interaction induced in the second term is given as

$$S_{[\phi\phi\phi]}^{(D-4)b} = (D-4)b \frac{\mu^{D-4}}{(4\pi)^{\frac{D}{2}}} \int d^D x \phi^2 \partial^4 \phi. \tag{3.32}$$

Here, note that the contribution of this interaction to UV divergences arises in two or higher than two loop corrections because of the presence of the $D-4$ factor.

Furthermore, expanding the metric $\bar{g}_{\alpha\beta}$ in each term with respect to the traceless tensor mode, we obtain the interactions between the conformal mode and the traceless tensor mode. From the $-2\phi\bar{\nabla}^2\bar{R}/3$ and $\bar{R}^2/18$ terms in the third terms of eq. (3.29), we obtain two quadratic interactions

$$S_{G[\phi h]}^{bt} = -\frac{2}{3} bt \frac{\mu^{D/2-2}}{(4\pi)^{D/2}} \int d^D x \partial^2 \partial^\alpha \partial^\beta \phi h_{\alpha\beta}, \tag{3.33}$$

$$S_{G[hh]}^{bt^2} = \frac{bt^2}{(4\pi)^{D/2}} \int d^D x \frac{1}{18} \partial_\alpha \chi^\alpha \partial_\beta \chi^\beta. \tag{3.34}$$

We should note that these interactions do not contribute to loop calculations in Landau gauge. Therefore, when we choose Landau gauge, we can reduce the number of Feynman diagrams.

The three-point interaction derived from the $2\phi\bar{\Delta}_4\phi$ term is given as

$$S_{G[\phi\phi h]}^{bt} = b \frac{\mu^{D-4}}{(4\pi)^{D/2}} \int d^D x 2\phi \bar{\Delta}_4 \phi|_{\mathcal{O}(t)} \tag{3.35}$$

$$= bt \frac{\mu^{D/2-2}}{(4\pi)^{D/2}} \int d^D x h^{\alpha\beta} \left(4\partial_\alpha \phi \partial_\beta \partial^2 \phi + \frac{8}{3} \partial^\gamma \partial_\alpha \phi \partial_\gamma \partial_\beta \phi - \frac{4}{3} \partial^\gamma \phi \partial_\alpha \partial_\beta \partial_\gamma \phi - 4\partial_\alpha \partial_\beta \phi \partial^2 \phi \right) \tag{3.36}$$

and the four-point interaction is

$$
\begin{aligned}
S^{bt^2}_{G[\phi\phi hh]} &= 2b\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int d^Dx\phi\bar{\Delta}_4\phi|_{\mathcal{O}(t^2)} \\
&= 2bt^2\frac{1}{(4\pi)^{D/2}}\int d^Dx\phi\Big[h^{\alpha\beta}\Big(\partial_\alpha\partial_\beta h^{\gamma\delta}\partial_\gamma\partial_\delta\phi + 2\partial_\alpha h^{\gamma\delta}\partial_\beta\partial_\gamma\partial_\delta\phi \\
&\quad + h^{\gamma\delta}\partial_\alpha\partial_\beta\partial_\gamma\partial_\delta\phi + \frac{1}{2}\partial_\alpha h_{\gamma\beta}\partial^\gamma\partial^2\phi + \frac{1}{2}\chi_\beta\partial_\alpha\partial^2\phi + \partial_\alpha\partial_\beta\chi^\gamma\partial_\gamma\phi \\
&\quad + 2\partial_\alpha\chi^\gamma\partial_\beta\partial_\gamma\phi + 2\chi^\gamma\partial_\alpha\partial_\beta\partial_\gamma\phi\Big) + \chi^\alpha\partial_\alpha h^{\gamma\delta}\partial_\gamma\partial_\delta\phi + \chi^\alpha\partial_\alpha\chi^\beta\partial_\beta\phi \\
&\quad + \chi^\alpha\chi^\beta\partial_\alpha\partial_\beta\phi + \frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\partial^2\phi \\
&\quad + \partial^2\left\{\frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\phi + \frac{1}{2}h^{\alpha\beta}\partial_\alpha h^\gamma{}_\beta\partial_\gamma\phi + \frac{1}{2}h^{\alpha\beta}\chi_\beta\partial_\alpha\phi\right\} \\
&\quad + 2\phi\Big\{-\frac{1}{2}\partial^\alpha\chi^\beta\partial_\alpha h_{\beta\gamma}\partial^\gamma\phi - \frac{1}{2}\partial_\alpha\chi_\beta\partial^\beta h^{\alpha\gamma}\partial_\gamma\phi + \frac{1}{2}\partial^\alpha\chi^\beta\partial^\gamma h_{\alpha\beta}\partial_\gamma\phi + \frac{1}{2}\partial^2 h^{\alpha\beta}\partial_\alpha h_{\beta\gamma}\partial^\gamma\phi \\
&\quad - \frac{1}{4}\partial^2 h^{\alpha\beta}\partial^\gamma h_{\alpha\beta}\partial_\gamma\phi - h^{\alpha\beta}\partial_\beta\chi^\gamma\partial_\alpha\partial_\gamma\phi - h^{\alpha\beta}\partial^\gamma\chi_\beta\partial_\alpha\partial_\gamma\phi + \frac{1}{2}h^{\alpha\beta}\partial^2 h_{\beta\gamma}\partial_\alpha\partial^\gamma\phi \\
&\quad - \frac{1}{2}\partial^\gamma h_{\alpha\beta}\partial^\beta h_{\gamma\delta}\partial^\alpha\partial^\delta\phi - \frac{1}{4}\partial^\gamma h_{\alpha\beta}\partial^\delta h^{\alpha\beta}\partial_\gamma\partial_\delta\phi \\
&\quad - \frac{1}{2}\partial^\alpha(h_{\alpha\beta}\partial^\gamma h^{\beta\delta})\partial_\gamma\partial_\delta\phi + \frac{1}{2}\partial_\gamma(h_{\alpha\beta}\partial_\delta h^{\beta\gamma})\partial^\alpha\partial^\delta\phi + \frac{1}{2}\partial^\alpha(h_{\alpha\beta}\partial^\beta h_{\gamma\delta})\partial^\gamma\partial^\delta\phi\Big\} \\
&\quad - \frac{2}{3}\phi\Big\{-\chi^\alpha\Big(\partial_\beta\chi^\beta\partial_\alpha\phi + \frac{1}{2}\chi_\alpha\partial^2\phi\Big) - h^{\alpha\beta}(\partial_\alpha\chi_\beta\partial^2\phi + \partial_\gamma\chi^\gamma\partial_\alpha\partial_\beta\phi) - \frac{1}{4}\partial^\gamma h^{\alpha\beta}\partial_\gamma h_{\alpha\beta}\partial^2\phi\Big\} \\
&\quad + \frac{1}{3}\phi\Big\{-\frac{1}{2}\partial^\gamma h^{\alpha\beta}\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\delta\phi - \frac{1}{2}\partial_\alpha(\chi_\beta\chi^\beta)\partial^\alpha\phi - \partial_\gamma(h^{\alpha\beta}\partial_\alpha\chi_\beta)\partial^\gamma\phi - h^{\alpha\beta}\partial_\beta\partial_\gamma\chi^\gamma\partial_\alpha\phi\Big\}\Big].
\end{aligned}
\tag{3.37}
$$

Furthermore, we need the following interactions in order to calculate the two-loop quantum gravity corrections for cosmological constant in next chapter. The three-point interaction with bt^2 follows from $\phi(\bar{G}_4 - 2\bar{\nabla}^2\bar{R}/3)$ in the second term of eq. (3.29) is given as

$$
\begin{aligned}
S^{bt^2}_{G[\phi hh]} &= b\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int d^Dx\phi\left(\bar{G}_4 - \frac{2}{3}\bar{\nabla}^2\bar{R}\right)\Big|_{\mathcal{O}(t^2)} \\
&= bt^2\frac{1}{(4\pi)^{D/2}}\int d^Dx\phi\Big[\frac{4}{3}\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\gamma\partial^\delta h^{\alpha\beta} - 2\delta^{\beta\gamma}\partial_\epsilon\partial^\delta h_{\alpha\beta}\partial^\epsilon\partial^\alpha h_{\gamma\delta} \\
&\quad + \partial^\gamma\partial^\delta h_{\alpha\beta}\partial^\alpha\partial^\beta h_{\gamma\delta} - 2\delta^{\beta\delta}\partial_\epsilon\partial^\alpha h_{\alpha\beta}\partial^\epsilon\partial^\gamma h_{\gamma\delta} - 2\partial^\beta\partial^\gamma h_{\alpha\beta}\partial^\alpha\partial^\delta h_{\gamma\delta} \\
&\quad - \delta^{\alpha\gamma}\delta^{\beta\delta}\partial^2 h_{\alpha\beta}\partial^2 h_{\gamma\delta} + 4\delta^{\beta\delta}\partial^2 h_{\alpha\beta}\partial^\alpha\partial^\delta h_{\gamma\delta} + \partial^\alpha\partial^\beta h_{\alpha\beta}\partial^\gamma\partial^\delta h_{\gamma\delta} \\
&\quad + \frac{1}{3}\delta^{\alpha\gamma}\delta^{\beta\delta}\partial^\epsilon h_{\alpha\beta}\partial^2\partial_\epsilon h_{\gamma\delta} + \frac{1}{3}\delta^{\alpha\delta}\partial^2(\partial^\beta h_{\alpha\beta}\partial^\gamma h_{\gamma\delta}) + \frac{2}{3}\delta^{\beta\delta}\partial^2(h_{\alpha\beta}\partial^\alpha\partial^\gamma h_{\gamma\delta}) \\
&\quad + \frac{2}{3}h_{\alpha\beta}\partial^\alpha\partial^\beta\partial^\gamma\partial^\delta h_{\gamma\delta} + \frac{2}{3}\partial^\beta h_{\alpha\beta}\partial^\alpha\partial^\gamma\partial^\delta h_{\gamma\delta}\Big].
\end{aligned}
\tag{3.38}
$$

The three-point interaction with $(D-4)bt$ obtained by expanding the quadratic part of ϕ in the third terms of eq. (3.29) up to the order $\mathcal{O}(t)$ is given as

$$
\begin{aligned}
S_{G[\phi\phi h]}^{(D-4)bt} &= (D-4)b\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int d^Dx\left[\frac{1}{2}\bar{G}_4\phi^2+3\phi\bar{\nabla}^4\phi+\bar{R}^{\alpha\beta}\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi-\frac{14}{9}\phi\bar{R}\bar{\nabla}^2\phi+\frac{10}{9}\phi\bar{\nabla}^\gamma\bar{R}\bar{\nabla}_\gamma\phi\right]\Bigg|_{\mathcal{O}(t)} \\
&= -(D-4)bt\frac{\mu^{D/2-2}}{(4\pi)^{D/2}}\int d^Dx\phi\Big[2\partial^2h^{\alpha\beta}\partial_\alpha\partial_\beta\phi+6h^{\alpha\beta}\partial_\alpha\partial_\beta\partial^2\phi+6\chi^\alpha\partial_\alpha\partial^2\phi \\
&\qquad -4\partial^\alpha\chi^\beta\partial_\alpha\partial_\beta\phi-\frac{10}{9}\partial_\alpha\partial_\beta\chi^\alpha\partial^\beta\phi+\frac{14}{9}\partial_\alpha\chi^\alpha\partial^2\phi\Big]. \qquad (3.39)
\end{aligned}
$$

Moreover, expanding up to $\mathcal{O}(t^2)$, we obtain the following four-point interaction:

$$
\begin{aligned}
S_{G[\phi\phi hh]}^{(D-4)bt^2} &= (D-4)b\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int d^Dx\left[\frac{1}{2}\bar{G}_4\phi^2+3\phi\bar{\nabla}^4\phi+\bar{R}^{\alpha\beta}\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi-\frac{14}{9}\phi\bar{R}\bar{\nabla}^2\phi+\frac{10}{9}\phi\bar{\nabla}^\gamma\bar{R}\bar{\nabla}_\gamma\phi\right]\Bigg|_{\mathcal{O}(t^2)} \\
&= (D-4)bt^2\frac{1}{(4\pi)^{D/2}}\int d^Dx\Big[\frac{1}{2}(\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\gamma\partial^\delta h^{\alpha\beta}-2\partial_\gamma\partial_\beta h_{\alpha\delta}\partial^\gamma\partial^\alpha h^{\beta\delta} \\
&\quad +\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^{\alpha\beta}h^{\gamma\delta}-2\partial_\alpha\chi_\beta\partial^\alpha\chi^\beta-2\partial_\alpha\chi_\beta\partial^\alpha\chi^\beta-2\partial_\alpha\chi_\beta\partial^\beta\chi^\alpha \\
&\quad -\partial^2h_{\alpha\beta}\partial^2h^{\alpha\beta}+4\partial_\alpha\chi_\beta\partial^2h^{\alpha\beta}+\partial_\alpha\chi^\alpha\partial_\beta\chi^\beta)\phi^2 \\
&\quad +3\phi\Big\{\partial^2\left(\frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\phi+\frac{1}{2}h^{\alpha\beta}\partial_\alpha h^\gamma{}_\beta\partial_\gamma\phi+\frac{1}{2}h^{\alpha\beta}\chi_\beta\partial_\alpha\phi\right) \\
&\quad +h^{\alpha\beta}\Big(\partial_\alpha\partial_\beta h^{\gamma\delta}\partial_\gamma\partial_\delta\phi+2\partial_\alpha h^{\gamma\delta}\partial_\beta\partial_\gamma\partial_\delta\phi+h^{\gamma\delta}\partial_\alpha\partial_\beta\partial_\gamma\partial_\delta\phi+\frac{1}{2}\partial_\alpha h^\gamma{}_\beta\partial_\gamma\partial^2\phi \\
&\quad +\frac{1}{2}\chi_\beta\partial_\alpha\partial^2\phi+\partial_\alpha\partial_\beta\chi^\gamma\partial_\gamma\phi+2\partial_\alpha\chi^\gamma\partial_\beta\partial_\gamma\phi+2\chi^\gamma\partial_\alpha\partial_\beta\partial_\gamma\phi\Big) \\
&\quad +\chi^\alpha\partial_\alpha h^{\gamma\delta}\partial_\gamma\partial_\delta\phi+\chi^\alpha\partial_\alpha\chi^\beta\partial_\beta\phi+\chi^\alpha\chi^\beta\partial_\alpha\partial_\beta\phi+\frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\partial^2\phi\Big\} \\
&\quad +4\phi\Big\{-\frac{1}{2}\partial^\alpha\chi^\beta\partial_\alpha h_{\beta\gamma}\partial^\gamma\phi-\frac{1}{2}\partial_\alpha\chi_\beta\partial^\beta h^{\alpha\gamma}\partial_\gamma\phi+\frac{1}{2}\partial^\alpha\chi^\beta\partial^\gamma h_{\alpha\beta}\partial_\gamma\phi \\
&\quad +\frac{1}{2}\partial^2h^{\alpha\beta}\partial_\alpha h_{\beta\gamma}\partial^\gamma\phi-\frac{1}{4}\partial^2h^{\alpha\beta}\partial^\gamma h_{\alpha\beta}\partial_\gamma\phi-h^{\alpha\beta}\partial_\beta\chi^\gamma\partial_\alpha\partial_\gamma\phi-h^{\alpha\beta}\partial^\gamma\chi_\beta\partial_\alpha\partial_\gamma\phi \\
&\quad +\frac{1}{2}h^{\alpha\beta}\partial^2h_{\beta\gamma}\partial_\alpha\partial^\gamma\phi-\frac{1}{2}\partial^\gamma h_{\alpha\beta}\partial^\beta h_{\gamma\delta}\partial^\alpha\partial^\delta\phi-\frac{1}{4}\partial^\gamma h_{\alpha\beta}\partial^\delta h^{\alpha\beta}\partial_\gamma\partial_\delta\phi \\
&\quad -\frac{1}{2}\partial^\alpha(h_{\alpha\beta}\partial^\gamma h^{\beta\delta})\partial_\gamma\partial_\delta\phi+\frac{1}{2}\partial_\gamma(h_{\alpha\beta}\partial_\delta h^{\beta\gamma})\partial^\alpha\partial^\delta\phi+\frac{1}{2}\partial^\alpha(h_{\alpha\beta}\partial^\beta h_{\gamma\delta})\partial^\gamma\partial^\delta\phi\Big\} \\
&\quad -\frac{14}{9}\phi\Big\{-\chi^\alpha\left(\partial_\beta\chi^\beta\partial_\alpha\phi+\frac{1}{2}\chi_\alpha\partial^2\phi\right)-h^{\alpha\beta}(\partial_\alpha\chi_\beta\partial^2\phi+\partial_\gamma\chi^\gamma\partial_\alpha\partial_\beta\phi)-\frac{1}{4}\partial^\gamma h^{\alpha\beta}\partial_\gamma h_{\alpha\beta}\partial^2\phi\Big\} \\
&\quad +\frac{10}{9}\phi\Big\{-\frac{1}{2}\partial^\gamma h^{\alpha\beta}\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\delta\phi-\frac{1}{2}\partial_\alpha(\chi_\beta\chi^\beta)\partial^\alpha\phi-\partial_\gamma(h^{\alpha\beta}\partial_\alpha\chi_\beta)\partial^\gamma\phi \\
&\quad -h^{\alpha\beta}\partial_\beta\partial_\gamma\chi^\gamma\partial_\alpha\phi\Big\}\Big]. \qquad (3.40)
\end{aligned}
$$

3.4 Calculations of two-points functions

We present some results of the renormalization factors for loop diagrams with gravitational internal lines. Some of them have already been calculated elsewhere [11, 13]. We here add new calculations in arbitrary gauge as well [14].

First, we mention how to treat IR divergences. In fourth-order theories, in general, IR divergences become stronger than those in the usual second-order field theories. Further, since the Einstein term and the cosmological constant term have the exponential factor of ϕ, these terms cannot be considered as usual mass terms. So, we have to regularize IR divergences by introducing an infinitesimal mass parameter z in the propagators (3.25) and (3.31) as

$$\frac{1}{k^4} \to \frac{1}{k_z^4} = \frac{1}{(k^2+z^2)^2}, \tag{3.41}$$

while we do not introduce z in the tensor part $I^{(\zeta)}_{\alpha\beta,\gamma\delta}(k)$ to preserve the transverse and traceless properties. Since this mass parameter violates diffeomorphism invariance, it is a virtual parameter that should be canceled out at the end. This means that a massive graviton is not gauge invariant. In the first place, the ordinary particle picture itself is not true.

In Feynman diagrams, the conformal mode ϕ and the traceless tensor mode $h_{\alpha\beta}$ are respectively denoted by a solid line and a wave line.

3.4.1 Beta function

First, let us calculate the beta function of the coupling α_t defined by eq. (3.10). We here calculate the contribution from the two-point function of $h_{\alpha\beta}$ with an internal ϕ-line denoted by Figure.3.1, as an example.

Using the three-point interaction $S^{bt}_{\mathrm{G}[\phi\phi h]}$(3.36) with the momentum function $V^3_{\alpha\beta}$ (E.1), we can calculate the contribution from the diagram (a) in Figure.3.1 as

$$\begin{aligned}\Gamma_1^{\mathrm{W}} &= -\frac{\mu^{4-D}}{16}t^2 \int \frac{d^Dk}{(2\pi)^D} h_{\alpha\beta}(k)h_{\gamma\delta}(-k) \int \frac{d^Dp}{(2\pi)^D}\frac{1}{p_z^4(p+k)_z^4} \\ &\quad \times V^3_{\alpha\beta}(p,-p-k)V^3_{\gamma\delta}(-p,p+k) \\ &= \frac{\alpha_t}{4\pi}\int \frac{d^Dk}{(2\pi)^D} h_{\alpha\beta}(k)h_{\gamma\delta}(-k)\left\{\frac{1}{30}\left(\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}k^4 - \delta_{\alpha\gamma}k_\beta k_\delta k^2 \right.\right. \\ &\quad \left.\left. + \frac{1}{3}k_\alpha k_\beta k_\gamma k_\delta\right)\left(-\frac{2}{D-4} - \gamma + \ln 4\pi - \ln\frac{k^2}{\mu^2} + \frac{229}{60}\right) - \frac{1}{270}k_\alpha k_\beta k_\gamma k_\delta\right\},\end{aligned} \tag{3.42}$$

where there is no b dependence and IR divergences cancel out within this diagram. On the other hand, the tadpole diagram (b) coming from the four-point interaction $S^{bt^2}_{\mathrm{G}[\phi\phi hh]}$ (3.37) gives no contributions because the tadpole integral vanishes at the limit $z \to 0$ due to the presence of derivatives on the ϕ field in the interaction.

R.H.S of the above equation can be combined into the D-dimensional Weyl form and thus the effective action from Figure.3.1 is given by

$$\Gamma_1^{\rm W} = \frac{\alpha_t}{4\pi} \int \frac{d^D k}{(2\pi)^D} h_{\alpha\beta}(k) h_{\gamma\delta}(-k) \biggl\{ -\frac{1}{30} \left(\frac{2}{D-4} - \gamma + \ln 4\pi + \ln \frac{k^2}{\mu^2} - \frac{289}{60} \right) \\ \times \left[\frac{D-3}{D-2} \left(\delta_{\alpha\gamma}\delta_{\beta\delta} k^4 - 2\delta_{\alpha\gamma} k_\beta k_\delta k^2 \right) + \frac{D-3}{D-1} k_\alpha k_\beta k_\gamma k_\delta \right] \biggr\}. \quad (3.43)$$

This divergence can be removed using the field renormalization factor Z_h defined in (3.7) such that $Z_h - 1$ is taken to be $(1/15)(\alpha_t/4\pi)/(D-4)$. Since this diagram is gauge invariant, it has a relationship with the renormalization factor Z_t (3.7) such as $Z_t Z_h^{1/2} = 1$. Thus, we obtain the contribution to $Z_t - 1$ from Figure.3.1 to be $(-1/30)(\alpha_t/4\pi)/(D-4)$. This result is consistent with the previous calculation using the DeWitt-Schwinger method in four dimensions [17].

(a) (b)

Figure 3.1: The two-point function of $h_{\alpha\beta}$ corrected with the conformal mode ϕ

In general, the renormalization factor for the coupling constant is given by [32, 33, 34]

$$Z_t = 1 + \left[\frac{1}{240} \left(N_S + 6N_F + 12N_A \right) + \frac{197}{60} \right] \frac{\alpha_t}{4\pi} \frac{1}{D-4} + \mathcal{O}(\alpha_t^2). \quad (3.44)$$

For the contribution from the traceless tensor mode, we here quote the result [9, 17, 19] obtained by using the background field method [45] as follows. Introducing the background traceless tensor mode as $\hat{g}_{\alpha\beta} = (e^{t\hat{h}})_{\alpha\beta}$ and calculating the two-point function of the background $\hat{h}_{\alpha\beta}$, we obtain the contribution 199/60 for Z_t using the relation $Z_t Z_{\hat{h}}^{1/2} = 1$ ensured by the gauge invariance of the background, where $Z_{\hat{h}}$ is the renormalization factor of the background $\hat{h}_{\alpha\beta}$ (see footnote 1). The sum of this value and $-1/30$ from the conformal mode calculated above gives the last term at $\mathcal{O}(\alpha_t)$. Thus, we obtain the beta function (2.7) that has the negative value. The coupling α_t indicates the asymptotic freedom, which guarantees that we develop the perturbation theory about conformally flat spacetime.

Here, note that the asymptotic limit does not mean the realization of a picture in which free gravitons are propagating in the flat spacetime because the conformal mode is still non-perturbative and so the spacetime totally fluctuates quantum mechanically. And also, it indicates that scalar-like fluctuations by the conformal mode are much more dominant than tensor fluctuations at very high energies.

3.4.2 Non-renormalization theorem

In this subsection we demonstrate that the non-renormalization theorem $Z_{\phi=1}$ (3.8) indeed holds at $\mathcal{O}(\alpha_t)$ in arbitrary gauge. Figure 3.2 shows Feynman diagrams for the two-point function of

(a) Γ_1 (b) Γ_2

Figure 3.2: One loop correction of the conformal mode with respect to the traceless tensor mode

the conformal mode with an internal line of the traceless tensor mode.

These corrections are calculated as follows

$$\Gamma_1^R = 2b\frac{\alpha_t}{4\pi}\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int\frac{d^Dp}{(2\pi)^D}\phi(p)\phi(-p)p^4\Bigg[\frac{10}{3(D-4)}+\frac{5}{3}\left(\gamma-\ln 4\pi+\ln\frac{z^2}{\mu^2}\right)-\frac{43}{18} + \zeta\left\{\frac{8}{3(D-4)}+\frac{4}{3}\left(\gamma-\ln 4\pi+\ln\frac{z^2}{\mu^2}\right)-\frac{10}{9}\right\}\Bigg], \quad (3.45)$$

$$\Gamma_2^R = 2b\frac{\alpha_t}{4\pi}\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int\frac{d^Dp}{(2\pi)^D}\phi(p)\phi(-p)p^4\Bigg[-\frac{10}{3(D-4)}-\frac{5}{3}\left(\gamma-\ln 4\pi+\ln\frac{z^2}{\mu^2}\right) + \zeta\left\{-\frac{8}{3(D-4)}-\frac{4}{3}\left(\gamma-\ln 4\pi+\ln\frac{z^2}{\mu^2}\right)+\frac{13}{9}\right\}\Bigg]. \quad (3.46)$$

From these calculations, the one-loop correction for the two-point function is expressed as

$$\Gamma_{1+2}^R = 2b\frac{\alpha_t}{4\pi}\frac{\mu^{D-4}}{(4\pi)^{D/2}}\int\frac{d^Dp}{(2\pi)^D}\phi(p)\phi(-p)p^4\left[-\frac{25}{12}+\frac{1}{3}\zeta\right]. \quad (3.47)$$

The UV and IR divergences are respectively canceled and we find that the conformal mode is not renormalized. Thus, the renormalization factor of ϕ becomes $Z_\phi = 1$ at $\mathcal{O}(\alpha_t)$.

The other nontrivial test of he non-renormalization theorem have been done in the quantum conformal gravity coupled to QED at $\mathcal{O}(\alpha_e^3)$ and at $\mathcal{O}(\alpha_e^3/b)$ with a internal ϕ-line, where α_e is the fine structure constant of QED [11].

In the following calculations, we also see that the renormalization can be carried out with holding $Z_\phi = 1$.

Chapter 4

Renormalization of mass parameters

Let us calculate the anomalous dimensions of the Planck mass and the cosmological constant with holding $Z_\phi = 1$ [13, 14], including a consistency check of the renormalizability. We first calculate them at $t = 0$ and demonstrate that the results agree with the exact solution derived using the BRST conformal symmetry. We then calculate quantum corrections at the order of α_t.

4.1 Definitions of anomalous dimension

The bare Planck mass is represented with the renormalization factor Z_{EH} as

$$M_0^2 = \mu^{D-4} Z_{EH} M^2. \tag{4.1}$$

The anomalous dimension of the Planck mass is defined as

$$\gamma_{\rm EH} \equiv -\frac{\mu}{M^2}\frac{dM^2}{d\mu} = D - 4 + \bar{\gamma}_{\rm EH}, \tag{4.2}$$

where $\bar{\gamma}_{\rm EH} \equiv \mu\frac{d}{d\mu}\ln Z_{\rm EH}$.

The cosmological constant is represented with using the renormalization factor Z_Λ and the pure-pole factor L_M by

$$\Lambda_0 = \mu^{D-4} Z_\Lambda \left(\Lambda + L_M M^4\right). \tag{4.3}$$

Its anomalous dimension is then defined as

$$\gamma_\Lambda \equiv -\frac{\mu}{\Lambda}\frac{d\Lambda}{d\mu} = D - 4 + \bar{\gamma}_\Lambda + \frac{M^4}{\Lambda}\bar{\delta}_\Lambda, \tag{4.4}$$

where $\bar{\gamma}_\Lambda$ and $\bar{\delta}_\Lambda$ are respectively defined by

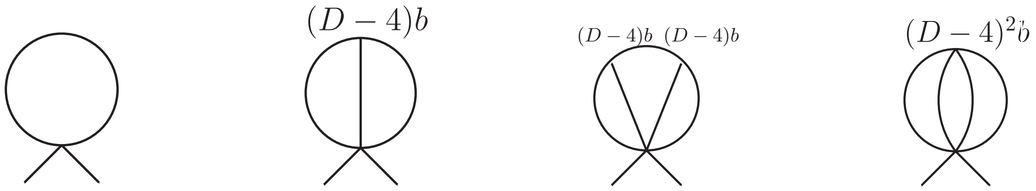
$$\bar{\gamma}_\Lambda = \mu\frac{d}{d\mu}\ln Z_\Lambda, \tag{4.5}$$

$$\bar{\delta}_\Lambda = \mu\frac{dL_M}{d\mu} - (D-4)L_M + (\bar{\gamma}_\Lambda - 2\bar{\gamma}_{EH})L_M. \tag{4.6}$$

In general, these anomalous dimensions are expanded in power series of $1/b_c$. Indeed, in the classical limit defined as $b_c \to \infty$, quantum corrections with respect to conformal mode should vanish since the conformal mode does not propagate in this limit. Therefore, the anomalous dimension of the cosmological constant should vanish at the classical limit even at $\alpha_t \neq 0$. On the other hand, the anomalous dimension of the Planck mass has a quantum correction by α_t that does not vanish at the classical limit.

4.2 Anomalous dimensions and BRST conformal symmetry

The anomalous dimension of the cosmological constant at $t = 0$ have been calculated up to the order of $1/b_c^3$. The corresponding Feynman diagrams are shown in Figure 4.1. These diagrams are evaluated with particular attention to the dependence of fictitious mass scale z. We then extract UV divergences only, which all yield simple poles, while IR divergences are ignored for the moment, which are discussed in the next chapter.



(a) (b) (c) (d)

Figure 4.1: Diagrams for the cosmological constant with respect to ϕ up to order $\mathcal{O}(\frac{1}{b^3})$

The UV divergences can be renormalized by taking the renormalization factor as

$$Z_\Lambda = 1 + \left(\frac{4}{b} + \frac{4}{b^2} + \frac{20}{3}\frac{1}{b^3} \right) \frac{1}{D-4}. \tag{4.7}$$

Using eq. (4.7), we thus obtain the following expression:

$$\bar{\gamma}_\Lambda = \frac{4}{b} + \frac{8}{b^2} + \frac{20}{b^3}. \tag{4.8}$$

This value vanishes at the large b limit, which is consistent with the classical limit.

Substituting b for b_c at last, we obtain it in four dimension at four dimensions. This result agrees with the first three terms of the $1/b_c$ expansion of the exact solution

$$\gamma_\Lambda = 2b_c \left(1 - \sqrt{1 - \frac{4}{b_c}} \right) - 4 \tag{4.9}$$

derived by using the BRST conformal symmetry.

We next present the anomalous dimension for the Planck mass and $\bar{\delta}_\Lambda$ in the mass-dependent part of (4.4). The Feynman diagrams for these quantities are shown in Figure.4.2, in which the

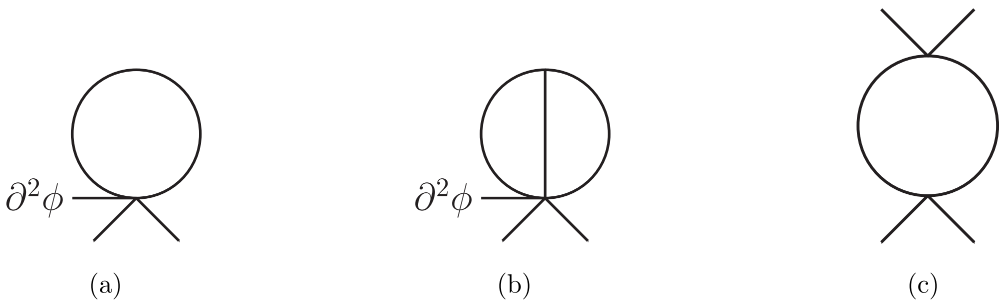


Figure 4.2: Diagrams for the Einstein Hilbert term with respect to ϕ up to order $\mathcal{O}(\frac{1}{b^2})$

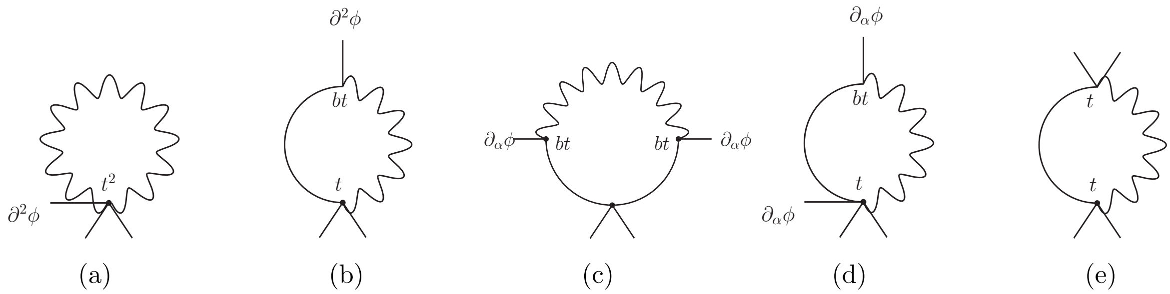


Figure 4.3: The first four diagrams are for Z_{EH} at $\mathcal{O}(\alpha_t)$ and the last one is for L_M at $\mathcal{O}(\alpha_t/b)$

first two contribute to Z_{EH} and the third contributes to L_M. We then obtain the renormalization factors up to the second order of $1/b$ as

$$Z_{EH} = 1 + \left(\frac{1}{b} + \frac{1}{2b^2}\right)\frac{1}{D-4}, \quad L_M = -\frac{9}{8}\frac{(4\pi)^2}{b^2}\frac{1}{D-4}. \tag{4.10}$$

From the first one, we obtain the following expression

$$\bar{\gamma}_{EH} = \frac{1}{b} + \frac{1}{b^2}. \tag{4.11}$$

By taking $b = b_c$ at last, we obtain the anomalous dimension of the Planck mass at four dimensions. This value also agrees with the exact solution $\bar{\gamma}_{EH} = 2b_c(1 - \sqrt{1 - 2/b_c}) - 2$ derived by using the BRST conformal symmetry. The pole term L_M gives the contribution

$$\bar{\delta}_M = -\frac{9(4\pi)^2}{8b^2}. \tag{4.12}$$

4.3 Anomalous dimensions at order of α_t

Next, we consider the contributions at $\mathcal{O}(\alpha_t)$ to the anomalous dimensions of the mass parameters.

Planck mass First, we consider quantum corrections to the Planck mass. The potentially divergent $\mathcal{O}(\alpha_t)$ Feynman diagrams in Landau gauge are shown in Figure. 4.3, in which the first four diagrams contribute to Z_{EH}. However, the three diagrams (4.3b), (4.3c) and (4.3d) have no UV divergences in Landau gauge. The last diagram (4.3e) that contributes to L_M also has no UV divergences. Thus, only (4.3a) gives the contribution as

$$Z_{EH} = 1 + \frac{5}{4}\frac{\alpha_t}{4\pi}\frac{1}{D-4}. \tag{4.13}$$

Combining with the coupling-independent part, we obtain the anomalous dimension of the Planck mass as

$$\bar{\gamma}_{EH} = \frac{1}{b} + \frac{1}{b^2} + \frac{5}{4}\frac{\alpha_t}{4\pi} \tag{4.14}$$

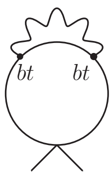
with $b = b_c$.

Cosmological constant The anomalous dimension of the cosmological constant at $\mathcal{O}(\alpha_t)$ arises at the two-loop level, which is exactly given as the function at $\mathcal{O}(\alpha_t/b)$ that vanishes at the classical limit of $b \to \infty$.

In Landau gauge, five Feynman diagrams contribute to the anomalous dimensions. In what follows, we calculate UV divergences of the respective diagrams in order with leaving gauge parameter ζ, which is taken to be zero when we calculate the anomalous dimensions at last.

We first calculate the contribution from the two-loop Feynman diagram that includes the diagram (D.1a) calculated in Section 3.4.2 and in Appendix E, which gives

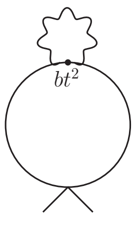
$\Gamma_1^{\Lambda} =$



$$= \frac{t^2}{b}\Lambda(4\pi)^{\frac{D}{2}-4}\mu^{4-D}(z^2)^{D-4}\int d^Dx e^{D\phi}$$
$$\times\left[\frac{40}{3(D-4)^2} + \frac{6}{D-4} + \zeta\left(\frac{32}{3(D-4)^2} + \frac{4}{D-4}\right)\right], \tag{4.15}$$

where the integrand F_1 is defined by (D.2). Two-loop Feynman diagram with the diagram (D.1b) is also calculated as

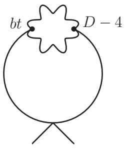
$\Gamma_2^{\Lambda} =$



$$= -\frac{t^2}{b}\Lambda\mu^{4-D}(z^2)^{D-4}(4\pi)^{\frac{D}{2}-4}\int d^Dx e^{D\phi}$$
$$\times\left[\frac{40}{3(D-4)^2} + \frac{124}{9(D-4)} + \zeta\left\{\frac{32}{3(D-4)^2} + \frac{47}{9(D-4)}\right\}\right], \tag{4.16}$$

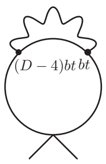
where the integrand F_2 is defined by (D.4). As a consequence of the non-renormalization theorem of ϕ discussed in Chapter 3, the double poles in Γ_1^Λ and Γ_2^Λ cancel out.

Besides, there are three two-loop Feynman diagrams that yield simple poles only. One of them is given as

$\Gamma_3^\Lambda =$ 

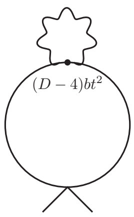
$$= \frac{t^2}{b}\mu^{4-D}\Lambda\mu^{4-D}\mu^{4-D}(z^2)^{D-4}(4\pi)^{\frac{D}{2}-4}\int d^Dx e^{D\phi}\left[\frac{25}{3(D-4)}+\frac{85}{6}\right], \quad (4.17)$$

where the integrand F_3 is defined by (D.6). This is independent of the gauge parameter. The others are given as

$\Gamma_4^\Lambda =$ 

$$= -\frac{t^2}{b}\Lambda\mu^{4-D}(z^2)^{D-4}(4\pi)^{\frac{D}{2}-4}\int d^Dx e^{D\phi}\left[\frac{20}{D-4}+\zeta\frac{16}{D-4}\right], \quad (4.18)$$

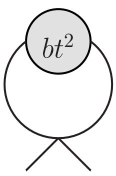
where F_4 is defined by (D.8) and

$\Gamma_5^\Lambda =$ 

$$= -\frac{t^2}{b}\Lambda\mu^{4-D}(z^2)^{D-4}(4\pi)^{\frac{D}{2}-4}\int d^Dx e^{D\phi}\left[\frac{20}{D-4}+\zeta\frac{16}{D-4}\right], \quad (4.19)$$

where F_5 is defined by (D.10).

Combining these five contributions, we finally obtain the following simple pole divergence:

$\Gamma^\Lambda =$ 

$$= \frac{t^2}{b}\Lambda(4\pi)^{\frac{D}{2}-4}\mu^{D-4}(z^2)^{D-4}\int d^Dx e^{D\phi}\left(\frac{155}{9(D-4)}+\zeta\frac{127}{9(D-4)}\right). \quad (4.20)$$

Thus, the two-loop renormalization factor of the cosmological constant given at the order of α_t/b is calculated in Landau gauge as

$$Z_\Lambda = 1 - \frac{155}{9b}\frac{\alpha_t}{4\pi}\frac{1}{D-4} \quad (4.21)$$

by taking $\zeta = 0$ at last.

The anomalous dimension of the cosmological constant is finally calculated as

$$\gamma_\Lambda = \frac{4}{b}+\frac{8}{b^2}+\frac{20}{b^3}-\frac{9(4\pi)^2}{8b^2}\frac{M^4}{\Lambda}-\frac{310}{9b}\frac{\alpha_t}{4\pi}, \quad (4.22)$$

where the last term is the two-loop contribution and the α_t-independent contributions calculated before are also added. This expression is physically acceptable since it vanishes at the classical limit.

Here, we have calculated the anomalous dimension in Landau gauge in order to reduce the number of Feynman diagrams and also to obtain physically acceptable results directly. It is because in arbitrary gauge the interaction $S^{bt}_{G[\phi h]}$ becomes effective and then yields contributions with positive power of b that do not vanish in the classical limit. Of course, such a unphysical behavior should disappear at last, but it is difficult to show that at present.

Chapter 5

Physical cosmological constant

Recall that the gravitational theories based on the Einstein action can not go beyond the Planck scale. So, the Planck mass scale gives the UV cutoff of such classical or quantum theories. And also, the existence of the UV cutoff is one of the reason behind the cosmological constant problem [43]. On the other hand, our theory does not have UV cutoff, and so we can discuss this problem free from it.

So, in this chapter, we first consider what the physical cosmological constant is [44]. Here, recall that when we define a physical mass of particle in quantum field theories, we usually adopt the on-shell renormalization scheme. On the other hand, such a scheme is not known for certain for renormalization of the cosmological constant in quantum gravity. Thus, it is the matter what the physical quantity is. In order to answer it, we here examine the effective action of the cosmological term and its behavior under the renormalization group flow.

As for considering the cosmological term, it is sufficient to consider the effective action that depends on the conformal mode background σ only, which is expanded in a power series as

$$\begin{aligned}\Gamma(\sigma) &= \sum_n \frac{1}{n!}\int d^Dx_1\cdots d^Dx_n \Gamma^{(n)}(x_1,\cdots,x_n)\sigma(x_1)\cdots\sigma(x_n) \\ &= \sum_n \frac{1}{n!}\int \frac{d^Dk_1}{(2\pi)^D}\cdots\frac{d^Dk_n}{(2\pi)^D}(2\pi)^D\delta^{(D)}(k_1+\cdots+k_n) \\ &\qquad \times \Gamma^{(n)}(k_1,\cdots,k_n)\sigma(k_1)\cdots\sigma(k_n), \end{aligned} \tag{5.1}$$

where $\Gamma^{(n)}$ is the n-point Green function given as the sum of all 1PI Feynman diagrams with n external legs of σ. In what follows, we first study the renormalization group equation for the n-point Green function.

The renormalization group analysis of $\Gamma^{(n)}$ can be carried out as in the case of the φ^4-theory [46, 47, 48, 49, 50]. One of the crucial difference is that the conformal mode is not renormalized such that $Z_\phi = 1$. The background field σ is also not renormalized (see footnote 1). Therefore, the renormalized $\Gamma^{(n)}$ is the same as the bare one, and thus $\mu d\Gamma^{(n)}/d\mu = 0$ is satisfied.

The effective potential V is given by the zero momentum part of $\Gamma^{(n)}(k_1,\cdots,k_n)$, which is expressed as

$$V(\sigma) = \sum_n \Gamma^{(n)}(0, \cdots, 0) \int d^D x \sigma^n(x). \tag{5.2}$$

The diffeomorphism invariance implies that $\Gamma^{(n)}(0, \cdots, 0) = vD^n$ and thus the effective potential has the form

$$V(\sigma) = v \int d^D x e^{D\sigma(x)}. \tag{5.3}$$

The renormalization group equation implies that v is scale invariant as

$$\mu \frac{d}{d\mu} v = 0. \tag{5.4}$$

We thus find that the effective potential is the physical cosmological constant, which can be observed cosmologically.

Before calculating the physical cosmological constant at the one-loop level explicitly, we first discuss the renormalization structure of the effective action, which will gives a renormalization group improvement of it.

5.1 Renormalization group structure

The renormalization group equation is derived from the condition $\mu d\Gamma^{(n)}/d\mu = 0$, which gives the following equation:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_t \alpha_t \frac{\partial}{\partial \alpha_t} - \gamma_\Lambda \Lambda \frac{\partial}{\partial \Lambda} - \gamma_{EH} M^2 \frac{\partial}{\partial M^2} \right) \Gamma^{(n)} \left(k_j, \alpha_t, \Lambda, M^2, \mu \right) = 0, \tag{5.5}$$

where we take $D = 4$ and thus the bar on the renormalization group quantities are suppressed here, which are used to define the running couplings later, and also the differential term $(D - 4)\bar{\beta}_b \partial/\partial b$ is removed.

Changing the momentum variable as $k_j \to \lambda k_j$ and do the dimensional analysis. Then, we find that $\Gamma^{(n)}$ has the following form:

$$\begin{aligned} \Gamma^{(n)}\left(\lambda k_j, \alpha_t, \Lambda, M^2, \mu\right) = \mu^4 \Omega_1^{(n)} \left(\frac{\lambda k_j}{\mu}, \alpha_t, \frac{\Lambda}{\mu^4}, \frac{M^2}{\mu^2} \right) + \Lambda \Omega_2^{(n)} \left(\frac{\lambda k_j}{\mu}, \alpha_t, \frac{\Lambda}{\mu^4}, \frac{M^2}{\mu^2} \right) \\ + M^4 \Omega_3^{(n)} \left(\frac{\lambda k_j}{\mu}, \alpha_t, \frac{\Lambda}{\mu^4}, \frac{M^2}{\mu^2} \right). \end{aligned} \tag{5.6}$$

This implies that $\Gamma^{(n)}$ satisfies the differential equation

$$\left(\mu \frac{\partial}{\partial \mu} + 4\Lambda \frac{\partial}{\partial \Lambda} + 2M^2 \frac{\partial}{\partial M^2} + \lambda \frac{\partial}{\partial \lambda} - 4 \right) \Gamma^{(n)} \left(\lambda k_j, \alpha_t, \Lambda, M^2, \mu \right) = 0. \tag{5.7}$$

Therefore, combining (5.5) with (5.7) and removing the partial derivative with respect to μ, we obtain the expression

$$\begin{aligned}\Big[-\lambda\frac{\partial}{\partial\lambda}+\beta_t(\alpha_t)\alpha_t\frac{\partial}{\partial\alpha_t}-(4+\gamma_\Lambda(\alpha_t,\Lambda,M^2))\Lambda\frac{\partial}{\partial\Lambda}\\ -(2+\gamma_{EH}(\alpha_t))M^2\frac{\partial}{\partial M^2}+4\Big]\Gamma^{(n)}(\lambda k_j,\alpha_t,\Lambda,M^2,\mu)=0.\end{aligned} \quad (5.8)$$

Here, we introduce the running coupling constant $\bar{\alpha}_t(\lambda)$, the running cosmological constant $\bar{\Lambda}(\lambda)$ and the running Planck mass $\bar{M}(\lambda)$, which are defined using the following differential equations:

$$\begin{aligned}-\lambda\frac{d}{d\lambda}\bar{\alpha}_t(\lambda)&=\beta_t\big(\bar{\alpha}_t(\lambda)\big)\bar{\alpha}_t(\lambda),\\ -\lambda\frac{d}{d\lambda}\bar{\Lambda}(\lambda)&=-\left[4+\gamma_\Lambda\big(\bar{\alpha}_t(\lambda),\bar{\Lambda}(\lambda),\bar{M}^2(\lambda)\big)\right]\bar{\Lambda}(\lambda),\\ -\lambda\frac{d}{d\lambda}\bar{M}^2(\lambda)&=-\left[2+\gamma_{EH}\big(\bar{\alpha}_t(\lambda)\big)\right]\bar{M}^2(\lambda).\end{aligned} \quad (5.9)$$

If we replace α_t, Λ and M with the corresponding running quantities in equation (5.8), we find that this equation can be written with the help of the defining equations (5.9) as

$$\left(-\lambda\frac{d}{d\lambda}+4\right)\Gamma^{(n)}\big(\lambda k_j,\bar{\alpha}_t(\lambda),\bar{\Lambda}(\lambda),\bar{M}^2(\lambda),\mu\big)=0. \quad (5.10)$$

Here, we should note that this equation is written in terms of total differential with respect to λ, not partial one. The solution of this renormalization group equation is thus given as

$$\Gamma^{(n)}\big(\lambda k_j,\bar{\alpha}_t(\lambda),\bar{\Lambda}(\lambda),\bar{M}^2(\lambda),\mu\big)=\lambda^4\Gamma^{(n)}(k_j,\alpha_t,\Lambda,M^2,\mu) \quad (5.11)$$

by setting the conditions of $\bar{\alpha}_t(1)=\alpha_t$, $\bar{\Lambda}(1)=\Lambda$ and $\bar{M}^2(1)=M^2$.

From the solution of the renormalization group equations (5.11) with $k_j=0$ and the expression (5.2), we obtain the following equation:

$$V\big(\bar{\alpha}_t(\lambda),\bar{\Lambda}(\lambda),\bar{M}^2(\lambda),\mu\big)=\lambda^4V(\alpha_t,\Lambda,M^2,\mu). \quad (5.12)$$

Thus, the physical cosmological constant improved by renormalization group equation is given as

$$V=\bar{v}(\lambda)e^{4\sigma},\qquad \bar{v}(\lambda)=\lambda^{-4}v(\bar{\alpha}_t(\lambda),\bar{\Lambda}(\lambda),\bar{M}^2(\lambda),\mu), \quad (5.13)$$

which does not depend on the renormalization group parameter such that $\lambda d\bar{v}(\lambda)/d\lambda=0$.

5.2 Explicit form of the physical cosmological constant

Having seen above, the effective potential gives the physical cosmological constant that is independent of the renormalization group flow. Here, we calculate explicit form of it at the one-loop level, in which the background field σ is taken to be a constant. We then consider the large b limit, while the ratios Λ/b, M^2/b and bt^2 are taken to be the order of unity. In this limit, the one-loop approximation becomes valid and loop corrections to the effective potential are written by a function of these ratios.

The conformal mode is here separated into the constant background and quantum field $\tilde{\phi}$ as

$$\phi = \sigma + \tilde{\phi}. \tag{5.14}$$

Expanding the gravitational action up to the second order of the quantum fields $\tilde{\phi}$ and $h_{\alpha\beta}$ in Landau gauge, we obtain the following action:

$$S^{\mathrm{kin}} = S_{\phi^2} + S_{h^2} + S_c, \tag{5.15}$$

where each term is given as

$$\begin{aligned}
S_{\phi^2} &= \int d^D x \bigg\{ \frac{\mu^{D-4}}{(4\pi)^{D/2}} \big[2b\tilde{\phi}\partial^4\tilde{\phi} + (D-4)b(2\sigma+3)\tilde{\phi}\partial^4\tilde{\phi} \big] \\
&\quad + \frac{(D-1)(D-2)}{2}\mu^{D-4}M^2 e^{(D-2)\sigma}\tilde{\phi}\partial^2\tilde{\phi} + \mu^{D-4}\Lambda e^{D\sigma}\left(1+\frac{D^2}{2}\tilde{\phi}^2\right)\bigg\}, \\
S_{h^2} &= \int d^D x \bigg\{ \frac{1}{2} h_{\alpha\beta} K^{(0)\alpha\beta,\gamma\delta} + (D-4)\frac{D-3}{D-2}\sigma h_{\alpha\beta}\partial^4 h^{\alpha\beta} \\
&\quad - \frac{t^2}{8}M^2 e^{(D-2)\sigma} h_{\alpha\beta}\partial^2 h^{\alpha\beta} \bigg\}, \\
S_c &= \int d^D x \mu^{D-4} \big[(Z_\Lambda - 1)\Lambda + L_M M^4 \big] e^{D\sigma},
\end{aligned} \tag{5.16}$$

where $K^{(0)\alpha\beta,\gamma\delta}$ is the differential operator in Landau gauge whose momentum representation is given by (3.23). The renormalization factors in the last counterterm are given by those calculated in the previous chapter.

As mentioned before, in fourth order theories, IR divergences become strong. In the following calculations, we take care of IR divergences and show that they indeed disappear at last.

Contributions from the conformal mode We first calculate the contribution from $\tilde{\phi}$ to the effective potential. In order to normalize the action, we rescale the quantum field $\tilde{\phi}$ as

$$\tilde{\phi} = \sqrt{\frac{(4\pi)^{\frac{D}{2}}}{4b\mu^{D-4}\left\{1+(D-4)\left(\frac{3}{2}+\sigma\right)\right\}}}\varphi. \tag{5.17}$$

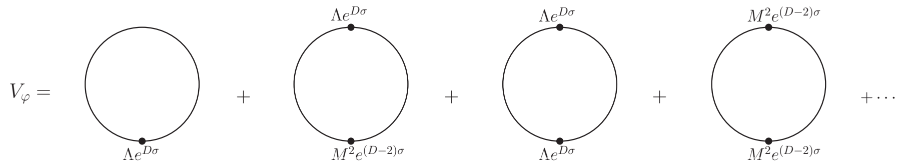


Figure 5.1: One-loop contributions from the conformal mode

We then obtain the following expression

$$S_{\phi^2} = \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}\varphi(k)\mathcal{D}_\varphi \varphi(-k) = \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}\varphi(k)\big[k^4 - Ak^2 + B\big]\varphi(-k), \tag{5.18}$$

where A and B are defined as

$$\begin{aligned} A &= \frac{(4\pi)^{\frac{D}{2}}(D-1)(D-2)}{4b\left[1+(D-4)\left(\sigma+\frac{3}{2}\right)\right]}M^2 e^{(D-2)\sigma}, \\ B &= \frac{(4\pi)^{\frac{D}{2}}D^2}{4b\left[1+(D-4)\left(\sigma+\frac{3}{2}\right)\right]}\Lambda e^{D\sigma}. \end{aligned} \tag{5.19}$$

The one-loop correction to the effective potential is then expressed as follows:

$$\begin{aligned} V_\phi &= -\ln\big[\det(\mathcal{D}_0^{-1}\mathcal{D}_\varphi)\big]^{-\frac{1}{2}} \\ &= \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}\ln\left(1 - \frac{A}{k^2} + \frac{B}{k^4}\right), \end{aligned} \tag{5.20}$$

where $\mathcal{D}_0 = k^4$ is the inverse of the propagator of the rescaled field φ. The corresponding diagrams are shown in Figure. 5.1. Expanding the logarithmic function in a power series of A and B, we obtain the following expression:

$$\begin{aligned} V_\phi &= \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\int \frac{d^D k}{(2\pi)^D}\left(-\frac{A}{k^2} + \frac{B}{k^4}\right)^n \\ &= \frac{1}{2}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\frac{(-1)^{n-1}}{n}\frac{n!}{m!(n-m)!}(-A)^m B^{n-m} I_{2n-m}(z), \end{aligned} \tag{5.21}$$

where the loop integral I_ℓ is defined as

$$I_\ell(z) = \int \frac{d^D k}{(2\pi)^D}\frac{1}{(k^2+z^2)^\ell}, \tag{5.22}$$

which is evaluated by introducing infinitesimal fictitious mass z as mentioned before. The integral I_ℓ has UV and IR divergences for $\ell \le 2$, while for $\ell > 2$ it has only IR divergences. After carrying out the calculation, we take the vanishing limit of the mass z.

The integral I_1 vanishes at the limit $z \to 0$, while I_2 has both UV and IR divergences as

$$I_2(z) = \frac{1}{(4\pi)^2}\left(-\frac{2}{D-4} - \gamma + \ln 4\pi - \ln z^2\right). \tag{5.23}$$

The integral with $\ell > 2$ has IR divergences only written in a negative powers of z as

$$I_\ell(z) = \frac{1}{(4\pi)^2}\frac{1}{(\ell-1)(\ell-2)}\left(\frac{1}{z^2}\right)^{\ell-2}.$$

Substituting these results into (5.21), we obtain the following expression:

$$V_\phi = \frac{1}{(4\pi)^2}\left[\left(\frac{B}{2}-\frac{A^2}{4}\right)\left(-\frac{2}{D-4}-\gamma+\ln 4\pi-\ln z^2\right)+\frac{AB}{4}\frac{1}{z^2}-\frac{B^2}{24}\frac{1}{z^4}\right]$$
$$+\frac{1}{2(4\pi)^2}\sum_{n=3}^{\infty}\sum_{m=0}^{n}\frac{(-1)^{n-1}}{n}\frac{n!}{m!(n-m)!}(-1)^m A^m B^{n-m}\frac{(z^2)^{2-2n+m}}{(2n-m-1)(2n-m-2)}. \quad (5.24)$$

The sum of the infinite series part can be evaluated using the formula given in Appendix G. We can then take the limit of $z \to 0$ and show that IR divergences indeed cancel out. In this way, we obtain the expression that has UV divergences only as follows:

$$V_\phi = \frac{1}{(4\pi)^2}\left(\frac{B}{2}-\frac{A^2}{4}\right)\left(-\frac{2}{D-4}-\gamma+\ln 4\pi\right)$$
$$+\frac{1}{(4\pi)^2}\left[\frac{1}{8}(2B-A^2)(3-\ln B)-\frac{A}{4}\sqrt{4B-A^2}\arccos\left(\frac{A}{2\sqrt{B}}\right)\right]. \quad (5.25)$$

Substituting the expression of A and B, the part with the pole is now expanded as follows:

$$\frac{1}{(4\pi)^2}\left(\frac{A^2}{2}-B\right)\frac{1}{D-4} = \frac{9\pi^2 M^4 e^{D\sigma}}{b^2}\left(\frac{2}{D-4}+2\ln 4\pi-2\sigma-\frac{8}{3}\right)$$
$$-\frac{2\Lambda e^{D\sigma}}{b}\left(\frac{2}{D-4}+\ln 4\pi-2\sigma-2\right). \quad (5.26)$$

The UV divergences are subtracted by the counterterms in the MS scheme. Taking $D=4$ and combining all finite terms, we obtain the following effective potential:

$$V_\phi = e^{4\sigma}\left[\frac{\Lambda}{b}(7-2\ln 4\pi)-\frac{9\pi^2 M^4}{2b}\left(\frac{25}{3}-4\ln 4\pi\right)\right.$$
$$\left.-\left(\frac{\Lambda}{b}-\frac{9\pi^2 M^4}{2b^2}\right)\ln\frac{64\pi^2\Lambda}{b\mu^4}-\frac{6\pi M^2}{b}\sqrt{\frac{\Lambda}{b}-\frac{9\pi^2 M^4}{4b^2}}\arccos\left(\frac{3\pi M^2}{2\sqrt{b\Lambda}}\right)\right]. \quad (5.27)$$

Here, note that there is no σ-dependence apart from the overall factor.

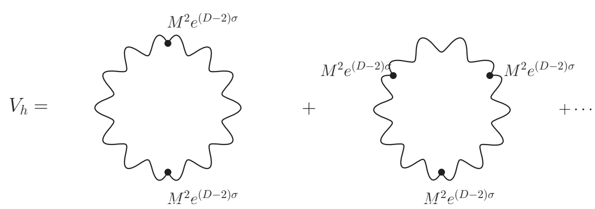


Figure 5.2: One-loop contributions from the traceless tensor mode

Contributions from the traceless tensor mode We next calculate the contribution from the traceless tensor mode for the effective potential in Landau gauge, whose diagrams are shown in Figure. 5.2 We here rewrite the action in the following form

$$S_{h^2} = \frac{1}{2}\int \frac{d^D k}{(2\pi)^D} h_{\alpha\beta}(k) K_h^{\alpha\beta,\gamma\delta}(k) h_{\gamma\delta}(-k). \tag{5.28}$$

The function $K_h^{\alpha\beta,\gamma\delta}$ of momentum is given as

$$K_h^{\alpha\beta,\gamma\delta} = K^{(0)\alpha\beta}{}_{,\mu\nu}\left[I_H^{\mu\nu,\gamma\delta} + \left\{(D-4)\sigma + \frac{C}{k^2}\right\} I^{(0)\mu\nu,\gamma\delta}\right], \tag{5.29}$$

$$C = t^2 \frac{D-2}{8(D-3)} M^2 e^{(D-2)\sigma}, \tag{5.30}$$

where $K^{(0)\alpha\beta,\gamma\delta} = K^{(\zeta)\alpha\beta,\gamma\delta}|_{\zeta\to 0}$. The one-loop correction to the effective potential is then given as

$$\begin{aligned} V_h &= -\ln\left[\det\left((K^{(0)-1})^{\alpha\beta}{}_{,\mu\nu} K_h^{\mu\nu,\gamma\delta}\right)\right]^{-\frac{1}{2}} \\ &= \frac{(D+1)(D-2)}{4}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left[C^n I_n(z) + (D-4)n\sigma C^{n-1} I_{n-1}(z)\right], \end{aligned} \tag{5.31}$$

where $\left(K^{(0)-1}\right)^{\alpha\beta,\gamma\delta}$ is the free propagator of the traceless tensor mode in Landau gauge and we use $\mathrm{Tr}[(I^{(0)})^n] = \mathrm{Tr}[I^{(0)}] = (D+1)(D-2)/2$. Evaluating loop integrals as mentioned above, we obtain the following expression:

$$\begin{aligned} V_h = \frac{(D+1)(D-2)}{4(4\pi)^2}\Bigg[&\left(\frac{1}{2} - (D-4)\sigma\right) C^2 \left(\frac{2}{D-4} - \gamma + \ln 4\pi + \ln z^2\right) \\ &+ z^4 g\left(\frac{C}{z^2}\right) + (D-4)\sigma C^2 h\left(\frac{C}{z^2}\right)\Bigg], \end{aligned} \tag{5.32}$$

where the functions g and h are defined as (G.1) and (G.2) in Appendix G. This also becomes finite in the $z \to 0$ limit and thus all IR divergences cancel out.

Substituting the expressions of C into the expression above and expanding around four dimensions, we can separate it into the UV divergent part and the finite part at four dimensions. After subtracting the UV divergence using the counterterm, we obtain the one-loop contribution from the traceless tensor mode as

$$V_h = \frac{5}{128} e^{4\sigma} \frac{t^4}{(4\pi)^2} M^4 \left[\ln\frac{t^4 M^4}{16\mu^4} - \frac{21}{5}\right]. \tag{5.33}$$

This also becomes independent of σ apart from the overall factor.

Combining two contributions calculated above, we finally obtain the effective potential for the cosmological constant term, which is expressed as

$$V = \Lambda e^{4\sigma} + V_\phi + V_h = v(\alpha_t, \Lambda, M^2, \mu) e^{4\sigma}, \tag{5.34}$$

where

$$
\begin{aligned}
v = \Lambda + \frac{\Lambda}{b}(7 - 2\ln 4\pi) - \frac{9\pi^2 M^4}{2b}\left(\frac{25}{3} - 4\ln 4\pi\right) \\
- \left(\frac{\Lambda}{b} - \frac{9\pi^2 M^4}{2b^2}\right)\ln\frac{64\pi^2\Lambda}{b\mu^4} - \frac{6\pi M^2}{b}\sqrt{\frac{\Lambda}{b} - \frac{9\pi^2 M^4}{4b^2}}\arccos\left(\frac{3\pi M^2}{2\sqrt{b\Lambda}}\right) \\
+ \frac{5}{128}\alpha_t^2 M^4\left[\ln\frac{(4\pi)^2\alpha_t^2 M^4}{16\mu^4} - \frac{21}{5}\right].
\end{aligned}
\tag{5.35}
$$

Thus, we obtain the physical cosmological constant v as a function of the renormalized quantities of the cosmological constant and the Planck mass and the coupling constant. What v becomes independent of σ reflects the invariance under the renormalization group flow discussed before. This indicates that if we take the physical cosmological constant small or zero initially at $\bar{\alpha}_t = 0$, the value is preserved even at low energies.

This result also indicates in the view point of quantum field theory as follows. The cosmological constant in the action should be positive and not so small since the action is bounded from below enough for the path integral to be stable. Nevertheless, we can take the physical cosmological constant any values even though the cosmological constant in the action is not small.

Chapter 6

Summary and discussion

We studied the renormalizable quantum conformal gravity and its renormalization structure using dimensional regularization. It is a UV complete quantum gravity theory without UV cutoff and thus it can describe spacetime dynamics beyond the Planck scale. The gravitational actions are defined by the square of the D-dimensional Weyl tensor F_D (3.2) and the D-dimensional extension of the Euler density G_D (3.3), which are determined through the analysis of Hathrell's renormalization group equations (see appendix B). The theory has a single dimensionless coupling t that is introduced in front of the Weyl action and thus the dynamics of the traceless tensor mode is handled in the perturbation theory. In contrast, the conformal factor of the metric field is treated exactly in the exponential form of the conformal mode in which we do not introduce the coupling constant. The dynamics of this mode is governed by the Wess-Zumino actions induced quantum mechanically. Especially, its kinetic term is given by the Riegert action at the zeroth order of the coupling t. The coefficient of the Riegert action denoted by b_c is given by the one-loop coefficient of the conformal anomaly proportional to the Euler term, and thus quantum corrections by the conformal mode is expanded by the inverse of b_c.

We first provided the two-point functions of gravitational fields, including the calculation of the beta function and the demonstration of the non-renormalization theorem represented by $Z_\phi = 1$. We then studied the conformal gravity system with adding the Einstein-Hilbeert action and the cosmological term, and calculated the anomalous dimensions of the cosmological constant and the Planck mass. Even at the zeroth order of t, there are loop corrections for these mass parameters. The anomalous dimension for the cosmological constant has been calculated at three-loop level up to the order of $1/b_c^3$ and that for the Planck mass was calculated up to the order of $1/b_c^2$. As a consistency check, it has been found that their results agree with the exact solutions obtained by imposing the BRST conformal invariance.

One of the main calculations in this thesis is the calculation of the anomalous dimensions up to the order of $\alpha_t = t^2/4\pi$. In order to reduce the number of Feynman diagrams and to avoid some indeterminate factors, we choose Landau gauge. We then found that the anomalous dimension with respect to the Planck mass is positive, but that of the cosmological constant, which is given by two-loop diagrams, becomes negative as given in (4.22). Field-theoretically, it gives a non-trivial examination of two-loop renormalizability of our quantum gravity theory. At this time, we would like to emphasize the fact that there is no other renormalizable quantum gravity theories that are well-defined at the higher loop level.

We also studied the renormalization group equation of the effective action with respect to the cosmological term that depends on the conformal mode only. Due to the diffeomorphism invariance and the non-renormalizability of the conformal mode, we showed that the effective potential becomes invariant under the renormalization group flow. Thus, it was found that the effective potential gives the physical cosmological constant we can observe.

The physical cosmological constant was calculated at the one-loop level explicitly. It consists of two renormalized parameters of the Planck mass and the cosmological constant so that we can take its value small actually without suffering from the instability of the path integral as mentioned in the last of Chapter 5. So, we take the value small and it will be passed on to the low energy effective theory of gravity given by an expansion in derivatives of the metric field [52]. It will give a new perspective on the cosmological constant problem free from UV cutoff.

In fourth order quantum field theory, IR divergences become stronger than those of usual second order theories. So, throughout our calculations, we introduce a fictitious small mass to regularize IR divergences, which violates diffeomorphism invariance. As a consistency check, we showed that all IR divergences indeed cancel out, especially in the calculation of the effective potential in which more strong IR divergences arise.

Finally, we discuss the physical meanings of which the beta function of the coupling constant t becomes negative. This indicates that at very high energies fluctuations of the traceless tensor mode become less dominant, while those of the conformal mode are still filled in spacetime non-perturbatively, which results in the background-metric independent nature, called the BRST conformal symmetry. The negativity of the beta function also indicates that there is a new dynamical IR scale denoted by Λ_{QG} where the conformal invariance breaks down. Thus, at this scale, the phase transition occurs and spacetime changes from the conformally invariant phase to the present classical phase in which gravitons and elementary particles propagates in the fixed background. We can construct an inflationary scenario that begins at the Planck mass scale M_{P} and ends at the dynamical scale Λ_{QG} if we take their order as $M_{\mathrm{P}} \geq \Lambda_{\mathrm{QG}}$ [51, 52, 53].

What is the physical quantity in our quantum gravity theory which can be observed through cosmological experiments? The physical cosmological constant is one of them. The physical Planck mass also will be defined in the same way through the effective action whose form is fixed by diffeomorphism invariance. On the other hand, we can not define the S-matrix as a physical quantity since spacetime still fully fluctuates even at $t = 0$ so that there is no flat spacetime to define the asymptotic state.

Cosmologically, the primordial power spectrum of the early universe is one of the physical observables. In a linear approximation which becomes valid at the large b_c limit, the primordial spectrum is given by the two-point function of the conformal mode, which provides the scale-invariant spectrum what is called Harrison-Zel'dovich spectrum with positive amplitude $1/b_c$ [51, 52, 53]. In general, however, there is no systematical argument yet to derive observables or full power spectra from Green functions among physical diffeomorphism invariant operators such as the LSZ reduction formula in the S-matrix. So, we can not discuss the detail of the spectrum beyond the linear approximation at present. It is left as a future issue.

A further direction of the study of renormalizability is that we develop the analysis of renormalization group equations by Hathrell into our quantum gravity system in order to verify whether our renormalization procedure is going well at all orders and to clarify the renormalization structure such as the gauge-parameter dependence in arbitrary gauge [54, 55] and so on. Further research on these things would clarify the structure of quantum gravity and of the dynamics of the early universe.

Appendix A

Gravitational formulae

A.1 Curvature tensor and their properties

Riemann tensor is defined through the equation

$$[\nabla_\gamma, \nabla_\delta]A^\alpha = R^\alpha_{\beta\gamma\delta}A^\beta. \tag{A.1}$$

Besides, Ricci tensor defined by contracting the first and the third indices of Riemann tensor satisfies

$$[\nabla_\alpha, \nabla_\beta]A^\alpha = R_{\alpha\beta}A^\alpha. \tag{A.2}$$

The definitions of Christoffel symbol, Riemann tensor, Ricci tensor, Ricci scalar are written as follows:

$$\begin{aligned}
\Gamma^\alpha_{\beta\gamma} &= \frac{1}{2}g^{\alpha\mu}(\partial_\gamma g_{\mu\beta} + \partial_\beta g_{\mu\gamma} - \partial_\mu g_{\beta\gamma}), \\
R^\alpha_{\beta\gamma\delta} &= \partial_\gamma \Gamma^\alpha_{\beta\delta} - \partial_\delta \Gamma^\alpha_{\beta\gamma} + \Gamma^\alpha_{\mu\gamma}\Gamma^\mu_{\beta\delta} - \Gamma^\alpha_{\mu\delta}\Gamma^\mu_{\beta\gamma}, \\
R_{\alpha\beta} &= R^\mu{}_{\alpha\mu\beta}, \\
R &= g^{\alpha\beta}R_{\alpha\beta}.
\end{aligned}$$

Bianchi identity is

$$\nabla_\lambda R^\mu{}_{\alpha\nu\beta} + \nabla_\nu R^\mu{}_{\alpha\beta\lambda} + \nabla_\beta R^\mu{}_{\alpha\lambda\nu} = 0.$$

We can derive following relation with above formula.

$$\begin{aligned}
\nabla_\beta R^{\mu\alpha\nu\beta} &= -\nabla^\mu R^{\nu\alpha} + \nabla^\alpha R^{\nu\mu}, \\
\nabla_\alpha \nabla_\beta R^{\mu\alpha\nu\beta} &= -\frac{1}{2}\nabla^\mu \nabla^\nu R + \nabla^2 R^{\mu\nu} + R^{\mu\alpha\nu\beta}R_{\alpha\beta} - R^{\mu\alpha}R^\nu{}_\alpha, \\
\nabla_\mu R^{\mu\nu} &= \frac{1}{2}\nabla^\nu R, \\
\nabla_\alpha \nabla^2 A - \nabla^2 \nabla_\alpha A &= R_{\alpha\beta}\nabla^\beta A.
\end{aligned}$$

A.2 Variations of the gravitational quantities

We here present various variations in what follows.

$$\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}, \tag{A.3}$$

$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu} = -\frac{1}{2}\sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu}, \tag{A.4}$$

$$\delta\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} (\nabla_\gamma \delta g_{\mu\beta} + \nabla_\beta \delta g_{\mu\gamma} - \nabla_\mu \delta g_{\beta\gamma}), \tag{A.5}$$

$$\begin{aligned}\delta R^{\alpha}{}_{\beta\gamma\delta} = \frac{1}{2} g^{\alpha\mu} (&\nabla_\gamma \nabla_\delta \delta g_{\mu\beta} + \nabla_\gamma \nabla_\beta \delta g_{\mu\delta} - \nabla_\gamma \nabla_\mu \delta g_{\beta\delta} - \nabla_\delta \nabla_\gamma \delta g_{\mu\beta} \\ &- \nabla_\delta \nabla_\beta \delta g_{\mu\gamma} + \nabla_\delta \nabla_\mu \delta g_{\beta\gamma}),\end{aligned} \tag{A.6}$$

$$\begin{aligned}\delta R^{\alpha\beta\gamma\delta} = &-R^{\alpha\mu\gamma\delta} g^{\beta\nu} \delta g_{\mu\nu} - R^{\alpha\beta\mu\delta} g^{\gamma\nu} \delta g_{\mu\nu} - R^{\alpha\beta\gamma\mu} g^{\delta\nu} \delta g_{\mu\nu} \\ &+ \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} g^{\gamma\lambda} g^{\delta\sigma} \big(\nabla_\lambda \nabla_\sigma \delta g_{\mu\nu} + \nabla_\lambda \nabla_\nu \delta g_{\mu\sigma} - \nabla_\lambda \nabla_\mu \delta g_{\nu\sigma} \\ &- \nabla_\sigma \nabla_\lambda \delta g_{\mu\nu} - \nabla_\sigma \nabla_\nu \delta g_{\mu\lambda} + \nabla_\sigma \nabla_\mu \delta g_{\nu\lambda}\big),\end{aligned} \tag{A.7}$$

$$\begin{aligned}\delta R_{\alpha\beta} = &\frac{1}{2} \left\{ \nabla_\alpha \nabla^\mu \delta g_{\mu\beta} + \nabla_\beta \nabla^\mu \delta g_{\mu\alpha} - \nabla^2 \delta g_{\alpha\beta} - \nabla_\alpha \nabla_\beta (g^{\mu\nu} \delta g_{\mu\nu}) \right\} \\ &- R^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta} \delta g_{\mu\nu} + \frac{1}{2} (R^{\mu}_{\alpha} \delta g_{\mu\beta} + R^{\mu}_{\beta} \delta g_{\mu\alpha}),\end{aligned} \tag{A.8}$$

$$\delta R = -R^{\alpha\beta} \delta g_{\alpha\beta} + \nabla^\alpha \nabla^\beta \delta g_{\alpha\beta} - \nabla^2 (g^{\alpha\beta} \delta g_{\alpha\beta}). \tag{A.9}$$

Also, the variations of a scalar field with derivatives can be written as follows:

$$\delta(\nabla_\alpha A) = \nabla_\alpha \delta A, \tag{A.10}$$

$$\delta(\nabla_\alpha \nabla_\beta A) = \nabla_\alpha \nabla_\beta \delta A - \frac{1}{2} \nabla^\gamma A (\nabla_\beta \delta g_{\gamma\alpha} + \nabla_\alpha \delta g_{\gamma\beta} - \nabla_\gamma \delta g_{\alpha\beta}), \tag{A.11}$$

$$\delta(\nabla^2 A) = \nabla^2 \delta A - \nabla^\alpha \nabla^\beta A \delta g_{\alpha\beta} - \nabla^\alpha A \nabla^\beta \delta g_{\alpha\beta} + \frac{1}{2} \nabla^\gamma A \nabla_\gamma (g^{\alpha\beta} \delta g_{\alpha\beta}), \tag{A.12}$$

$$\begin{aligned}\delta(\nabla^4 A) = &\nabla^4 \delta A - \nabla^2 (\delta g_{\mu\nu} \nabla^\mu \nabla^\nu A) - \nabla^2 (\nabla^\mu A \nabla^\nu \delta g_{\mu\nu}) \\ &+ \frac{1}{2} \nabla^2 (\nabla^\lambda A \nabla_\lambda (g^{\mu\nu} \delta g_{\mu\nu})) - \delta g_{\mu\nu} \nabla^\mu \nabla^\nu \nabla^2 A \\ &- \nabla^\mu \nabla^2 A \nabla^\nu \delta g_{\mu\nu} + \frac{1}{2} \nabla^\lambda \nabla^2 A \nabla_\lambda (g^{\mu\nu} \delta g_{\mu\nu}).\end{aligned} \tag{A.13}$$

Using above formulae, the variations of scalar quantities with curvatures are given as follows:

$$\delta(R^2) = 2R\{-R^{\mu\nu}\delta g_{\mu\nu} + \nabla^\mu\nabla^\nu\delta g_{\mu\nu} - \nabla^2(g^{\mu\nu}\delta g_{\mu\nu})\}, \tag{A.14}$$

$$\delta(R^{\alpha\beta}R_{\alpha\beta}) = 2R^{\alpha\mu}\nabla_\alpha\nabla^\nu\delta g_{\mu\nu} - R^{\mu\nu}\nabla^2\delta g_{\mu\nu} - R^{\alpha\beta}\nabla_\alpha\nabla_\beta(g^{\mu\nu}\delta g_{\mu\nu}) - 2R_{\alpha\beta}R^{\mu\alpha\nu\beta}\delta g_{\mu\nu}, \tag{A.15}$$

$$\delta(R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}) = 2R^{\mu\beta\gamma\delta}R^\nu_{\ \beta\gamma\delta}\delta g_{\mu\nu} + 2R^{\alpha\beta\gamma\delta}(\nabla_\gamma\nabla_\beta\delta g_{\alpha\delta} - \nabla_\gamma\nabla_\alpha\delta g_{\beta\delta}), \tag{A.16}$$

$$\begin{aligned}\delta(\nabla^2 R) = &-\nabla^\mu\nabla^\nu R\delta g_{\mu\nu} - \nabla_\lambda R\nabla^\mu\big(g^{\nu\lambda}\delta g_{\mu\nu}\big) + \frac{1}{2}\nabla_\lambda R\nabla^\lambda(g^{\mu\nu}\delta g_{\mu\nu}) \\ &- \nabla^2(R^{\mu\nu}\delta g_{\mu\nu}) + \nabla^2\nabla^\mu\nabla^\nu\delta g_{\mu\nu} - \nabla^4(g^{\mu\nu}\delta g_{\mu\nu}),\end{aligned} \tag{A.17}$$

$$\begin{aligned}\delta(R^{\alpha\beta}\nabla_\alpha\nabla_\beta A) = &R^\beta_{\ \gamma}\nabla_\alpha\nabla_\beta A\delta g^{\alpha\gamma} + R^\alpha_{\ \delta}\nabla_\alpha\nabla_\beta A\delta g^{\beta\delta} \\ &+ \frac{1}{2}\nabla^\delta\nabla^\beta\big(g^{\alpha\gamma}\delta g_{\gamma\delta}\big)\nabla_\alpha\nabla_\beta A + \frac{1}{2}\nabla^\gamma\nabla^\alpha(g^{\beta\delta}\delta g_{\delta\gamma}\nabla_\alpha\nabla_\beta A \\ &- \frac{1}{2}\nabla^\alpha\nabla^\beta A\nabla^2\delta g_{\alpha\beta} - \frac{1}{2}\nabla^\alpha\nabla^\beta\big(g^{\gamma\delta}\delta g_{\gamma\delta}\big)\nabla_\alpha\nabla_\beta A \\ &+ R^{\alpha\beta}\nabla_\alpha\nabla_\beta\delta A - \frac{1}{2}R^{\alpha\beta}\nabla^\gamma A\big(\nabla_\beta\delta g_{\gamma\alpha} + \nabla_\alpha\delta g_{\gamma\beta} - \nabla_\gamma\delta g_{\alpha\beta}\big),\end{aligned} \tag{A.18}$$

$$\begin{aligned}\delta(R\nabla^2 A) = &\Big[-R^{\alpha\beta}\delta g_{\alpha\beta} + \nabla^\alpha\nabla^\beta\delta g_{\alpha\beta} - \nabla^2(g^{\alpha\beta}\delta g_{\alpha\beta})\Big]\nabla^2 A \\ &+ R\Big[\nabla^2\delta A - \nabla^\alpha\nabla^\beta A\delta g_{\alpha\beta} - \nabla^\alpha A\nabla^\beta\delta g_{\alpha\beta} + \frac{1}{2}\nabla^\gamma A\nabla_\gamma(g^{\alpha\beta}\delta g_{\alpha\beta})\Big],\end{aligned} \tag{A.19}$$

$$\begin{aligned}\delta(\nabla^\alpha R\nabla_\alpha A) = &\nabla^\alpha R\nabla^\beta A\delta g_{\alpha\beta} - \nabla^\alpha A\nabla_\alpha R^{\gamma\delta}\delta g_{\gamma\delta} + \nabla^\alpha A\nabla_\alpha\nabla^\gamma\nabla^\delta\delta g_{\gamma\delta} \\ &- \nabla^\alpha A\nabla_\alpha\nabla^2(g^{\gamma\delta}\delta g_{\gamma\delta}) + \nabla^\alpha R\nabla_\alpha\delta A.\end{aligned} \tag{A.20}$$

A.2.1 Energy-momentum tensors

Energy-momentum tensor is defined by

$$\theta^{\alpha\beta} \equiv \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\alpha\beta}}S, \tag{A.21}$$

The variations of gravitational quantities are presented in what follows.

$$\begin{aligned}\frac{2}{\sqrt{-g}}\int d^D x\frac{\delta(\sqrt{-g}F_D)}{\delta g_{\mu\nu}} = &F_D g^{\mu\nu} - 4\bigg(R^{\mu\beta\gamma\delta}R^\nu_{\ \beta\gamma\delta} - \frac{4}{D-2}R_{\alpha\beta}R^{\mu\alpha\nu\beta} + \frac{2}{(D-1)(D-2)}RR^{\mu\nu}\bigg) \\ &- 8\nabla_\alpha\nabla_\beta R^{\mu\alpha\nu\beta} - \frac{8}{D-1}\nabla^\nu\nabla^\mu R + \frac{8}{D-2}\nabla^2 R^{\mu\nu} + \frac{4(D-3)}{(D-1)(D-2)}\nabla^2 R g^{\mu\nu},\end{aligned} \tag{A.22}$$

$$\frac{2}{\sqrt{-g}}\int d^Dx\frac{\delta(\sqrt{-g}G_4)}{\delta g_{\mu\nu}} = G_4 g^{\mu\nu} - 4\big(R^{\mu\beta\gamma\delta}R^{\nu}_{\ \beta\gamma\delta} - 4R^{\mu\alpha}R^{\nu}_{\ \alpha} + RR^{\mu\nu}\big) - 8(R^{\mu\alpha}R^{\nu}_{\ \alpha} - R^{\mu\alpha\nu\beta}R_{\alpha\beta}), \quad \text{(A.23)}$$

$$\frac{2}{\sqrt{-g}}\int d^Dx\frac{\delta(\sqrt{-g}R^2)}{\delta g_{\mu\nu}} = R^2 g^{\mu\nu} - 4RR^{\mu\nu} + 4\nabla^\nu\nabla^\mu R - 4\nabla^2 R g^{\mu\nu}, \quad \text{(A.24)}$$

$$\frac{2}{\sqrt{-g}}\int d^Dx g_{\mu\nu}\frac{\delta\sqrt{-g}R}{\delta g_{\mu\nu}} = Rg^{\mu\nu} - 2R^{\mu\nu}. \quad \text{(A.25)}$$

The trace of the energy momentum tensor is defined by

$$\theta \equiv g_{\alpha\beta}\theta^{\alpha\beta} = g_{\alpha\beta}\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\alpha\beta}}S = \frac{\delta S}{\delta\Omega} \quad \text{(A.26)}$$

and the conformal variation of the respective quantities are given by

$$\int d^Dx g_{\mu\nu}\frac{\delta(\sqrt{-g}F_D)}{\delta\Omega} = (D-4)F_D, \quad \text{(A.27)}$$

$$\int d^Dx g_{\mu\nu}\frac{\delta(\sqrt{-g}G_4)}{\delta\Omega} = (D-4)G_4, \quad \text{(A.28)}$$

$$\int d^Dx g_{\mu\nu}\frac{\delta(\sqrt{-g}R^2)}{\delta\Omega} = (D-4)R^2 - 4(D-1)\nabla^2 R. \quad \text{(A.29)}$$

A.2.2 Local conformal variations of gravitational quantities

$$\frac{\delta}{\delta\Omega(y)}R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}(x) = \frac{-4}{\sqrt{-g}}\left\{R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}\delta(x-y) + 2R^{\alpha\beta}\nabla_\alpha\nabla_\beta\delta(x-y)\right\}, \quad \text{(A.30)}$$

$$\frac{\delta}{\delta\Omega(y)}R^{\alpha\beta}R_{\alpha\beta}(x) = -\frac{2}{\sqrt{-g}}\left\{(D-2)R^{\alpha\beta}\nabla_\alpha\nabla_\beta\delta(x-y) + 2R^{\alpha\beta}R_{\alpha\beta}\delta(x-y) - R\nabla^2\delta(x-y)\right\}, \quad \text{(A.31)}$$

$$\frac{\delta}{\delta\Omega(y)}R^2(x) = \frac{-4}{\sqrt{-g}}\left\{R^2\delta(x-y) + (D-1)R\nabla^2\delta(x-y)\right\}, \quad \text{(A.32)}$$

$$\frac{\delta}{\delta\Omega(y)}R_{\alpha\beta}(x) = \frac{-1}{\sqrt{-g}}\left\{(D-2)\nabla_\alpha\nabla_\beta\delta(x-y) + g_{\alpha\beta}\nabla^2\delta(x-y)\right\}, \quad \text{(A.33)}$$

$$\frac{\delta}{\delta\Omega(y)}R(x) = \frac{-2}{\sqrt{-g}}\left\{R\delta(x-y) + (D-1)\nabla^2\delta(x-y)\right\}, \quad \text{(A.34)}$$

$$\frac{\delta}{\delta\Omega(y)}\nabla^2 R(x) = \frac{2}{\sqrt{-g}}\bigg\{-2\nabla^2 R\delta(x-y) + \left(\frac{D}{2}-3\right)\nabla_\lambda R\nabla^\lambda\delta(x-y) - R\nabla^2\delta(x-y) - (D-1)\nabla^4\delta(x-y)\bigg\}. \quad \text{(A.35)}$$

A.2.3 Conformal mode dependence of the gravitational quantities

We separate the gravitational field into the conformal mode and the traceless tensor mode such as $g_{\alpha\beta} = e^{2\phi}\bar{g}_{\alpha\beta}$, the gravitational quantities are then expanded in what follows.

$$\begin{aligned} R^{\alpha}_{\beta\gamma\delta} &= \bar{R}^{\alpha}_{\beta\gamma\delta} + \bar{g}_{\beta\gamma}\big(\bar{\nabla}_\delta\bar{\nabla}^\alpha\phi - \bar{\nabla}^\alpha\phi\bar{\nabla}_\delta\phi\big) - \bar{g}_{\beta\delta}(\bar{\nabla}_\gamma\bar{\nabla}^\alpha\phi - \bar{\nabla}^\alpha\phi\bar{\nabla}_\gamma\phi) \\ &\quad - \bar{g}_{\mu\nu}(\bar{\nabla}_\delta\bar{\nabla}^\alpha\phi - \bar{\nabla}^\alpha\phi\bar{\nabla}_\delta\phi + \bar{g}_{\beta\delta}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi) + \bar{\delta}^\alpha_\delta(\bar{\nabla}_\beta\bar{\nabla}_\gamma\phi - \bar{\nabla}_\beta\bar{\nabla}_\gamma\phi + \bar{\nabla}_\beta\bar{\nabla}_\gamma\phi), \end{aligned} \tag{A.36}$$

$$R_{\alpha\beta} = \bar{R}_{\alpha\beta} - (D-2)\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi + (D-2)\bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi - \bar{g}_{\alpha\beta}\{\bar{\nabla}^2\phi + (D-2)\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi\}, \tag{A.37}$$

$$R = e^{-2\phi}[\bar{R} - 2(D-1)\bar{\nabla}^2\phi - (D-1)(D-2)\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi], \tag{A.38}$$

$$\begin{aligned} R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} &= e^{-4\phi}\Big[\bar{R}^{\alpha\beta\gamma\delta}\bar{R}_{\alpha\beta\gamma\delta} - 8\bar{R}^{\alpha\beta}(\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi - \bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi) - 4\bar{R}\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi \\ &\quad + 4(\bar{\nabla}^2\phi)^2 + 8(D-2)\bar{\nabla}^2\phi\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi + 4(D-2)\bar{\nabla}^\alpha\bar{\nabla}^\beta\phi\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi \\ &\quad - 8(D-2)\bar{\nabla}^\alpha\bar{\nabla}^\beta\phi\bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi + 2(D-1)(D-2)\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi\bar{\nabla}^\beta\bar{\nabla}_\beta\phi, \end{aligned} \tag{A.39}$$

$$\begin{aligned} R^{\alpha\beta}R_{\alpha\beta} &= e^{-4\phi}\Big[\bar{R}^{\alpha\beta}\bar{R}_{\alpha\beta} - 2(D-2)\bar{R}^{\alpha\beta}(\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi - \bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi) \\ &\quad - 2\bar{R}\big\{\bar{\nabla}^2\phi + (D-2)\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi\big\} + (3D-4)(\bar{\nabla}^2\phi)^2 \\ &\quad + 2(D-2)(2D-3)\bar{\nabla}^2\phi\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi + (D-2)^2\bar{\nabla}^\alpha\bar{\nabla}^\beta\phi\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi \\ &\quad - 2(D-2)^2\bar{\nabla}^\alpha\bar{\nabla}^\beta\phi\bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi + (D-1)(D-2)^2\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi\bar{\nabla}^\beta\phi\bar{\nabla}_\beta\phi\Big], \end{aligned} \tag{A.40}$$

$$\begin{aligned} R^2 &= e^{-4\phi}[\bar{R}^2 - 2(D-1)\bar{R}\{2\bar{\nabla}^2\phi + (D-2)\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi\} + 4(D-1)^2(\bar{\nabla}^2\phi)^2 \\ &\quad + 4(D-1)^2(D-2)\bar{\nabla}^2\phi\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi + (D-1)^2(D-2)^2\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi\bar{\nabla}^\beta\phi\bar{\nabla}_\beta\phi], \end{aligned} \tag{A.41}$$

$$\begin{aligned} \nabla^2 R &= e^{-4\phi}\Big[\bar{\nabla}^2\bar{R} - 2(D-1)(D-2)\bar{R}^{\mu\nu}\bar{\nabla}_\mu\phi\bar{\nabla}_\nu\phi - 2\big(\bar{\nabla}^2 phi + (D-4)\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi\big)\bar{R} \\ &\quad + (D-6)\bar{\nabla}_\mu\bar{R}\bar{\nabla}^\mu\phi - 2(D-1)\bar{\nabla}^4\phi + 4(D-1)(\bar{\nabla}^2\phi)^2 \\ &\quad + 2(D-1)(3D-10)\bar{\nabla}^2\phi\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi - 2(D-1)(D-2)(D-6)\bar{\nabla}^\mu\bar{\nabla}^\nu\phi\bar{\nabla}_\mu\phi\bar{\nabla}_\nu\phi \\ &\quad - 2(D-1)(D-2)\bar{\nabla}^\mu\bar{\nabla}^\nu\phi\bar{\nabla}_\mu\bar{\nabla}_\nu\phi - 4(D-1)(D-4)\bar{\nabla}^\mu\bar{\nabla}^2\phi\bar{\nabla}_\mu\phi \\ &\quad + 2(D-1)(D-2)(D-4)\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi\bar{\nabla}^\nu\phi\bar{\nabla}_\nu\phi\Big], \end{aligned} \tag{A.42}$$

$$\begin{aligned} G_4 &= e^{-4\phi}\Big[\bar{G}_4 + 8(D-3)\bar{R}^{\alpha\beta}\big(\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi - \bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi\big) - 2(D-3)\bar{R}\big\{2\bar{\nabla}^2\phi + (D-4)\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi\big\} \\ &\quad + 4(D-2)(D-3)(\bar{\nabla}^2\phi)^2 + 4(D-2)(D-3)^2\bar{\nabla}^2\phi\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi \\ &\quad - 4(D-2)(D-3)\bar{\nabla}^\alpha\bar{\nabla}^\beta\phi\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi + 8(D-2)(D-3)\bar{\nabla}^\alpha\bar{\nabla}^\beta\phi\bar{\nabla}_\alpha\phi\bar{\nabla}_\beta\phi \\ &\quad + (D-1)(D-2)(D-3)(D-4)\bar{\nabla}^\alpha\phi\bar{\nabla}_\alpha\phi\bar{\nabla}^\beta\phi\bar{\nabla}_\beta\phi\Big]. \end{aligned} \tag{A.43}$$

A.3 Perturbations in traceless tensor mode

The metric tensor with the bar is expanded in terms of the traceless tensor mode as

$$\bar{g}_{\alpha\beta} = (\hat{g}e^{h})_{\alpha\beta} = \hat{g}_{\alpha\beta} + h_{\alpha\beta} + \frac{1}{2}(h^2)_{\alpha\beta} + \frac{1}{6}(h^3)_{\alpha\beta} + \mathcal{O}(h^4). \tag{A.44}$$

Christoffel symbol with the bar is expanded as

$$\begin{aligned}\bar{\Gamma}^{\alpha}{}_{\beta\gamma} &= \frac{1}{2}\bar{g}^{\alpha\mu}(\partial_\gamma \bar{g}_{\mu\beta} + \partial_\beta \bar{g}_{\mu\gamma} - \partial_\mu \bar{g}_{\beta\gamma}) \\ &= \hat{\Gamma}^{\alpha}{}_{\beta\gamma} + H^{\alpha}{}_{\beta\gamma} + (H^2)^{\alpha}{}_{\beta\gamma} + (H^3)^{\alpha}{}_{\beta\gamma},\end{aligned} \tag{A.45}$$

where $H^{\alpha}_{\beta\gamma}$, $(H^2)^{\alpha}{}_{\beta\gamma}$ and $(H^3)^{\alpha}{}_{\beta\gamma}$ are defined as

$$H^{\alpha}{}_{\beta\gamma} = \hat{\nabla}_{(\beta}h^{\alpha}_{\gamma)} - \frac{1}{2}\hat{\nabla}^{\alpha}h_{\beta\gamma}, \tag{A.46}$$

$$(H^2)^{\alpha}{}_{\beta\gamma} = \frac{1}{2}\hat{\nabla}_{(\beta}(h^2)^{\alpha}_{\gamma)} - \frac{1}{4}\hat{\nabla}^{\alpha}(h^2)_{\beta\gamma} - h^{\alpha}_{\mu}\hat{\nabla}_{(\beta}h^{\mu}{}_{\gamma)} + \frac{1}{2}h^{\alpha}_{\mu}\hat{\nabla}^{\mu}h_{\beta\gamma}, \tag{A.47}$$

$$\begin{aligned}(H^3)^{\alpha}{}_{\beta\gamma} = &\frac{1}{2}(h^2)^{\alpha}_{\mu}\left(\hat{\nabla}_{(\beta}h^{\mu}_{\gamma)} - \frac{1}{2}\hat{\nabla}^{\mu}h_{\beta\gamma}\right) - \frac{1}{2}h^{\alpha}_{\mu}(\hat{\nabla}_{(\beta}(h^2)^{\mu}_{\gamma)} \\ &- \frac{1}{2}\hat{\nabla}^{\mu}(h^2)_{\beta\gamma}) + \frac{1}{6}(\hat{\nabla}_{(\beta}(h^3)^{\alpha}_{\gamma)} - \frac{1}{2}\hat{\nabla}^{\alpha}(h^3)_{\beta\gamma}).\end{aligned} \tag{A.48}$$

Riemann tensor, Ricci tensor and Ricci scalar are expanded as follows:

$$\begin{aligned}\bar{R}^{\alpha}{}_{\beta\gamma\delta} = \hat{R}^{\alpha}{}_{\beta\gamma\delta} &+ \hat{\nabla}_\gamma\Big\{\hat{\nabla}_{(\beta}h^{\alpha}{}_{\delta)} - \frac{1}{2}\hat{\nabla}^{\alpha}h_{\beta\delta} + \frac{1}{2}\hat{\nabla}_{(\beta}(h^2)^{\alpha}{}_{\delta)} - \frac{1}{4}\hat{\nabla}^{\alpha}(h^2)_{\beta\delta} \\ &\qquad - h^{\alpha}{}_{\mu}\hat{\nabla}_{(\beta}h^{\mu}{}_{\delta)} + \frac{1}{2}h^{\alpha}{}_{\mu}\hat{\nabla}^{\mu}h_{\beta\delta}\Big\} \\ &+ \hat{\nabla}_\delta\Big\{\hat{\nabla}_{(\beta}h^{\alpha}{}_{\gamma)} - \frac{1}{2}\hat{\nabla}^{\alpha}h_{\beta\gamma} + \frac{1}{2}\hat{\nabla}_{(\beta}(h^2)^{\alpha}{}_{\gamma)} \\ &\qquad - \frac{1}{4}\hat{\nabla}^{\alpha}(h^2)_{\beta\gamma} - h^{\alpha}{}_{\mu}\hat{\nabla}_{(\beta}h^{\mu}{}_{\gamma)} + \frac{1}{2}h^{\alpha}{}_{\mu}\hat{\nabla}^{\mu}h_{\beta\gamma}\Big\} \\ &+ \left(\hat{\nabla}_{(\mu}h^{\alpha}_{\gamma)} - \frac{1}{2}\hat{\nabla}^{\alpha}h_{\mu\gamma}\right)\left(\hat{\nabla}_{(\beta}h^{\mu}{}_{\delta)} - \frac{1}{2}\hat{\nabla}^{\mu}h_{\beta\delta}\right) \\ &- \left(\hat{\nabla}_{(\mu}h^{\alpha}_{\delta)} - \frac{1}{2}\hat{\nabla}^{\alpha}h_{\mu\delta}\right)\left(\hat{\nabla}_{(\beta}h^{\mu}{}_{\gamma)} - \frac{1}{2}\hat{\nabla}^{\mu}h_{\beta\gamma}\right),\end{aligned} \tag{A.49}$$

$$\begin{aligned}\bar{R}_{\alpha\beta} = \hat{R}_{\alpha\beta} &- \hat{R}^{\mu}{}_{\alpha\nu\beta}h^{\nu}{}_{\mu} + \hat{R}_{\mu(\alpha}h^{\alpha}{}_{\beta)} + \hat{\nabla}_{(\alpha}\hat{\nabla}^{\mu}h_{\beta)\mu} \\ &- \frac{1}{2}\hat{\nabla}^2 h_{\alpha\beta} - \frac{1}{2}h^{\mu}{}_{(\alpha}\hat{\nabla}^2 h_{\beta)\mu} - \frac{1}{2}(\hat{\nabla}_\mu h_{\nu(\alpha}\hat{\nabla}^{\nu}h_{\beta)}^{\mu}) - \frac{1}{4}\hat{\nabla}_\alpha h^{\mu}{}_{\nu}\hat{\nabla}_\beta h^{\nu}{}_{\mu} \\ &- \frac{1}{2}\hat{\nabla}_\mu(h^{\mu}{}_{\nu}\hat{\nabla}_{(\alpha}h^{\nu}{}_{\beta)} + \frac{1}{2}\hat{\nabla}_\mu(h^{\nu}{}_{(\alpha}\hat{\nabla}_{\beta)}h^{\mu}{}_{\nu}) + \frac{1}{2}\hat{\nabla}_\nu(h^{\mu}{}_{\nu}\hat{\nabla}^{\nu}h_{\alpha\beta}),\end{aligned} \tag{A.50}$$

$$\bar{R} = \hat{R} - \hat{R}_{\mu\nu}h^{\mu\nu} + \hat{\nabla}_\mu\hat{\nabla}_\nu h^{\mu\nu} + \frac{1}{2}\hat{R}^\mu{}_{\alpha\nu\beta}h^\nu{}_\mu h^{\alpha\beta} - \frac{1}{4}\hat{\nabla}_\lambda h^\mu{}_\nu\hat{\nabla}^\lambda h^\nu{}_\mu + \frac{1}{2}\hat{\nabla}_\mu h^\mu{}_\lambda\hat{\nabla}_\nu h^{\nu\lambda} - \hat{\nabla}_\mu(h^\mu{}_\nu\hat{\nabla}^\lambda h^\nu{}_\lambda). \quad \text{(A.51)}$$

In the following, we take the flat background metric. The expansion of the curvature-squared quantities are then given as follows:

$$\bar{R}^{\alpha\beta\gamma\delta}\bar{R}_{\alpha\beta\gamma\delta} = \partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\gamma\partial^\delta h^{\alpha\beta} - 2\partial_\gamma\partial_\beta h_{\alpha\delta}\partial^\gamma\partial^\alpha h^{\beta\delta} + \partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\alpha\partial^\beta h^{\gamma\delta}, \quad \text{(A.52)}$$

$$\bar{R}^{\alpha\beta}\bar{R}_{\alpha\beta} = \frac{1}{2}\partial_\alpha\chi_\beta\partial^\alpha\chi^\beta + \frac{1}{2}\partial_\alpha\chi_\beta\partial^\beta\chi^\alpha + \frac{1}{4}\partial^2 h_{\alpha\beta}\partial^2 h^{\alpha\beta} - \partial_\alpha\chi_\beta\partial^2 h^{\alpha\beta}, \quad \text{(A.53)}$$

$$\bar{R}^2 = \partial_\alpha\chi^\alpha\partial_\beta\chi^\beta, \quad \text{(A.54)}$$

$$\bar{\nabla}^2\bar{R} = \partial^2\partial_\alpha\chi^\alpha - \frac{1}{2}\partial_\gamma h_{\alpha\beta}\partial^2\partial^\gamma h^{\alpha\beta} - \frac{1}{2}\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\gamma\partial^\delta h^{\alpha\beta} - \frac{1}{2}\partial^2(\chi_\alpha\chi^\alpha) - \partial^2(h^{\alpha\beta}\partial_\alpha\chi_\beta) - h^{\alpha\beta}\partial_\alpha\partial_\beta\partial_\gamma\chi^\gamma - \chi^\alpha\partial_\alpha\partial_\beta\chi^\beta, \quad \text{(A.55)}$$

where $\chi_\alpha = \partial^\beta h_{\alpha\beta}$. Besides, the expansions of the quantities with the conformal mode which are necessary in the text are given in what follows.

$$\bar{\nabla}^2\phi = \partial^2\phi - h^{\alpha\beta}\partial_\alpha\partial_\beta\phi - \chi^\alpha\partial_\alpha\phi + \frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\phi + \frac{1}{2}h^{\alpha\beta}\partial_\alpha h^\gamma{}_\beta\partial_\gamma\phi + \frac{1}{2}h^{\alpha\beta}\chi_\beta\partial_\alpha\phi, \quad \text{(A.56)}$$

$$\begin{aligned}\bar{\nabla}^4\phi = \partial^4\phi &- \{\partial^2(h^{\alpha\beta}\partial_\alpha\partial_\beta\phi + \chi^\alpha\partial_\alpha\phi) + h^{\alpha\beta}\partial_\alpha\partial_\beta\partial^2\phi + \chi^\alpha\partial_\alpha\partial^2\phi\} \\ &+ \partial^2\left\{\frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\phi + \frac{1}{2}h^{\alpha\beta}\partial_\alpha h^\gamma{}_\beta\partial_\gamma\phi + \frac{1}{2}h^{\alpha\beta}\chi_\beta\partial_\alpha\phi\right\} \\ &+ h^{\alpha\beta}\Big(\partial_\alpha\partial_\beta h^{\gamma\delta}\partial_\gamma\partial_\delta\phi + 2\partial_\alpha h^{\gamma\delta}\partial_\beta\partial_\gamma\partial_\delta\phi + h^{\gamma\delta}\partial_\alpha\partial_\beta\partial_\gamma\partial_\delta\phi \\ &\quad + \frac{1}{2}\partial_\alpha h^\gamma{}_\beta\partial_\gamma\partial^2\phi + \frac{1}{2}\chi_\beta\partial_\alpha\partial^2\phi + \partial_\alpha\partial_\beta\chi^\gamma\partial_\gamma\phi \\ &\quad + 2\partial_\alpha\chi^\gamma\partial_\beta\partial_\gamma\phi + 2\chi^\gamma\partial_\alpha\partial_\beta\partial_\gamma\phi\Big) \\ &+ \chi^\alpha\partial_\alpha h^{\gamma\delta}\partial_\gamma\partial_\delta\phi + \chi^\alpha\partial_\alpha\chi^\beta\partial_\beta\phi + \chi^\alpha\chi^\beta\partial_\alpha\partial_\beta\phi + \frac{1}{2}(h^2)^{\alpha\beta}\partial_\alpha\partial_\beta\partial^2\phi, \quad \text{(A.57)}\end{aligned}$$

$$
\begin{aligned}
\bar{R}^{\alpha\beta}\bar{\nabla}_\alpha\bar{\nabla}_\beta\phi = {} & \partial_\alpha\chi_\beta\partial^\alpha\partial^\beta\phi - \frac{1}{2}\partial^2 h_{\alpha\beta}\partial^\alpha\partial^\beta\phi \\
& - \frac{1}{2}\partial^\alpha\chi^\beta\partial_\alpha h_{\beta\gamma}\partial^\gamma\phi - \frac{1}{2}\partial_\alpha\chi_\beta\partial^\beta h^{\alpha\gamma}\partial_\gamma\phi + \frac{1}{2}\partial^\alpha\chi^\beta\partial^\gamma h_{\alpha\beta}\partial_\gamma\phi \\
& + \frac{1}{2}\partial^2 h^{\alpha\beta}\partial_\alpha h_{\beta\gamma}\partial^\gamma\phi - \frac{1}{4}\partial^2 h^{\alpha\beta}\partial^\gamma h_{\alpha\beta}\partial_\gamma\phi - h^{\alpha\beta}\partial_\beta\chi^\gamma\partial_\alpha\partial_\gamma\phi \\
& - h^{\alpha\beta}\partial^\gamma\chi_\beta\partial_\alpha\partial_\gamma\phi + \frac{1}{2}h^{\alpha\beta}\partial^2 h_{\beta\gamma}\partial_\alpha\partial^\gamma\phi - \frac{1}{2}\partial^\gamma h_{\alpha\beta}\partial^\beta h_{\gamma\delta}\partial^\alpha\partial^\delta\phi \\
& - \frac{1}{4}\partial^\gamma h_{\alpha\beta}\partial^\delta h^{\alpha\beta}\partial_\gamma\partial_\delta\phi - \frac{1}{2}\partial^\alpha(h_{\alpha\beta}\partial^\gamma h^{\beta\delta})\partial_\gamma\partial_\delta\phi \\
& + \frac{1}{2}\partial_\gamma(h_{\alpha\beta}\partial_\delta h^{\beta\gamma})\partial^\alpha\partial^\delta\phi + \frac{1}{2}\partial^\alpha(h_{\alpha\beta}\partial^\beta h_{\gamma\delta})\partial^\gamma\partial^\delta\phi, \qquad \text{(A.58)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{R}\bar{\nabla}^2\phi = {} & \partial_\alpha\chi^\alpha\partial^2\phi - \chi^\alpha\left(\partial_\beta\chi^\beta\partial_\alpha\phi + \frac{1}{2}\chi_\alpha\partial^2\phi\right) \\
& - h^{\alpha\beta}(\partial_\alpha\chi_\beta\partial^2\phi + \partial_\gamma\chi^\gamma\partial_\alpha\partial_\beta\phi) - \frac{1}{4}\partial^\gamma h^{\alpha\beta}\partial_\gamma h_{\alpha\beta}\partial^2\phi, \qquad \text{(A.59)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{\nabla}^\alpha\bar{R}\bar{\nabla}_\alpha\phi = {} & \partial_\alpha\partial_\beta\chi^\alpha\partial^\beta\phi - \frac{1}{2}\partial^\gamma h^{\alpha\beta}\partial_\gamma\partial_\delta h_{\alpha\beta}\partial^\delta\phi \\
& - \frac{1}{2}\partial_\alpha(\chi_\beta\chi^\beta)\partial^\alpha\phi - \partial_\gamma(h^{\alpha\beta}\partial_\alpha\chi_\beta)\partial^\gamma\phi - h^{\alpha\beta}\partial_\beta\partial_\gamma\chi^\gamma\partial_\alpha\phi. \qquad \text{(A.60)}
\end{aligned}
$$

Appendix B

Determination of the gravitational action

The advantages of using dimensional ragularization are that it does not break diffeomorphism invariance and the theory becomes independent of the choice of the path integral measure owing to $\int d^Dp = \delta^{(D)}(0) = 0$. In this case, the contribution from the measure such as conformal anomaly directly expressed as a D dependence of the action. However, when we generalize a four dimensional action to D-dimensional one, many ambiguities emerge, differently from the case of ordinary gauge field action. In order to settle the ambiguities, we begin our discussion by considering the determination of the forms of gravitational counterterms [40, 14] based on QCD in curved space as an example of conformally coupled quantum field theory.

B.1 QCD on curved space

QCD action on curved space is defined as follows

$$S = \int d^Dx\sqrt{g}\bigg\{\frac{1}{g_0^2}\left[\frac{1}{4}{F_0}^a_{\alpha\beta}{F_0}^{a\alpha\beta} + \frac{1}{2\xi_0}(\nabla^\alpha {A_0}^a_\alpha)^2\right] + i\bar{\psi}_0\gamma^\alpha D_\alpha\psi_0 \\ - i\partial^\alpha\tilde{c}_0^a(\partial_\alpha {c_0}^a - f^{abc}{A_0}^b_\alpha {c_0}^c) + a_0F_D + b_0G_4 + c_0H^2\bigg\}, \tag{B.1}$$

where

$$\begin{aligned} {F_0}^a_{\alpha\beta} &= \partial_\alpha {A_0}^a_\beta - \partial_\beta {A_0}^a_\beta - f^{abc}{A_0}^b_\alpha {A_0}^c_\beta, \\ D_\alpha &= \partial_\alpha + \omega_{\alpha\mu\nu}\frac{\Sigma_{\mu\nu}}{2} - {A_0}^a_\alpha T^a, \quad \Sigma_{\mu\nu} = -\frac{1}{4}[\gamma_\mu, \gamma_\nu] \\ \omega_{\alpha\mu\nu} &= e^\beta_\mu(\partial_\alpha e_{\beta\nu} - \Gamma^\gamma_{\alpha\beta}e_{\gamma\nu}) \end{aligned} \tag{B.2}$$

and $\omega_{\alpha\mu\nu}$, e^α_μ, $\Sigma_{\mu\nu}$ are respectively the spin connection and the vielbein and the Lorentz generator. Only in this chapter, Euclidean indices are written in μ and ν and the gamma matrix can be expressed as $\gamma^\alpha = e^\alpha_\mu\gamma^\mu$. The algebra for gamma matrices is defined as $\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$.

The generators of Lie group are normalized as

$$Tr(T^aT^b) = -T_R\delta^{ab} \quad \text{and} \quad f^{acd}f^{bcd} = C_G\delta^{ab}. \tag{B.3}$$

For the moment, we consider three kinds of gravitational counterterms F_D, G_4 and H^2 which are every possible combination of the fourth-derivative terms.. F_D is the square of D-dimensional Weyl tensor $C_{\alpha\beta\gamma\delta}$. , which is defined as

$$C_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} - \frac{2}{D-2}(g_{\alpha[\gamma}R_{\delta]\beta} - g_{\beta[\gamma}R_{\delta]\alpha}) + \frac{1}{(D-1)(D-2)}g_{\alpha[\gamma}g_{\delta]\beta}R, \tag{B.4}$$

and the square of it is

$$F_D = C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta} = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - \frac{4}{D-2}R^{\alpha\beta}R_{\alpha\beta} + \frac{2}{(D-1)(D-2)}R^2. \tag{B.5}$$

The quantity G_4 is four dimensional Euler density and H is a rescaled Ricci scalar defined as follows

$$G_4 = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - 4R^{\alpha\beta}R_{\alpha\beta} + R^2, \quad H = \frac{R}{D-1}. \tag{B.6}$$

In the following, we will show that the two counter terms with b_0 and c_0 are combined into one at last through various renormalization group equations.

The renormalization factors for quantum fields and coupling constant, gauge parameter are defined as

$$A_{0\alpha}^{\,a} = \mu^{2-\frac{D}{2}}Z_A^{\frac{1}{2}}A_\alpha^a, \quad \psi_0 = Z_2^{\frac{1}{2}}\psi, \tag{B.7}$$

$$g_0 = \mu^{2-\frac{D}{2}}Z_g g, \quad \xi_0 = Z_A Z_g^{-2}\xi, \tag{B.8}$$

where μ is a mass scale to make some quantities dimensionless.

The renormalization group equations are derived from what bare quantities does not depend on the mass scale, and now the differential operator with respect to μ is expressed as

$$\mu\frac{d}{d\mu} = \mu\frac{\partial}{\partial\mu} + \mu\frac{d\alpha_g}{d\mu}\frac{\partial}{\partial\alpha_g} + \mu\frac{d\xi}{d\mu}\frac{\partial}{\partial\xi} + \cdots, \tag{B.9}$$

where $\alpha_g = g^2/4\pi$. We first find the beta function of α_g. The renormalization group equation is

$$0 = \mu\frac{d}{d\mu}\frac{g_0^2}{4\pi} = \mu^{4-D}\left\{(4-D)Z_g^2\alpha_g + 2Z_g\mu\frac{dZ_g}{d\mu}\alpha_g + Z_g^2\mu\frac{d\alpha_g}{d\mu}\right\} \tag{B.10}$$

When we define $\bar{\beta}_g$ as

$$\bar{\beta}_g(\alpha_g) = -2\mu\frac{d}{d\mu}\ln Z_g, \tag{B.11}$$

the beta function of α_g is expressed as follows

$$\beta_g(\alpha_g, D) \equiv \frac{\mu}{\alpha_g}\frac{d\alpha_g}{d\mu} = D - 4 + \bar{\beta}_g(\alpha_g). \tag{B.12}$$

If we expand the renormalization factor as

$$\ln Z_g = \sum_{n=1}^{\infty} \frac{f_n(\alpha_g)}{(D-4)^n}, \tag{B.13}$$

we obtain

$$\bar{\beta}_g = -2\alpha_g \frac{\partial f_1}{\partial \alpha_g}, \quad \frac{\partial f_{n+1}}{\partial \alpha_g} + \bar{\beta}_g \frac{\partial f_n}{\partial \alpha_g} = 0. \tag{B.14}$$

Besides, the anomalous dimensions of the fields A^a_α, ψ are defined as

$$\gamma_A = \mu \frac{d \ln Z_A}{d\mu}, \quad \gamma_2 = \mu \frac{d \ln Z_2}{d\mu}. \tag{B.15}$$

The coefficients of the Weyl term, Euler term and modified Ricci scalar squared are given as

$$\begin{aligned} a_0 &= \mu^{D-4}(a + L_a) \\ b_0 &= \mu^{D-4}(b + L_b) \\ c_0 &= \mu^{D-4}(c + L_c). \end{aligned} \tag{B.16}$$

These $L_{a,b,c}$ are pole term which take the form as follows

$$\begin{aligned} L_a &= \sum_{n=1}^{\infty} \frac{a_n}{(D-4)^n} \\ L_b &= \sum_{n=1}^{\infty} \frac{b_n}{(D-4)^n}. \\ L_c &= \sum_{n=1}^{\infty} \frac{c_n}{(D-4)^n}. \end{aligned}$$

The beta functions of these coefficients are expressed in the same form as

$$\begin{aligned} \beta_a &= \mu \frac{da}{d\mu} = -(D-4)a + \bar{\beta}_a, \\ \beta_b &= \mu \frac{db}{d\mu} = -(D-4)b + \bar{\beta}_b, \\ \beta_c &= \mu \frac{dc}{d\mu} = -(D-4)c + \bar{\beta}_c, \end{aligned} \tag{B.17}$$

We can also obtain the recurrence relation in a manner similar to the beta function of α_g as

$$\begin{aligned}
\bar{\beta}_a &= -\frac{\partial(\alpha_g a_1)}{\partial\alpha_g}, \quad \frac{\partial}{\partial\alpha_g}(\alpha_g a_{n+1}) + \bar{\beta}\alpha_g\frac{\partial a_n}{\partial\alpha_g} = 0,\\
\bar{\beta}_b &= -\frac{\partial(\alpha_g b_1)}{\partial\alpha_g}, \quad \frac{\partial}{\partial\alpha_g}(\alpha_g b_{n+1}) + \bar{\beta}\alpha_g\frac{\partial b_n}{\partial\alpha_g} = 0,\\
\bar{\beta}_c &= -\frac{\partial(\alpha_g c_1)}{\partial\alpha_g}, \quad \frac{\partial}{\partial\alpha_g}(\alpha_g c_{n+1}) + \bar{\beta}\alpha_g\frac{\partial c_n}{\partial\alpha_g} = 0.
\end{aligned} \tag{B.18}$$

B.1.1 Normal products

When we discuss the finiteness condition of the theory, we significantly use finite composite operators called normal products in curved space [36, 37, 38, 39, 40, 14], which is described as [operator]. In what follows, we first derive the normal product of square of the gauge field strength $[F^a_{\alpha\beta}F^{a\alpha\beta}]$. Then, using it, we rewrite the energy-momentum tensor in terms of normal products.

We here consider the renormalized correlation function given by

$$\begin{aligned}
&\left\langle \prod_{j=1}^{N_A} A^a_{\mu_j}(x_j)\prod_{k=1}^{N_\psi}(\psi\mathrm{and}\bar{\psi})(x_k)\right\rangle\\
&= (\mu^{D-4}Z_A)^{-\frac{N_A}{2}}Z_2^{-\frac{N_\psi}{2}}\int dA_0 d\psi_0 d\bar{\psi}_0\prod_{j=1}^{N_A}A^a_{0\mu_j}(x_j)\prod_{k=1}^{N_\psi}(\psi_0\mathrm{and}\bar{\psi}_0)(x_k)e^{-S}.
\end{aligned} \tag{B.19}$$

First, we introduce the equation-of-motion operators for gauge field and fermion field as

$$\begin{aligned}
E_A &= \frac{1}{\sqrt{g}}A^a_\alpha\frac{\delta S}{\delta A^a_\alpha}\\
&= -\frac{1}{g_0^2}\left(A_{0\beta}^{\ a}\nabla_\alpha F_0^{\ a\alpha\beta} + f^{abc}A_{0\alpha}^{\ b}A_{0\beta}^{\ c}F_0^{\ a\alpha\beta}\right) - i\bar{\psi}_0\gamma^\alpha A_{0\alpha}^{\ a}T^a\psi_0\\
&\quad - \frac{1}{g_0^2\xi_0}A_{0\alpha}^{\ a}\nabla^\alpha\nabla^\beta A_{0\beta}^{\ a} - f^{abc}(\nabla^\alpha\bar{c}_0^a)A_{0\alpha}^{\ b}c_0^{\ c},\\
E_\psi &= \frac{\delta S}{\delta\chi} \equiv \frac{1}{\sqrt{g}}\psi_a\frac{\delta S}{\delta\psi_a} + \frac{1}{\sqrt{g}}\bar{\psi}_a\frac{\delta S}{\delta\bar{\psi}_a}\\
&= i\bar{\psi}_0(\gamma^\alpha\vec{D}_\alpha - \overleftarrow{D}_\alpha\gamma^\alpha)\psi_0.
\end{aligned} \tag{B.20}$$

Inserting the equation-of-motion operator into the renormalized correlation function and performing the partial integration, we obtain the following equations.

$$\begin{aligned}
\left\langle E_{0A}(x)\prod_{j=1}^{N_A}A^a_{\mu_j}(x_j)\right\rangle &= \sum_{j=1}^{N_A}\frac{1}{\sqrt{g}}\delta^{(D)}(x-x_j)\left\langle\prod_{j=1}^{N_A}A^a_{\mu_j}(x_j)\right\rangle,\\
\left\langle E_{0\psi}(x)\prod_{j=1}^{N_\psi}(\psi\mathrm{and}\bar{\psi})(x_j)\right\rangle &= \sum_{j=1}^{N_\psi}\frac{1}{\sqrt{g}}\delta^{(D)}(x-x_j)\left\langle\prod_{j}^{N_\psi}(\psi\mathrm{and}\bar{\psi})(x_j)\right\rangle.
\end{aligned} \tag{B.21}$$

Here, note that there is no term from functional differentials at the same point since it is dimensionally regularized to zero as $\delta A^a_{0\mu}(x)/\delta A^b_{0\nu}(x) = \delta^a_b \delta^\mu_\nu \delta(0) = 0$. The R.H.S is finite and thus the L.H.S is finite. Therefore, the equation-of-motion operators are finite operators and we can write them in terms of normal products as

$$E_{0A} = [E_A], \quad E_{0\psi} = [E_\psi]. \tag{B.22}$$

From eq.(B.21), $\int \sqrt{g} E_{0A}$ and $\int \sqrt{g} E_{0\psi}$ can be respectively replaced with the numbers N_A and N_ψ in correlation functions.

In order to derive the normal product of $[F^a_{\alpha\beta} F^{a\alpha\beta}]$, we consider the finite function obtained by applying the differential operators $\alpha_g \frac{\partial}{\partial \alpha_g}$ and $\xi \frac{\partial}{\partial \xi}$ to the correlation function. We here consider the following combination:

$$\begin{aligned}
&\left(\alpha_g \frac{\partial}{\partial \alpha_g} - \xi \frac{\partial}{\partial \xi}\right) \left\langle \prod_{j=1}^{N_A} A_{\mu_j}(x_j) \prod_{k=1}^{N\psi} (\psi \text{and} \bar{\psi})(x_k) \right\rangle = \text{finite} \\
&= \left(\alpha_g \frac{\partial}{\partial \alpha_g} - \xi \frac{\partial}{\partial \xi}\right) \left\{ -\frac{N_A}{2} \ln Z_A - \frac{N_\psi}{2} \ln Z_2 \right\} \left\langle \prod_{j=1}^{N_A} A_{\mu_j}(x_j) \prod_{k=1}^{N\psi} (\psi \text{and} \bar{\psi})(x_k) \right\rangle \\
&\quad - \left\langle \left(\alpha_g \frac{\partial}{\partial \alpha_g} - \xi \frac{\partial}{\partial \xi}\right) \prod_{j=1}^{N_A} A_{\mu_j}(x_j) \prod_{k=1}^{N\psi} (\psi \text{and} \bar{\psi})(x_k) \right\rangle \\
&= \left\langle \int d^D x \sqrt{g} \left\{ -[E_A] \frac{\tilde{\gamma}_A}{2\beta_g} - [E_\psi] \frac{\tilde{\gamma}_2}{2\beta_g} + \frac{D-4}{\beta_g} \frac{1}{4 g_0^2} F^a_{0\alpha\beta} F_0^{a\alpha\beta} + \frac{1}{\beta_g} \frac{\tilde{\gamma}_A}{2 g_0^2 \xi_0} (\nabla^\alpha A^a_{0\alpha})^2 \right.\right. \\
&\quad \left. + \frac{D-4}{\beta_g} \mu^{D-4} \left[\left(L_a + \frac{\bar{\beta}_a}{D-4} \right) F_D + \left(L_b + \frac{\bar{\beta}_b}{D-4} \right) G_4 + \left(L_c + \frac{\bar{\beta}_c}{D-4} \right) H^2 \right] \right\} \\
&\qquad \left. \times \prod_{j=1}^{N_A} A_{\mu_j}(x_j) \prod_{k=1}^{N\psi} (\psi \text{and} \bar{\psi})(x_k) \right\rangle,
\end{aligned} \tag{B.23}$$

where

$$\tilde{\gamma}_A = \gamma_A + \left[-(D-4) + \gamma_A \right] \xi \frac{\partial}{\partial \xi} \ln Z_A, \quad \tilde{\gamma}_2 = \gamma_2 + \left[-(D-4) + \gamma_A \right] \xi \frac{\partial}{\partial \xi} \ln Z_2 \tag{B.24}$$

and we use the relations

$$\begin{aligned}
\alpha_g \frac{\partial g_0}{\partial \alpha_g} &= \frac{D-4}{2\beta_g} g_0, \\
\alpha_g \frac{\partial \xi_0}{\partial \alpha_g} &= \frac{1}{\beta_g} (\bar{\beta}_g + \gamma_A) \left(1 + \xi \frac{\partial}{\partial \xi} \right) \xi_0, \\
\alpha_g \frac{\partial a_0}{\partial \alpha_g} &= -\frac{D-4}{\beta_g} \mu^{D-4} \left(L_a + \frac{\bar{\beta}_a}{D-4} \right), \quad \alpha_g \frac{\partial b_0}{\partial \alpha_g} = -\frac{D-4}{\beta_g} \mu^{D-4} \left(L_b + \frac{\bar{\beta}_b}{D-4} \right), \\
\alpha_g \frac{\partial c_0}{\partial \alpha_g} &= -\frac{D-4}{\beta_g} \mu^{D-4} \left(L_c + \frac{\bar{\beta}_c}{D-4} \right).
\end{aligned}$$

The gauge-fixing term in the correlation function has the following form:

$$\int d^D x\sqrt{g}\frac{1}{g_0^2\xi_0}(\nabla^\alpha A_{0\alpha}^a) = \int d^D\sqrt{g}[E_c] + \text{BRST trivial} \tag{B.25}$$

under the on-shell BRST transformation, where $[E_c]$ is the equation-of-motion operator for the ghost field defined as in the case of fermion fields. Therefore, this term vanishes in physical correlation functions (B.19) in which ghost fields are not included. So, we obtain the finiteness condition

$$\begin{aligned}\Big\langle\int d^D x\sqrt{g}\Big\{&\frac{D-4}{\beta_g}\frac{1}{4g_0^2}F_{0\alpha\beta}^{\ a}F_0^{\ a\alpha\beta} - [E_A]\frac{\tilde{\gamma}_A}{2\beta_g} - [E_\psi]\frac{\tilde{\gamma}_2}{2\beta_g}\\ &+\frac{D-4}{\beta_g}\mu^{D-4}\Big[\Big(L_a+\frac{\bar{\beta}_a}{D-4}\Big)F_D + \Big(L_b+\frac{\bar{\beta}_b}{D-4}\Big)G_4 + \Big(L_c+\frac{\bar{\beta}_c}{D-4}\Big)H^2\Big]\Big\}\\ &\times\prod_{j=1}^{N_A}A_{\mu_j}(x_j)\prod_{k=1}^{N_\psi}(\psi\text{and}\bar{\psi})(x_k)\Big\rangle = \text{finite}.\end{aligned} \tag{B.26}$$

Here, noting the reciprocal of the beta function is expanded as

$$\frac{1}{\beta} = \frac{1}{D-4}\left(1+\sum_{n=1}^{\infty}\frac{(-\bar{\beta})^n}{(D-4)^n}\right), \tag{B.27}$$

we can see that the expression inside the bracket as the form of normal product of the square of gauge field strength which is required to be the form

$$\left[F_{\alpha\beta}^a F^{a\alpha\beta}\right] = \left(1+\sum\text{poles}\right)F_{0\alpha\beta}^a F_0^{a\alpha\beta} + \left(\sum\text{poles}\right)\times(\text{other operators}). \tag{B.28}$$

We thus find that the normal product

$$\begin{aligned}\frac{1}{4g^2}\left[F_{\alpha\beta}^a F^{a\alpha\beta}\right] &= \frac{D-4}{\beta_g}\frac{1}{4g_0^2}F_{0\alpha\beta}^a F_0^{a\alpha\beta} - E_{0A}\frac{\tilde{\gamma}_A}{2\beta_g} - E_{0\psi}\frac{\tilde{\gamma}_2}{2\beta_g}\\ &+\frac{D-4}{\beta_g}\mu^{D-4}\left[\left(L_a+\frac{\bar{\beta}_a}{D-4}\right)F_D+\left(L_b+\frac{\bar{\beta}_b}{D-4}\right)G_4+\left(L_c+\frac{\bar{\beta}_c}{D-4}\right)H^2-\frac{4(\sigma+L_\sigma)}{D-4}\nabla^2 H\right]\end{aligned} \tag{B.29}$$

up to the BRST trivial term. Moreover, we add the last term since the possible total derivative terms that is only $\nabla^2 H$, which has to determined by imposing the finiteness conditions for the energy-momentum tensor below.

The trace of energy-momentum tensor, namely conformal anomaly is defined by the conformal variation of the action as

$$\begin{aligned}\theta \equiv \frac{2}{\sqrt{g}}g_{\alpha\beta}\frac{\delta S}{\delta g_{\alpha\beta}} &= \frac{\delta S}{\delta\Omega}\\ &= \frac{D-4}{4}F_{0\alpha\beta}^{\ a}F_0^{\ a\alpha\beta} + \frac{D-1}{2}E_{0\psi} + (D-4)[a_0F_D + b_0G_4 + c_0H^2] - 4c_0\nabla^2 H\end{aligned} \tag{B.30}$$

up to the gauge-fixing term. This operator is one of the normal product since it satisfies

$$\frac{\delta}{\delta\Omega(x)}\left\langle\prod_{j=1}^{N_A}A^a_{\mu_j}(x_j)\prod_{k=1}^{N_\psi}(\psi\text{and}\bar{\psi})(x_k)\right\rangle=\left\langle\theta(x)\prod_{j=1}^{N_A}A^a_{\mu_j}(x_j)\prod_{k=1}^{N_\psi}(\psi\text{and}\bar{\psi})(x_k)\right\rangle=\text{finite}. \quad (B.31)$$

Using the expression of $[F^a_{\alpha\beta}F^{a\alpha\beta}]$, we can rewrite it in terms of various normal products and finite quantities as

$$\theta=\frac{\beta_g}{4g^2}\left[F^a_{\alpha\beta}F^{a\alpha\beta}\right]+\frac{\tilde{\gamma}_A}{2}[E_A]+\frac{1}{2}(D-1+\tilde{\gamma}_2)[E_\psi]-\mu^{D-4}[\beta_a F_D+\beta_b G_4+\beta_c H^2] \\ -4\mu^{D-4}(c-\sigma)\nabla^2 H, \quad (B.32)$$

where the last term is determined later.

In what follows, we consider the two-and three-point function of the trace of the energy-momentum tensor, which give us important conditions to determine the forms of gravitational action and also conformal anomaly.

B.2 Hathrell's renormalization group equations

Let us derive the non-trivial renormalization group equations which corrects the gravitational counterterms with bare constants b_0 and c_0 [37, 38, 39, 40, 14].

Two point function Since the partition function is finite, its gravitational variation should be finite too. Acting the conformal variation twice on the partition function, we obtain the following condition:

$$\langle\theta(x)\theta(y)\rangle-\left\langle\frac{\delta\theta(x)}{\delta\Omega(y)}\right\rangle=\text{finite} \quad (B.33)$$

Taking flat space and going to momentum space, we obtain the equation:

$$\langle\theta(p)\theta(-p)\rangle_{\text{flat}}-8c_0p^4=\text{finite}. \quad (B.34)$$

Now, we introduce the following expression.

$$\bar{\theta}\equiv\theta-\frac{D-1}{2}\left[E_\psi\right]. \quad (B.35)$$

In terms of this variable, the equation above can be expressed as

$$\left\langle\bar{\theta}(p)\bar{\theta}(-p)\right\rangle_{\text{flat}}-8p^4\mu^{D-4}L_c=\text{finite}. \quad (B.36)$$

Here, we use the fact that since one-point functions are dimensionally regularized to zero for a massless theory in flat space, the two-point function including the equation-of-motion operator vanishes, for instance, as

$$\langle E_\psi(x)P(y)\rangle|_{\text{flat}}=\left\langle\frac{\delta P(y)}{\delta\chi(x)}\right\rangle_{\text{flat}}=0, \quad (B.37)$$

where $P(y)$ is an composite field. We also introduce the composite operator defined as

$$\begin{aligned}\{A^2\} &\equiv \frac{D-4}{\beta_g}\frac{1}{4g_0^2}F_{0}{}^{a}_{\alpha\beta}F_0{}^{a\alpha\beta} \\ &= \frac{1}{4g^2}\left[F^a_{\alpha\beta}F^{a\alpha\beta}\right] + \frac{1}{\beta_g}\left(\tilde{\gamma}_A\left[E_A\right] + \tilde{\gamma}_2\left[E_\psi\right]\right).\end{aligned} \tag{B.38}$$

We then obtain the following relation:

$$\beta_g\left\{A^2\right\} = (D-4)\frac{1}{4g_0^2}F_{0}{}^{a}_{\alpha\beta}F_0{}^{a\alpha\beta} = \bar{\theta}\big|_{\text{flat}}. \tag{B.39}$$

Let us consider the two-point function of the composite operator $\{A^2\}$ defined as

$$\Gamma_{AA}(p^2) \equiv \left\langle\{A^2(p)\}\{A^2(-p)\}\right\rangle_{\text{flat}}. \tag{B.40}$$

This is written in terms of the two point function of the composite operator $[F^a_{\alpha\beta}F^{a\alpha\beta}]$ since the two-point function including equation-of-motion operator disappears. Also, the correlation function of renormalized operators does not involve non-local divergence. Therefore, the divergence of Γ_{AA} is written in terms of a pure pole as

$$\Gamma_{AA} - p^4\mu^{D-4}\left(\frac{D-4}{\beta_g}\right)^2 L_x = \text{finite}. \tag{B.41}$$

where the pure pole term L_x is defined as

$$L_x = \sum_{n=1}^{\infty}\frac{x_n}{(D-4)^n} \tag{B.42}$$

and the factor $(D-4)^2/\beta_g^2$ is introduced for later convenience.

Since $\beta_g^2\Gamma_{AA} = \langle\theta\bar{\theta}\rangle$, we can see that by combining eq.(B.36) and (B.41), the pure pole terms satisfy the following relation:

$$(D-4)^2L_x - 8L_c = \text{finite}. \tag{B.43}$$

It is obviously

$$c_n = \frac{1}{8}x_{n+2}. \tag{B.44}$$

This indicates that if the residue x_3 is calculated, we can find c_1 and then general c_n through the renormalization group equation if we calculate the residue x_3. So, we then have to derive the relationship between x_3 and x_1 using renormalization group equation.

So, we next derive the renormalization group equation that gives the relationship between the residues x_3 and x_1. In order to derive it, we here use the fact that if F is a finite quantity,

$$\frac{1}{\beta_g^n}\mu\frac{d}{d\mu}(\beta_g^nF) = \mu\frac{dF}{d\mu} + n\alpha_g\frac{\partial\bar{\beta}_g}{\partial\alpha_g} = \text{finite}. \tag{B.45}$$

Applying this fact of $n=2$ to eq. (B.41) as a finite quantity, we obtain the following condition:

$$\frac{1}{\beta_g^2}\mu\frac{d}{d\mu}\left\{\beta_g^2\left[\Gamma_{AA}-p^4\mu^{D-4}\left(\frac{D-4}{\beta_g}\right)^2 L_x\right]\right\}=\text{finite}. \tag{B.46}$$

Since $\beta_g\{A^2\}$ can be written in bare quantities, it satisfies $\mu d(\beta\{A^2\})/d\mu=0$. Thus, we obtain the following renormalization group equation:

$$\frac{1}{\beta_g^2}\mu\frac{d}{d\mu}\{\mu^{D-4}(D-4)^2 L_x\}=\text{finite}. \tag{B.47}$$

Expanding these equations and bringing out parts that poles are canceled, we obtain

$$\begin{aligned}&\frac{\partial}{\partial\alpha_g}(\alpha_g x_2)-\frac{\bar{\beta}_g}{\alpha_g}\frac{\partial}{\partial\alpha_g}(\alpha_g^2 x_1)=0,\\&\frac{\partial}{\partial\alpha_g}(\alpha_g x_3)-\frac{\bar{\beta}_g}{\alpha_g}\frac{\partial}{\partial\alpha_g}(\alpha_g^2 x_2)+\frac{\bar{\beta}_g^2}{\alpha_g^2}\frac{\partial}{\partial\alpha_g}(\alpha_g^3 x_1)=0.\end{aligned} \tag{B.48}$$

Using these equations, we can find the residues x_2 and x_3 from x_1.

Three point function Then, we consider the three point function of the energy-momentum tensor, which satisfies

$$\langle\theta(x)\theta(y)\theta(z)\rangle-\left\langle\frac{\delta\theta(x)}{\delta\Omega(y)}\theta(z)\right\rangle-\left\langle\frac{\delta\theta(y)}{\delta\Omega(z)}\theta(x)\right\rangle-\left\langle\frac{\delta\theta(z)}{\delta\Omega(x)}\theta(y)\right\rangle \\ +\left\langle\frac{\delta S}{\delta\Omega(x)\delta\Omega(y)\delta\Omega(z)}\right\rangle=\text{finite}. \tag{B.49}$$

It is convenient to define the following quantity.

$$\bar{\theta}(y,z)\equiv\frac{\delta\bar{\theta}(y)}{\delta\Omega(z)}-\frac{D-1}{2}\frac{\delta\bar{\theta}(y)}{\delta\chi(z)}, \tag{B.50}$$

which satisfies the symmetric condition $\bar{\theta}(y,z)=\bar{\theta}(z,y)$. In terms of $\bar{\theta}(x)$ and $\bar{\theta}(x,y)$, the condition of the three points function of θ can be written in flat space as

$$\langle\bar{\theta}(x)\bar{\theta}(y)\bar{\theta}(z)\rangle_{\text{flat}}-\langle\bar{\theta}(x)\bar{\theta}(y,z)\rangle_{\text{flat}}-\langle\bar{\theta}(y)\bar{\theta}(z,x)\rangle_{\text{flat}}-\langle\bar{\theta}(z)\bar{\theta}(x,y)\rangle_{\text{flat}} \\ +\left\langle\frac{\delta^3 S}{\delta\Omega(x)\delta\Omega(y)\delta\Omega(z)}\right\rangle_{\text{flat}}=\text{finite}. \tag{B.51}$$

The trace of energy-momentum tensor in flat space is $\bar{\theta}(x)|_{\text{flat}}=\beta_g\{A^2\}$, and this leads to the following equation

$$\bar{\theta}(x,y)|_{\text{flat}}=-4\beta_g\{A^2\}\delta^{(D)}(x-y)+8c_0\partial^4\delta^{(D)}(x-y). \tag{B.52}$$

Using these expressions, we obtain the following relations:

$$\begin{aligned} \langle\bar{\theta}(x)\bar{\theta}(y)\bar{\theta}(z)\rangle|_{\text{flat}} &= \beta_g^3\langle\{A^2(x)\}\{A^2(y)\}\{A^2(z)\}\rangle|_{\text{flat}}, \\ \langle\bar{\theta}(x)\bar{\theta}(y,z)\rangle|_{\text{flat}} &= -4\beta_g^2\langle\{A^2(x)\}\{A^2(y)\}\rangle|_{\text{flat}}\delta^{(D)}(x-y). \end{aligned} \tag{B.53}$$

Going to momentum space, and denoting the three-point function of $\{A^2\}$ by Γ_{AAA}, equation (B.51) is then expressed as follows:

$$\begin{aligned} \beta_g^3\Gamma_{AAA}(p_x^2,p_y^2,p_z^2) + 4\beta_g^2\{\Gamma_{AA}(p_x^2) + \Gamma_{AA}(p_y)^2 + \Gamma_{AA}(p_z^2)\} + b_0 B(p_x^2,p_y^2,p_z^2) + c_0 C(p_x^2,p_y^2,p_z^2) \\ = \text{finite}, \end{aligned} \tag{B.54}$$

where

$$\begin{aligned} B(p_x^2,p_y^2,p_z^2) &= -2(D-2)(D-3)(D-4)\left[p_x^4+p_y^4+p_z^4-2(p_x^2p_y^2+p_y^2p_z^2+p_z^2p_x^2)\right], \\ C(p_x^2,p_y^2,p_z^2) &= -4\left[(D+2)(p_x^4+p_y^4+p_z^4)+4(p_x^2p_y^2+p_y^2p_z^2+p_z^2p_x^2)\right]. \end{aligned} \tag{B.55}$$

These relations respectively take

$$\begin{aligned} B(p^2,p^2,0) &= 0, \\ C(p^2,p^2,0) &= -8(D+4)p^4 \end{aligned} \tag{B.56}$$

at $p_x^2 = p_y^2 = p^2 p_z^2 = 0$ and

$$\begin{aligned} B(p^2,0,0) &= -2(D-2)(D-3)(D-4)p^4, \\ C(p^2,0,0) &= -4(D+2)p^4 \end{aligned} \tag{B.57}$$

at $p_x^2 = p^2, p_y^2 = p_z^2 = 0$. Therefore, we obtain the following relations

$$\beta_g^3\Gamma_{AAA}(p^2,p^2,0) - 8(D-4)p^4\mu^{D-4}L_c = \text{finite} \tag{B.58}$$

and

$$\beta_g^3\Gamma_{AAA}(p^2,0,0) - p^4\mu^{D-4}[2(D-2)(D-3)(D-4)L_b + 4(D-6)L_c] = \text{finite}. \tag{B.59}$$

The three point function Γ_{AAA} generally has the following form:

$$\begin{aligned} \Gamma_{AAA}(p_x^2,p_y^2,p_z^2) - \sum(\text{poles}) \times \left[\Gamma_{AA}(p_x^2) + \Gamma_{AA}(p_y^2) + \Gamma_{AA}(p_z^2)\right] \\ - \mu^{D-4}\sum(\text{poles}) \times \{\text{terms of } p_i^2p_j^2\} = \text{finite}. \end{aligned} \tag{B.60}$$

Since three point functions involving $[E_\psi]$ do not vanish, the term $[E_\psi]/\beta_g$ in $\{A^2\}$ produces non-local poles by the presence of $1/\beta_g$. Thus, Γ_{AAA} has non-local poles unlike Γ_{AA}. The second term in (B.60) plays an important role to cancel out such non-local poles.

In order to determine the pure-pole factor in front of Γ_{AA} in (B.60), we consider the equation obtained by applying $\alpha_g\partial/\partial\alpha_g$ to (B.41). We then obtain the equation for $\Gamma_{AAA}(p^2,p^2,0)$ owing to $\alpha_g\partial\{A^2\}/\partial\alpha_g = -(\alpha_g/\beta_g)(\partial\bar{\beta}_g/\partial\alpha_g)\{A^2\}$ and $\alpha_g\partial S/\partial\alpha_g|_{\text{flat}} = -\int d^Dx\{A^2\}$ up to

BRST trivial terms. The pole factor can be extracted from this equation and fixed to be $(\alpha_g^2/\beta_g)\partial(\bar{\beta}_g/\alpha_g)/\partial\alpha_g$. Therefore, $\Gamma_{AAA}(p^2,0,0)$ has the following form:

$$\Gamma_{AAA}(p^2,0,0) - \frac{\alpha_g^2}{\beta_g}\frac{\partial}{\partial\alpha_g}\left(\frac{\bar{\beta}_g}{\alpha_g}\right)\Gamma_{AA}(p^2) - p^4\mu^{D-4}\left(\frac{D-4}{\beta_g}\right)^3 L_y = \text{finite}, \tag{B.61}$$

where

$$L_y = \sum_{n=1}^{\infty}\frac{y_n}{(D-4)^n}. \tag{B.62}$$

Multiplying β_g^3 to equation (B.61) and using the expression of (B.59), we obtain the following pole relation:

$$2(D-2)(D-3)(D-4)L_b + 4\left\{D-6-2\alpha_g^2\frac{\partial}{\partial\alpha_g}\left(\frac{\bar{\beta}_g}{\alpha_g}\right)\right\}L_c - (D-4)^3 L_y = \text{finite}. \tag{B.63}$$

Moreover, since $\mu\frac{d}{d\mu}(\beta_g^3\Gamma_{AAA}) = 0$, we can obtain the renormalization group equation connecting L_x and L_y by applying (B.45) at $n=3$ to (B.61) as a finite quantity as follows:

$$\left(\frac{D-4}{\beta_g}\right)^3\left[(D-4)L_y + \mu\frac{d}{d\mu}L_y\right] + \alpha_g^2\frac{\partial^2\bar{\beta}_g}{\partial\alpha_g^2}\left(\frac{D-4}{\beta_g}\right)^2 L_x = \text{finite}. \tag{B.64}$$

B.2.1 Determination of gravitational counterterms and conformal anomalies

Having clarified the relations between the respective residues, we will now discuss the determination of the gravitational counterterm [40, 14]. As we seen above, the pure pole terms L_b and L_c have a one-to-one connection. This means that we can unify the Euler term and the square of the rescaled Ricci scalar. In this case, the number of gravitational counterterm reduces to two. Then, we write the gravitational action as

$$S_g = \int d^Dx\sqrt{g}\{a_0F_D + b_0G_D\} \tag{B.65}$$

where

$$G_D \equiv G_4 + (D-4)\chi(D)H^2. \tag{B.66}$$

The coefficient $\chi(D)$ is a finite function with respect to D only. Using the counterterm (B.65), we can obtain the finiteness conditions that simply result in Hathrell's renormalization group equations, (B.43), (B.47), (B.63), and (B.64), under the relation

$$L_c - (D-4)\chi(D)L_b = \text{finite}. \tag{B.67}$$

Thus, we can make the theory finite using only two gravitational counterterms. We will show later that the function χ can be determined completely by solving the coupled renormalization group equations order by order.

On the other hand, we have to pay attention to the calculation of the finite quantities such as the expression of the conformal anomaly, since the gravitational counterterm (B.65) indicate that the finite parameter c is eliminated and extra finite terms are added.

When we use the counterterm (B.65) and repeat the calculation as before, we obtain the following normal product:

$$\frac{1}{4g^2}\left[F^a_{\alpha\beta}F^{a\alpha\beta}\right] = \frac{D-4}{\beta_g}\frac{1}{4g_0^2}F_0{}^a_{\alpha\beta}F_0{}^{a\alpha\beta} - [E_A]\frac{\tilde{\gamma}_A}{2\beta_g} - [E_\psi]\frac{\tilde{\gamma}_2}{2\beta_g}$$
$$+ \frac{D-4}{\beta_g}\mu^{D-4}\left[\left(L_a + \frac{\bar{\beta}_a}{D-4}\right)F_D + \left(L_b + \frac{\bar{\beta}_b}{D-4}\right)G_D - \frac{4\chi(D)(\sigma+L_\sigma)}{D-4}\nabla^2 H\right]. \quad \text{(B.68)}$$

Using this normal product, we can obtain the trace of energy-momentum tensor expressed in a manifestly finite form as

$$\begin{aligned}\theta &= \frac{D-4}{4g_0^2}F_0{}^a_{\alpha\beta}F_0{}^{a\alpha\beta} + \frac{D-1}{2}E_{0\psi} + (D-4)a_0F_D \\ &\quad + b_0[(D-4)G_4 + \chi(D)(D-4)\{(D-4)H^2 - 4\nabla^2 H\}] \\ &= \frac{\beta_g}{4g^2}\left[F^a_{\alpha\beta}F^{a\alpha\beta}\right] + \frac{\tilde{\gamma}_A}{2}[E_A] + \frac{D-1+\tilde{\gamma}_2}{2}[E_\psi] - \mu^{D-4}(\beta_a F_D + \beta_b G_D) \\ &\quad - 4\mu^{D-4}\chi(D)\left[(D-4)(b+L_b) - (\sigma+L_\sigma)\right]\nabla^2 H. \end{aligned} \quad \text{(B.69)}$$

The coefficient in the last term $\left[(D-4)(b+L_b) - (\sigma+L_\sigma)\right]$ is expressed as

$$(D-4)b - \sigma + b_1 + \sum_{n=1}^{\infty}\frac{b_{n+1}-\sigma_n}{(D-4)^n}. \quad \text{(B.70)}$$

This must be finite for the reason of the finiteness of the energy-momentum tensor such that

$$b_{n+1} = \sigma_n \quad (n \geq 1). \quad \text{(B.71)}$$

Acting $\frac{1}{\beta_g}\mu\frac{d}{d\mu}$ on the expression (B.69) and noting $\mu\frac{d\theta}{d\mu} = 0$, we obtain

$$\begin{aligned}\frac{1}{\beta_g}\mu\frac{d}{d\mu}\left(\left[\frac{\beta_g}{4g^2}F^a_{\alpha\beta}F^{a\alpha\beta}\right]\right) &= -\frac{1}{2\beta_g}\mu\frac{d}{d\mu}\{(D-1+\tilde{\gamma}_2)[E_\psi] + \tilde{\gamma}_A[E_A]\} \\ &\quad + \frac{1}{\beta_g}\mu\frac{d}{d\mu}[\mu^{D-4}(\beta_a F_D + \beta_b G_D)] + \frac{4(D-4)}{\beta_g}\chi(D)\mu^{D-4}(\bar{\beta}_b - \sigma + b_1)\nabla^2 H \\ &= -\frac{1}{2}\frac{\partial\tilde{\gamma}_2}{\partial\alpha_g}[E_\psi] - \frac{1}{2}\frac{\partial\tilde{\gamma}_A}{\partial\alpha_g}[E_A] + \mu^{D-4}\left(\alpha_g\frac{\partial\bar{\beta}_a}{\partial\alpha_g}F_D + \alpha_g\frac{\partial\bar{\beta}_b}{\partial\alpha_g}G_D\right) \\ &\quad + \frac{4(D-4)}{\beta_g}\chi(D)\mu^{D-4}(\bar{\beta}_b - \sigma + b_1)\nabla^2 H \end{aligned} \quad \text{(B.72)}$$

Since L.H.S is finite owing to (B.45), R.H.S should be also finite. The divergent term is only the last term, and therefore

$$\sigma = b_1 + \bar{\beta}_b. \quad \text{(B.73)}$$

Substituting this into the equation (B.69), we obtain the following form of the conformal anomaly

$$\theta = \frac{\beta_g}{4g^2}\left[F^a_{\alpha\beta}F^{a\alpha\beta}\right] + \frac{\tilde{\gamma}_A}{2}[E_A] + \frac{D-1+\tilde{\gamma}_2}{2}[E_\psi] - \mu^{D-4}(\beta_a F_D + \beta_b E_D), \tag{B.74}$$

where the last term is defined as

$$E_D = G_D - 4\chi(D)\nabla^2 H. \tag{B.75}$$

It has a desirable property similar to the other conformal anomalies F_D and $(F^a_{\alpha\beta})^2$, which is

$$\frac{\delta}{\delta\Omega}\int d^D x\sqrt{g}E_D = (D-4)E_D. \tag{B.76}$$

It is clear that the volume integral of E_D is nothing less than that of counterterm G_D.

Now, we determine the function $\chi(D)$ order by order. To determine the pole terms, we need the information of the QCD beta function and the simple-pole residues of L_x and L_y. In order to find respective residues, we expand the beta function $\bar{\beta}_g$ and the residues x_1 and y_1 in a power series with respect to α_g as follows

$$\begin{aligned}\bar{\beta}_g &= \beta_1\alpha_g + \beta_2\alpha_g^2 + \beta_3\alpha_g^3 + \mathcal{O}(\alpha_g^4),\\ x_1 &= X_1 + X_2\alpha_g + X_3\alpha_g^2 + \mathcal{O}(\alpha_g^3),\\ y_1 &= Y_1 + Y_2\alpha_g + Y_3\alpha_g^2 + \mathcal{O}(\alpha_g^2).\end{aligned} \tag{B.77}$$

Furthermore, we expand $\chi(D)$ in a power series of $D-4$ as

$$\chi(D) = \chi_1 + \chi_2(D-4) + \chi_3(D-4)^2 + \cdots. \tag{B.78}$$

The solution for the first three terms of χ is then given as

$$\chi_1 = \frac{1}{2}, \tag{B.79}$$

$$\chi_2 = 1 - \frac{Y_1}{4X_1}, \tag{B.80}$$

$$\chi_3 = \frac{1}{8}\left(2-\frac{Y_1}{X_1}\right)\left(3-\frac{Y_1}{X_1}\right) - \frac{1}{6}\frac{\beta_2}{\beta_1^2}\left(1-\frac{Y_1}{X_1}\right) + \frac{1}{6}\frac{X_2}{\beta_1 X_1}\left(\frac{Y_2}{X_2} - \frac{3}{2}\frac{Y_1}{X_1}\right). \tag{B.81}$$

The constants X_1 and Y_1 follow from the calculation of Γ_{AA} and Γ_{AAA} whose diagrams are shown in Figure B.1 up to one loop level. The results are

$$\Gamma_{AA}(p^2) = -\frac{r}{2}\frac{\mu^{D-4}}{(4\pi)^2}p^4\frac{1}{D-4} \tag{B.82}$$

$$\Gamma_{AAA}(p^3) = -\frac{r}{2}\frac{\mu^{D-4}}{(4\pi)^2}p^4\frac{1}{D-4} \tag{B.83}$$

We thus obtain that $X_1 = Y_1 = -\frac{r}{2(4\pi)^2}$ from the above equations where r is the dimension of Lie group. From this, we immediately find

$$\chi_2 = \frac{3}{4}. \tag{B.84}$$

(a) Two point function Γ_{AA} (b) Three point function Γ_{AAA}

Figure B.1: Feynman diagrams with respect to the composite operator $\{A^2\}$

In this way, we can see that at least χ_1 and χ_2 are the universal coefficients independent of the gauge group and the fermion representation. At present, it is not clear whether the coefficient $\chi_n (n \geq 3)$ has a universal value independent of the theories or not. Regardless of the theory, χ_n can be determined at all orders.

Then, we calculate the explicit value of b_1', which is the coupling-dependent part of b_1. From $\chi_1 = 1/2$, we obtain the relation $b_2 = 2c_1 + \mathcal{O}(\alpha_g^4)$. The residue $c_1 = x_3/8$ is obtained from x_1 using the renormalization equation (B.48). Since $x_1 = X_1 + \mathcal{O}(\alpha_g)$., we obtain $c_1 = -\beta_1\beta_2 X_1 \alpha_g^3/96 + \mathcal{O}(\alpha_g^4)$ and therefore we can find b_2. Further, using the renormalization group equation among b_n, we obtain

$$b_1' = \frac{\beta_2 X_1}{24}\alpha_g^2 + \mathcal{O}(\alpha_g^3). \tag{B.85}$$

Thus, the coupling dependent part of the residue b_1 starts from $\mathcal{O}(\alpha_g^2)$.

The expansion of the function $\chi(D)$ up to the second order of $D-4$ finally becomes

$$\chi(D) = \frac{1}{2} + \frac{3}{4}(D-4) + \cdots . \tag{B.86}$$

The above equation shows that the first two coefficient of $\chi(D)$ in QCD are in common with QED. In QED, the third coefficient has been calculated to be $\chi_3 = 1/3$.

At the $D \to 4$ limit, the conformal anomaly E_D reduces to the form

$$E_4 = G_4 - \frac{2}{3}\nabla^2 R. \tag{B.87}$$

This is exactly the combination proposed by Riegert in 1984 [15].

Appendix C

BRST transformations and gauge fixing terms

The action of the non-abelian gauge field and the kinetic term of the traceless tensor mode are given as follows:

$$S_{\text{YM + h}} = \int d^D x \left[\frac{1}{4g_0^2} F^a_{0\,\alpha\beta} F_0^{a\alpha\beta} + \frac{D-3}{D-2} (h_{0\alpha\beta} \partial^4 h_0{}^{\alpha\beta} + 2\chi_{0\alpha} \partial^2 \chi_0{}^\alpha) + \frac{D-3}{D-1} \partial_\alpha \chi_0{}^\alpha \partial_\beta \chi_0{}^\beta \right]. \quad \text{(C.1)}$$

In order to quantize them, we introduce the gauge fixing terms and the ghost actions as [9, 56]

$$S_{g.f+FP} = \int d^D x \delta_B \left\{ \tilde{c}_0^a \left(\partial^\alpha A_0{}^a_\alpha - \frac{g_0^2 \xi_0}{2} B_0^a \right) + \tilde{c}_{0\alpha} N^{\alpha\beta} \left(\chi_{0\beta} - \frac{\zeta_0}{2} B_{0\beta} \right) \right\}, \quad \text{(C.2)}$$

where δ_B is the BRST transformation defined soon below. $\tilde{c}_0^a$ and $\tilde{c}_0^\alpha$ are respectively the anti-ghost fields for the gauge and diffeomorphism transformations and B_0^a and $B_{0\alpha}$ are the corresponding Nakanishi-Lautrup fields.The differential operator $N_{\alpha\beta}$ is given as

$$N_{\alpha\beta} = \frac{2(D-3)}{D-2} \left(-2\eta_{\alpha\beta} \partial^2 + \frac{D-2}{D-1} \partial_\alpha \partial_\beta \right). \quad \text{(C.3)}$$

From the transformation laws of the traceless tensor mode and the gauge field, we can write down the BRST transformations of both fields with replacing gauge parameters into the corresponding Faddeev Popov ghosts c_0^a and $c_{0\alpha}$ as

$$\delta_B A_0{}^a_\alpha = \partial_\alpha c_0{}^a - f^{abc} A_0{}^b_\alpha c_0{}^c + t_0 (A^a_{0\gamma} \partial_\alpha c_0{}^\gamma + c_0{}^\gamma \partial_\gamma A^a_{0\alpha}), \quad \text{(C.4)}$$

$$\delta_B h_{0\alpha\beta} = \partial_\alpha c_{0\beta} + \partial_\beta c_{0\alpha} - \frac{2}{D} \eta_{\alpha\beta} \partial_\gamma c_0{}^\gamma + t c_0{}^\gamma \partial_\gamma h_{0\alpha\beta} + \frac{t_0}{2} h_{0\alpha\gamma} (\partial_\beta c_0{}^\gamma - \partial^\gamma c_{0\beta}) + \frac{t_0}{2} h_{0\beta\gamma} (\partial_\alpha c_0{}^\gamma - \partial^\gamma c_{0\alpha}) + \mathcal{O}(h^2). \quad \text{(C.5)}$$

Besides, the BRST transformation of the conformal mode is given as

$$\delta_B \phi = t_0 c^\gamma \hat{\nabla}_\gamma \phi + \frac{t_0}{D} \hat{\nabla} c_0{}^\gamma. \tag{C.6}$$

The transformation laws of Faddeev-Popov ghost fields and Nakanishi-Lautrup fields are determined from their nilpotency as

$$\begin{aligned} \delta_B c_0^a &= t_0 c_0{}^\gamma \partial_\gamma c_0^a, \quad \delta_B \tilde{c}_0^a = B_0^a, \quad \delta_B B_0^a = 0, \\ \delta_B c_{0\alpha} &= t_0 c_0{}^\gamma \partial_\gamma c_{0\alpha}, \quad \delta_B \tilde{c}_0^\alpha = B_0{}^\alpha, \quad \delta_B B_0{}^\alpha = 0. \end{aligned} \tag{C.7}$$

So, the gauge fixing terms and Faddeev-Popov ghost terms become

$$\begin{aligned} S_{g.f+FP} = \int d^D x \Big[& B_0^a \left(\partial^\alpha A_{0\alpha}^a - \frac{g_0^2 \xi_0}{2} B_0^a \right) - \tilde{c}_0^a \partial^\alpha \delta_B A_0^a \\ & + B_{0\alpha} N^{\alpha\beta} \left(\chi_{0\beta} - \frac{\zeta_0}{2} B_{0\beta} \right) - \tilde{c}_{0\alpha} N^{\alpha\beta} \partial^\gamma \delta_B h_{0\gamma\beta} \Big]. \end{aligned}$$

When we integrate out the Nakanishi-Lautrup fields, we finally obtain the gauge fixing terms of gauge field and the traceless tensor mode as follows:

$$S_{g.f} = \int d^D x \left[\frac{1}{2\xi_0} \partial^\alpha A_{0\alpha}^a \partial^\beta A_{0\beta}^a + \frac{1}{2\zeta_0} \chi_{0\alpha} N^{\alpha\beta} \chi_{0\beta} \right]. \tag{C.8}$$

Appendix D

Contractions of the traceless tensor mode

We need to contract the indices of the tensor mode propagator to find the integrand for loop calculations. Feynman diagrams which we need are as Figure D.1. We here present the results, which are calculated using MAXIMA sofware, in what follows.

$$\Gamma_1 = -bt^2 \frac{1}{(4\pi)^{D/2}} \frac{D-2}{4(D-3)} \int \frac{d^D p d^D q}{(2\pi)^{2D}} \phi(p)\phi(-p) \frac{F_1(p^2, q^2)}{(q^2+z^2)^2\{(p-q)^2+z^2\}^2}, \tag{D.1}$$

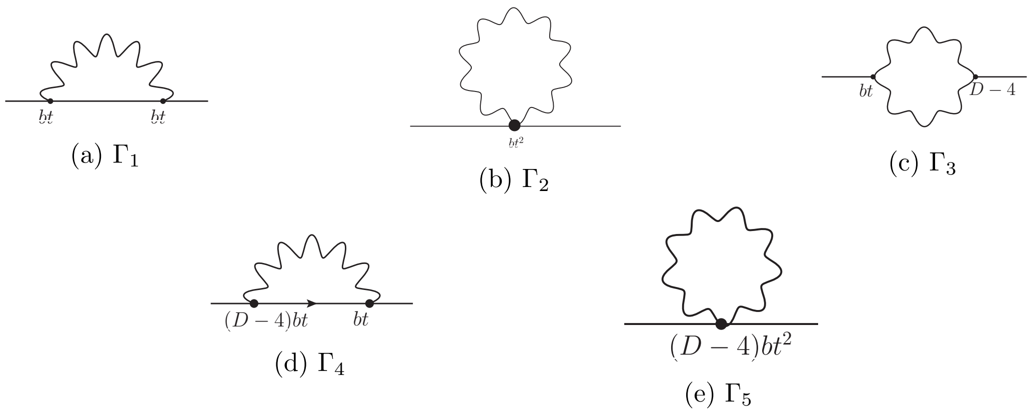


Figure D.1: Corrections of conformal mode with respect to traceless tensor mode

where

$$
\begin{aligned}
F_1(p^2,q^2) = \frac{16(D-2)}{D-1}\Bigg\{ & \left(p^2\right)^4 + 2\left(p^2\right)^3 q^2 + \left(p^2\right)^2\left(q^2\right)^2 - 2\left(\left(p^2\right)^2 q^2 + \left(p^2\right)^3\right)(p\cdot q) \\
& - \left(2\frac{(p^2)^3}{q^2} + 3(p^2)^2 + 2p^2q^2\right)(p\cdot q)^2 + 4\left(\frac{(p^2)^2}{q^2} + p^2\right)(p\cdot q)^3 \\
& + \left(\frac{(p^2)^2}{(q^2)^2} + 1\right)(p\cdot q)^4 - 2\left(\frac{p^2}{(q^2)^2} + \frac{1}{(q^2)}\right)(p\cdot q)^5 + \frac{(p\cdot q)^6}{(q^2)^2}\Bigg\} \\
& + \zeta\Bigg\{\frac{16}{D(D-1)}\left(p^2\right)^4 + \frac{8\left(3D^2-7D+16\right)}{3D(D-1)}\left(p^2\right)^3 q^2 \\
& + \frac{8\ \left(11\,D^2 - 25\,D + 32\right)}{9D(D-1)}\left(p^2\right)^2\left(q^2\right)^2 + 2\,p^2\left(q^2\right)^3 \\
& - \left(\frac{32(D^2-2D+2)}{D(D-1)}(p^2)^3 + \frac{16(11D^2-23D+18)}{3D(D-1)}\left(p^2\right)^2 q^2 - \frac{16(11D-8)}{9D}p^2\left(q^2\right)^2\right)(p\cdot q) \\
& + \left(\frac{32(D-2)}{D-1}\frac{(p^2)^3}{q^2} + \frac{8\left(43\,D^2 - 85\,D + 24\right)}{3D(D-1)}(p^2)^2\right. \\
& \left. + \frac{8\left(23D^2-43D+8\right)}{3D(D-1)}p^2q^2 - \frac{2(D+8)}{9D}(q^2)^2\right)(p\cdot q)^2 \\
& + \left(-\frac{64(D-2)}{D-1}\frac{(p^2)^2}{q^2} - \frac{16(13D-25)}{3(D-1)}p^2 + \frac{16}{3}q^2\right)(p\cdot q)^3 \\
& + \left(-\frac{16(D-2)}{D-1}\frac{(p^2)^2}{(q^2)^2} - \frac{16(4D-7)}{3(D-1)}\right)(p\cdot q)^4 \\
& + \left(\frac{32(D-2)}{D-1}\frac{1}{q^2} + \frac{32(D-2)}{D-1}\frac{p^2}{(q^2)^2}\right)(p\cdot q)^5 - \frac{16(D-2)}{D-1}\frac{(p\cdot q)^6}{(q^2)^2}\Bigg\}. \qquad \text{(D.2)}
\end{aligned}
$$

The contribution from the second diagram is

$$
\Gamma_2 = bt^2\frac{1}{(4\pi)^{D/2}}\frac{D-2}{2(D-3)}\int\frac{d^Dpd^Dq}{(2\pi)^{2D}}\phi(p)\phi(-p)\frac{F_2(p^2,q^2)}{(q^2+z^2)^2}, \qquad \text{(D.3)}
$$

where

$$
\begin{aligned}
F_2(p,q) = bt^2 & \frac{1}{(4\pi)^{D/2}}\frac{D-2}{2(D-3)}\int\frac{d^Dpd^Dq}{(2\pi)^{2D}}\phi(p)\phi(-p)\frac{1}{(q^2+z^2)^2} \\
& \times\Bigg[\frac{(D-2)(D+3)}{D-1}(p^2)^2 - \frac{(D-7)(D-2)(D+1)}{6(D-1)}p^2q^2 \\
& + \left(-\frac{(D-2)(D+5)}{D-1}\frac{p^2(p\cdot q)^2}{q^2} + \frac{(D-3)(D-2)(D+1)}{2(D-1)}\right) + \frac{2(D-2)}{D-1}\frac{(p\cdot q)^4}{(q^2)^2} \\
& + \zeta\Bigg\{\frac{D^2-D+4}{D(D-1)}(p^2)^2 + \frac{2(2D^2-5D+6)}{3D(D-1)}p^2q^2 \\
& + \left(\frac{(D-2)(D+5)}{D-1}\frac{p^2}{q^2} - \frac{2(D^2-4D+6)}{3D(D-1)}\right)(p\cdot q)^2 - \frac{2(D-2)}{D-1}\frac{(p\cdot q)^4}{(q^2)^2}\Bigg\}\Bigg]. \qquad \text{(D.4)}
\end{aligned}
$$

The contribution from the third diagram is

$$\Gamma_3 = -bt^2 \frac{1}{(4\pi)^{D/2}} \frac{(D-4)(D-2)^2}{2(D-3)^2} \int \frac{d^D p d^D q}{(2\pi)^{2D}} \phi(q)\phi(-q) \frac{F_3(p,q)}{(p^2+z^2)^2\{(p-q)^2+z^2\}^2}, \quad \text{(D.5)}$$

where

$$\begin{aligned} F_3(p,q) = &\frac{(D-3)(7D+19)}{12(D-1)}(p^2)^2(q^2)^2 - \frac{(D-3)(D+1)(7D-19)}{12(D-1)}(p^2)^3 q^2 \\ &+ \left\{ \frac{(D-3)(D+1)(5D-13)}{4(D-1)}(p^2)^2 q^2 - \frac{(D-3)(D+1)}{12(D-1)} p^2 (q^2)^2 \right\} (p\cdot q) \\ &+ \left\{ \frac{(D-3)^2(D+1)}{2(D-1)}(p^2)^2 - \frac{(D-3)(9D^2+D+16)}{12(D-1)} p^2 q^2 \right\} (p\cdot q)^2 \\ &+ \left\{ -\frac{(D-3)^2(D+1)}{D-1} p^2 + \frac{(D-3)D(D+1)}{12(D-1)} q^2 \right\} (p\cdot q)^3 + (p\cdot q)^4 \frac{D(D-3)}{2}. \end{aligned} \quad \text{(D.6)}$$

The contribution from the fourth diagram is

$$\Gamma_4 = bt^2 \frac{1}{(4\pi)^{D/2}} \frac{(D-2)(D-4)}{2(D-3)} \int \frac{d^D p d^D q}{(2\pi)^{2D}} \phi(p)\phi(-p) \frac{F_4(p^2,q^2)}{(q^2+z^2)^2\{(q-p)^2+z^2\}^2}, \quad \text{(D.7)}$$

where

$$\begin{aligned} F_4(p^2,q^2) = &\frac{D-2}{D-1}\Big\{ -24(p^2)^4 - 44(p^2)^3 q^2 - 20(p^2)^2(q^2)^2 + \left(48(p^2)^3 + 44(p^2)^2 q^2\right)(p\cdot q) \\ &+ \left(\frac{48(p^2)^3}{q^2} + 64(p^2)^2 + 40p^2q^2\right)(p\cdot q)^2 + \left(-\frac{96(p^2)^2}{q^2} - 88p^2\right)(p\cdot q)^3 \\ &+ \left(-\frac{24(p^2)^2}{(q^2)^2} + \frac{4p^2}{q^2} - 20\right)(p\cdot q)^4 + \left(\frac{48p^2}{(q^2)^2} + \frac{44}{q^2}\right)(p\cdot q)^5 - \frac{24}{(q^2)^2}(p\cdot q)^6 \Big\} \end{aligned}$$

$$
\begin{aligned}
&+\zeta\Big\{-\frac{24}{D(D-1)}(p^2)^4-\frac{4(27D^2-59D+131)}{9D(D-1)}(p^2)^3q^2-\frac{20(19D^2-41D+49)}{27D(D-1)}(p^2)^2(q^2)^2\\
&\quad-\frac{31D-16}{9D}p^2(q^2)^3+\bigg(\frac{48(D^2-2D+2)}{D(D-1)}(p^2)^3+\frac{4(194D^2-398D+303)}{9D(D-1)}(p^2)^2q^2\\
&\qquad+\frac{52(16D-13)}{27D}p^2(q^2)^2+\frac{4(D-1)}{9D}(q^2)^3\bigg)(p\cdot q)\\
&\qquad+\bigg(-\frac{48(D-2)}{D-1}\frac{(p^2)^3}{q^2}-\frac{4(393D^2-761D+234)}{9D(D-1)}(p^2)^2\\
&\qquad-\frac{8(107D^2-205D+53)}{9D(D-1)}p^2q^2-\frac{47D-92}{27D}(q^2)^2\bigg)(p\cdot q)^2\\
&\quad+\left(\frac{96(D-2)}{D-1}\frac{(p^2)^2}{q^2}+\frac{8(122D^2-239D+18)}{9D(D-1)}p^2-\frac{4(7D+6)}{9D}q^2\right)(p\cdot q)^3\\
&\quad+\left(\frac{24(D-2)}{D-1}\frac{(p^2)^2}{(q^2)^2}-\frac{4(D-2)}{D-1}\frac{p^2}{q^2}+\frac{4(58D-103)}{9D(D-1)}\right)(p\cdot q)^4\\
&\quad+\left(-\frac{48(D-2)}{D-1}\frac{p^2}{(q^2)^2}-\frac{44(D-2)}{D-1}\frac{1}{q^2}\right)(p\cdot q)^5+\frac{24(D-2)}{D-1}\frac{(p\cdot q)^6}{(q^2)^2}\Big\}\Bigg]. \qquad \text{(D.8)}
\end{aligned}
$$

Finally, The contribution from the fifth diagram is

$$
\Gamma_5=(D-4)bt^2\frac{1}{(4\pi)^{D/2}}\frac{D-2}{2(D-3)}\int\frac{d^Dpd^Dq}{(2\pi)^{2D}}\phi(p)\phi(-p)\frac{F_5(p^2,q^2)}{(q^2+z^2)^2}, \qquad \text{(D.9)}
$$

where

$$
\begin{aligned}
F_5(p^2,q^2)&=\frac{3(D-2)(D+3)}{2(D-1)}(p^2)^2-\frac{(7D-43)(D-2)(D+1)}{36(D-1)}p^2q^2\\
&+\left(-\frac{3(D-2)(D+5)}{2(D-1)}\frac{p^2}{q^2}+\frac{(D-3)(D-2)(D+1)}{2(D-1)}\right)(p\cdot q)^2+\frac{3(D-2)}{D-1}\frac{(p\cdot q)^4}{(q^2)^2}\\
&\qquad+\zeta\Big\{\frac{3(D^2-D+4)}{2D(D-1)}(p^2)^2+\frac{17D^2-38D+39}{9D(D-1)}p^2q^2\\
&\quad+\left(\frac{3(D-2)(D+5)}{2(D-1)}\frac{p^2}{q^2}+\frac{D^2+17D-54}{18D(D-1)}\right)(p\cdot q)^2-\frac{3(D-2)}{D-1}\frac{(p\cdot q)^4}{(q^2)^2}\Big\}\Bigg]. \qquad \text{(D.10)}
\end{aligned}
$$

Appendix E

Interactions in momentum space

We here present the interactions in momentum space with care of symmetric properties of indices in what follows.

$$S^{bt}_{G[\phi h]} = -\frac{2}{3}bt\frac{\mu^{D/2-2}}{(4\pi)^{D/2}}\int\frac{d^Dk}{(2\pi)^D}k^2k^\alpha k^\beta\phi(k)h_{\alpha\beta}(-k),$$

$$S^{bt}_{G[\phi\phi h]} = \frac{\mu^{D/2-2}}{(4\pi)^{D/2}}bt\int\frac{d^Dpd^Dq}{(2\pi)^{2D}}\phi(p)\phi(q)h_{\alpha\beta}(-p-q)V^3_{\alpha\beta}(p,q),$$

where

$$\begin{aligned} V^3_{\alpha\beta}(p,q) = {} & (p^\alpha q^\beta + q^\alpha p^\beta - 2q^\alpha q^\beta)p^2 + (q^\alpha p^\beta + p^\alpha q^\beta - 2p^\alpha p^\beta)q^2 \\ & - \frac{2}{3}(p^\alpha p^\beta - 2p^\alpha q^\beta - 2q^\alpha p^\beta + q^\alpha q^\beta)(p\cdot q). \end{aligned} \tag{E.1}$$

$$S^{(D-4)bt}_{G[\phi\phi h]} = -(D-4)bt\frac{\mu^{\frac{D}{2}-2}}{(4\pi)^{D/2}}\int\frac{d^Dpd^Dq}{(2\pi)^{2D}}\phi(p)\phi(q)h_{\alpha\beta}(-p-q)T^3_{\alpha\beta}(p,q), \tag{E.2}$$

where

$$\begin{aligned} T^3_{\alpha\beta}(p,q) = {} & \frac{1}{6}(14q^\alpha q^\beta - 7p^\alpha q^\beta - 7q^\alpha p^\beta + 2p^\alpha p^\beta)p^2 + \frac{1}{6}(2q^\alpha q^\beta - 7p^\alpha q^\beta - 7q^\alpha p^\beta + 14p^\alpha p^\beta)q^2 \\ & + \frac{2}{9}(5q^\alpha q^\beta - 4p^\alpha q^\beta - 4q^\alpha p^\beta + 5p^\alpha p^\beta)(p\cdot q). \end{aligned} \tag{E.3}$$

$$S^{bt^2}_{G[\phi hh]} = bt^2\frac{1}{(4\pi)^{D/2}}\int\frac{d^Dpd^Dq}{(2\pi)^{2D}}h_{\alpha\beta}(p)h_{\gamma\delta}(q)\phi(-p-q)S^3_{\alpha\beta,\gamma\delta}(p,q), \tag{E.4}$$

$$
\begin{aligned}
S^3_{\alpha\beta,\gamma\delta}(p,q) = &\frac{2}{3}(\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma})(p\cdot q)^2 - \frac{1}{2}(\delta^{\alpha\gamma}p^\delta q^\beta + \delta^{\alpha\delta}p^\gamma q^\beta + \delta^{\beta\gamma}p^\delta q^\alpha + \delta^{\beta\delta}p^\gamma q^\alpha)(p\cdot q)\\
&+ p^\gamma p^\delta q^\alpha q^\beta - \frac{1}{2}(\delta^{\alpha\gamma}p^\beta q^\delta + \delta^{\alpha\delta}p^\beta q^\gamma + p^\alpha\delta^{\beta\gamma}q^\delta + p^\alpha\delta^{\beta\delta}q^\gamma)(p\cdot q)\\
&- \frac{1}{2}(p^\alpha p^\gamma q^\beta q^\delta + p^\alpha p^\delta q^\beta q^\gamma + p^\beta p^\gamma q^\alpha q^\delta + p^\beta p^\delta q^\alpha q^\gamma) - \frac{1}{2}(\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma})p^2q^2\\
&+ \frac{1}{2}(\delta^{\alpha\gamma}q^\beta q^\delta + \delta^{\alpha\delta}q^\beta q^\gamma + \delta^{\beta\gamma}q^\alpha q^\delta + \delta^{\beta\delta}q^\alpha q^\gamma)p^2\\
&+ \frac{1}{2}(\delta^{\alpha\gamma}p^\beta p^\delta + \delta^{\alpha\delta}p^\beta p^\gamma + \delta^{\beta\gamma}p^\alpha p^\delta + \delta^{\beta\delta}p^\alpha p^\gamma)q^2\\
&+ p^\alpha p^\beta q^\gamma q^\delta + \frac{1}{12}(\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma})(p^2+q^2)(p\cdot q)\\
&+ \frac{1}{12}(\delta^{\alpha\gamma}p^\beta q^\delta + \delta^{\alpha\delta}p^\beta q^\gamma + \delta^{\beta\gamma}p^\alpha q^\delta + \delta^{\beta\delta}p^\alpha q^\gamma)(p+q)^2\\
&+ \frac{1}{12}(\delta^{\alpha\gamma}(p^\beta p^\delta + q^\beta q^\delta) + \delta^{\alpha\delta}(p^\beta p^\gamma + q^\beta q^\gamma)\\
&+ \delta^{\beta\gamma}(p^\alpha p^\delta + q^\alpha q^\delta) + \delta^{\beta\delta}(p^\alpha p^\gamma + q^\alpha q^\gamma))(p+q)^2\\
&+ \frac{1}{3}p^\alpha p^\beta p^\gamma p^\delta + \frac{1}{3}q^\alpha q^\beta q^\gamma q^\delta + \frac{1}{6}p^\alpha p^\beta(p^\gamma q^\delta + p^\delta q^\gamma) + \frac{1}{6}(p^\alpha q^\beta + p^\beta q^\alpha)q^\gamma q^\delta. \quad \text{(E.5)}
\end{aligned}
$$

$$
S^{(D-4)}_{F[\phi hh]} = (D-4)\int \frac{d^Dp d^Dq}{(2\pi)^{2D}} h_{\alpha\beta}(p)h_{\gamma\delta}(q)\phi(-p-q)W^3_{\alpha\beta,\gamma\delta}(p,q),
$$

where

$$
\begin{aligned}
W^3_{\alpha\beta,\gamma\delta}(p,q) = &\frac{1}{2}(p\cdot q)^2(\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma}) - \frac{1}{2}(p\cdot q)(q^\alpha p^\gamma\delta^{\beta\delta} + q^\alpha p^\delta\delta^{\beta\gamma} + q^\beta p^\gamma\delta^{\alpha\delta} + q^\beta p^\delta\delta^{\alpha\gamma}) + q^\alpha q^\beta p^\gamma p^\delta\\
&- \frac{1}{2(D-2)}\Big\{p^2q^2(\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma}) - p^2(q^\alpha q^\gamma\delta^{\beta\delta} + q^\alpha q^\delta\delta^{\beta\gamma} + q^\beta q^\gamma\delta^{\alpha\delta} + q^\beta q^\delta\delta^{\alpha\gamma})\\
&- q^2(p^\alpha p^\gamma\delta^{\beta\delta} + p^\alpha p^\delta\delta^{\beta\gamma} + p^\beta p^\gamma\delta^{\alpha\delta} + p^\beta p^\delta\delta^{\alpha\gamma})\\
&+ (p\cdot q)(p^\alpha q^\gamma\delta^{\beta\delta} + p^\alpha q^\delta\delta^{\beta\gamma} + p^\beta q^\gamma\delta^{\alpha\delta} + p^\beta q^\delta\delta^{\alpha\gamma})\\
&+ p^\alpha p^\gamma q^\beta q^\delta + p^\alpha p^\delta q^\beta q^\gamma + p^\beta p^\gamma q^\alpha q^\delta + p^\beta p^\delta q^\alpha q^\gamma\Big\}\\
&+ \frac{2}{(D-1)(D-2)}p^\alpha p^\beta q^\gamma q^\delta. \quad \text{(E.6)}
\end{aligned}
$$

$$
S^{bt^2}_{G[\phi\phi hh]} = bt^2\frac{1}{(4\pi)^{D/2}}\int \frac{d^Dp d^Dq d^Dr d^Ds}{(2\pi)^{4D}}(2\pi)^D\delta^D(p+q+r+s)\phi(p)\phi(s)h_{\alpha\beta}(q)h_{\gamma\delta}(r)V^{4\alpha\beta\gamma\delta}(q,r,s),
$$

where

$$
\begin{aligned}
V^{4\alpha\beta\gamma\delta}(q,r,s) = {} & \frac{\delta^{\alpha\gamma}\,\delta^{\beta\delta}\;(q\cdot r)\;s^2}{6} + \frac{\delta^{\alpha\delta}\,\delta^{\beta\gamma}\;(q\cdot r)\;s^2}{6} + \frac{s^\alpha\,\delta^{\beta\gamma} s^d s^2}{4} + \frac{r^\alpha\,\delta^{\beta\gamma} s^\delta s^2}{8} + \frac{q^\alpha\,\delta^{\beta\gamma} s^\delta s^2}{8} + \frac{\delta^{\alpha\gamma} s^\beta s^\delta s^2}{4} \\
& + \frac{\delta^{\alpha\gamma} r^\beta s^\delta s^2}{8} + \frac{\delta^{\alpha\gamma} q^\beta s^\delta s^2}{8} + \frac{s^\alpha \delta^{\beta\gamma} r^\delta s^2}{8} + \frac{r^\alpha \delta^{\beta\gamma} r^\delta s^2}{6} + \frac{q^\alpha \delta^{\beta\gamma} r^\delta s^2}{6} + \frac{\delta^{\alpha\gamma} s^\beta r^\delta s^2}{8} \\
& + \frac{\delta^{\alpha\gamma} r^\beta r^\delta s^2}{6} + \frac{\delta^{\alpha\gamma} q^\beta r^\delta s^2}{6} + \frac{s^\alpha \delta^{\beta\gamma} q^\delta s^2}{8} + \frac{q^\alpha \delta^{\beta\gamma} q^\delta s^2}{6} + \frac{\delta^{\alpha\gamma} s^\beta q^\delta s^2}{8} + \frac{\delta^{\alpha\gamma} q^\beta q^\delta s^2}{6} \\
& + \frac{s^\alpha \delta^{\beta\delta} s^\gamma s^2}{4} + \frac{r^\alpha \delta^{\beta\delta} s^\gamma s^2}{8} + \frac{q^\alpha \delta^{\beta\delta} s^\gamma s^2}{8} + \frac{\delta^{\alpha\delta} s^\beta s^\gamma s^2}{4} + \frac{\delta^{\alpha\delta} r^\beta s^\gamma s^2}{8} + \frac{\delta^{\alpha\delta} q^\beta s^\gamma s^2}{8} \\
& + \frac{s^\alpha \delta^{\beta\delta} r^\gamma s^2}{8} + \frac{r^\alpha \delta^{\beta\delta} r^\gamma s^2}{6} + \frac{q^\alpha \delta^{\beta\delta} r^\gamma s^2}{6} + \frac{\delta^{\alpha\delta} s^\beta r^\gamma s^2}{8} + \frac{\delta^{\alpha\delta} r^\beta r^\gamma s^2}{6} + \frac{\delta^{\alpha\delta} q^\beta r^\gamma s^2}{6} \\
& + \frac{s^\alpha \delta^{\beta\delta} q^\gamma s^2}{8} + \frac{q^\alpha \delta^{\beta\delta} q^\gamma s^2}{6} + \frac{\delta^{\alpha\delta} s^\beta q^\gamma s^2}{8} + \frac{\delta^{\alpha\delta} q^\beta q^\gamma s^2}{6} - \frac{\delta^{\alpha\gamma} \delta^{\beta\delta} (q\cdot s)(r\cdot s)}{2} - \frac{\delta^{\alpha\delta} \delta^{\beta\gamma} (q\cdot s)(r\cdot s)}{2} \\
& - \frac{\delta^{\alpha\gamma} \delta^{\beta\delta} (q\cdot r)(r\cdot s)}{12} - \frac{\delta^{\alpha\delta} \delta^{\beta\gamma} (q\cdot r)(r\cdot s)}{12} - \frac{\delta^{\alpha\gamma} \delta^{\beta\delta} q^2 (r\cdot s)}{4} - \frac{\delta^{\alpha\delta} \delta^{\beta\gamma} q^2 (r\cdot s)}{4} - \frac{r^\alpha \delta^{\beta\gamma} s^\delta (r\cdot s)}{4} \\
& - \frac{q^\alpha \delta^{\beta\gamma} s^\delta (r\cdot s)}{4} - \frac{\delta^{\alpha\gamma} r^\beta s^\delta (r\cdot s)}{4} - \frac{\delta^{\alpha\gamma} q^\beta s^\delta (r\cdot s)}{4} - \frac{s^\alpha \delta^{\beta\gamma} r^\delta (r\cdot s)}{4} - \frac{r^\alpha \delta^{\beta\gamma} r^\delta (r\cdot s)}{12} \\
& - \frac{q^\alpha \delta^{\beta\gamma} r^\delta (r\cdot s)}{12} - \frac{\delta^{\alpha\gamma} s^\beta r^\delta (r\cdot s)}{4} - \frac{\delta^{\alpha\gamma} r^\beta r^\delta (r\cdot s)}{12} - \frac{\delta^{\alpha\gamma} q^\beta r^\delta (r\cdot s)}{12} + \frac{s^\alpha \delta^{\beta\gamma} q^\delta (r\cdot s)}{4} \\
& + \frac{q^\alpha \delta^{\beta\gamma} q^\delta (r\cdot s)}{6} + \frac{\delta^{\alpha\gamma} s^\beta q^\delta (r\cdot s)}{4} + \frac{\delta^{\alpha\gamma} q^\beta q^\delta (r\cdot s)}{6} - \frac{r^\alpha \delta^{\beta\delta} s^\gamma (r\cdot s)}{4} - \frac{q^\alpha \delta^{\beta\delta} s^\gamma (r\cdot s)}{4} \\
& - \frac{\delta^{\alpha\delta} r^\beta s^\gamma (r\cdot s)}{4} - \frac{\delta^{\alpha\delta} q^\beta s^\gamma (r\cdot s)}{4} - \frac{s^\alpha \delta^{\beta\delta} r^\gamma (r\cdot s)}{4} - \frac{r^\alpha \delta^{\beta\delta} r^\gamma (r\cdot s)}{12} - \frac{q^\alpha \delta^{\beta\delta} r^\gamma (r\cdot s)}{12} \\
& - \frac{\delta^{\alpha\delta} s^\beta r^\gamma (r\cdot s)}{4} - \frac{\delta^{\alpha\delta} r^\beta r^\gamma (r\cdot s)}{12} - \frac{\delta^{\alpha\delta} q^\beta r^\gamma (r\cdot s)}{12} + \frac{s^\alpha \delta^{\beta\delta} q^\gamma (r\cdot s)}{4} + \frac{q^\alpha \delta^{\beta\delta} q^\gamma (r\cdot s)}{6} \\
& + \frac{\delta^{\alpha\delta} s^\beta q^\gamma (r\cdot s)}{4} + \frac{\delta^{\alpha\delta}\, q^\beta q^\gamma (r\cdot s)}{6} - \frac{\delta^{\alpha\gamma} \delta^{\beta\delta} (q\cdot s) r^2}{4} - \frac{\delta^{\alpha\delta} \delta^{\beta\gamma} (q\cdot s) r^2}{4} + \frac{s^\alpha \delta^{\beta\gamma} s^\delta r^2}{4} \\
& + \frac{\delta^{\alpha\gamma} s^\beta s^\delta r^2}{4} + \frac{s^\alpha\, \delta^{\beta\gamma} q^\delta r^2}{4} + \frac{\delta^{\alpha\gamma} s^\beta q^\delta r^2}{4} + \frac{s^\alpha \delta^{\beta\delta} s^\gamma r^2}{4} + \frac{\delta^{\alpha\delta} s^\beta s^\gamma r^2}{4} + \frac{s^\alpha \delta^{\beta\delta} q^\gamma r^2}{4} \\
& + \frac{\delta^{\alpha\delta} s^\beta q^\gamma r^2}{4} - \frac{\delta^{\alpha\gamma} \delta^{\beta\delta} (q\cdot r)(q\cdot s)}{12} - \frac{\delta^{\alpha\delta}\, \delta^{\beta\gamma}\;(q\cdot r)\;(q\cdot s)}{12} + \frac{r^\alpha\, \delta^{\beta\gamma}\, s^\delta (q\cdot s)}{4} - \frac{q^\alpha\, \delta^{\beta\gamma}\, s^\delta (q\cdot s)}{4} \\
& + \frac{\delta^{\alpha\gamma} r^\beta s^\delta (q\cdot s)}{4} - \frac{\delta^{\alpha\gamma} q^\beta s^\delta (q\cdot s)}{4} - \frac{s^\alpha \delta^{\beta\gamma} r^\delta (q\cdot s)}{4} + \frac{r^\alpha \delta^{\beta\gamma} r^\delta (q\cdot s)}{6} - \frac{q^\alpha \delta^{\beta\gamma} r^\delta (q\cdot s)}{12} \\
& - \frac{\delta^{\alpha\gamma} s^\beta r^\delta (q\cdot s)}{4} + \frac{\delta^{\alpha\gamma} r^\beta r^\delta (q\cdot s)}{6} - \frac{\delta^{\alpha\gamma} q^\beta r^\delta (q\cdot s)}{12} - \frac{s^\alpha \delta^{\beta\gamma} q^\delta (q\cdot s)}{4} - \frac{q^\alpha \delta^{\beta\gamma} q^\delta (q\cdot s)}{12} \\
& - \frac{\delta^{\alpha\gamma} s^\beta q^\delta (q\cdot s)}{4} - \frac{\delta^{\alpha\gamma} q^\beta\, q^\delta (q\cdot s)}{12} + \frac{r^\alpha \delta^{\beta\delta} s^\gamma\;(q\cdot s)}{4} - \frac{q^\alpha\, \delta^{\beta\delta}\, s^\gamma\;(q\cdot s)}{4} + \frac{\delta^{\alpha\delta}\, r^\beta\, s^\gamma\;(q\cdot s)}{4} \\
& - \frac{\delta^{\alpha\delta}\, q^\beta\, s^\gamma\;(q\cdot s)}{4} - \frac{s^\alpha\, \delta^{\beta\delta}\, r^\gamma\;(q\cdot s)}{4} + \frac{r^\alpha\, \delta^{\beta\delta}\, r^\gamma\;(q\cdot s)}{6} - \frac{q^\alpha\, \delta^{\beta\delta}\, r^\gamma\;(q\cdot s)}{12} - \frac{\delta^{\alpha\delta}\, s^\beta\, r^\gamma\;(q\cdot s)}{4} \\
& + \frac{\delta^{\alpha\delta}\, r^\beta\, r^\gamma\;(q\cdot s)}{6} - \frac{\delta^{\alpha\delta}\, q^\beta\, r^\gamma\;(q\cdot s)}{12} - \frac{s^\alpha\, \delta^{\beta\delta}\, q^\gamma\;(q\cdot s)}{4} - \frac{q^\alpha\, \delta^{\beta\delta}\, q^\gamma\;(q\cdot s)}{12} - \frac{\delta^{\alpha\delta}\, s^\beta\, q^\gamma\;(q\cdot s)}{4}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{\delta^{\alpha\delta}\, q^\beta\, q^\gamma\, (q\cdot s)}{12} - \frac{q^\alpha\, \delta^{\beta\gamma}\, s^\delta\, (q\cdot r)}{4} - \frac{\delta^{\alpha\gamma}\, q^\beta\, s^\delta\, (q\cdot r)}{4} - \frac{s^\alpha\, \delta^{\beta\gamma}\, r^\delta\, (q\cdot r)}{4} - \frac{\delta^{\alpha\gamma}\, s^\beta\, r^\delta\, (q\cdot r)}{4} \\
&- \frac{q^\alpha\, \delta^{\beta\delta}\, s^\gamma\, (q\cdot r)}{4} - \frac{\delta^{\alpha\delta}\, q^\beta\, s^\gamma\, (q\cdot r)}{4} - \frac{s^\alpha\, \delta^{\beta\delta}\, r^\gamma\, (q\cdot r)}{4} - \frac{\delta^{\alpha\delta}\, s^\beta\, r^\gamma\, (q\cdot r)}{4} + \frac{s^\alpha\, \delta^{\beta\gamma}\, s^\delta\, q^2}{4} \\
&+ \frac{r^\alpha\, \delta^{\beta\gamma}\, s^\delta\, q^2}{4} + \frac{\delta^{\alpha\gamma}\, s^\beta\, s^\delta\, q^2}{4} + \frac{\delta^{\alpha\gamma}\, r^\beta\, s^\delta\, q^2}{4} + \frac{s^\alpha\, \delta^{\beta\delta}\, s^\gamma\, q^2}{4} + \frac{r^\alpha\, \delta^{\beta\delta}\, s^\gamma\, q^2}{4} + \frac{\delta^{\alpha\delta}\, s^\beta\, s^\gamma\, q^2}{4} + \frac{\delta^{\alpha\delta}\, r^\beta\, s^\gamma\, q^2}{4} \\
&+ 2\, s^\alpha\, s^\beta\, s^\gamma\, s^\delta + r^\alpha\, s^\beta\, s^\gamma\, s^\delta + q^\alpha\, s^\beta\, s^\gamma\, s^\delta + s^\alpha\, r^\beta\, s^\gamma\, s^\delta + 2\, r^\alpha\, r^\beta\, s^\gamma\, s^\delta + q^\alpha\, r^\beta\, s^\gamma\, s^\delta \\
&+ s^\alpha\, q^\beta\, s^\gamma\, s^\delta + r^\alpha\, q^\beta\, s^\gamma\, s^\delta + \frac{2\, q^\alpha\, q^\beta\, s^\gamma\, s^\delta}{3} + s^\alpha\, s^\beta\, r^\gamma\, s^\delta + \frac{q^\alpha\, s^\beta\, r^\gamma\, s^\delta}{2} + \frac{r^\alpha\, r^\beta\, r^\gamma\, s^\delta}{2} \\
&+ \frac{q^\alpha\, r^\beta\, r^\gamma\, s^\delta}{4} + \frac{s^\alpha\, q^\beta\, r^\gamma\, s^\delta}{2} + \frac{r^\alpha\, q^\beta\, r^\gamma\, s^\delta}{4} + \frac{q^\alpha\, q^\beta\, r^\gamma\, s^\delta}{3} + s^\alpha\, s^\beta\, q^\gamma\, s^\delta - \frac{r^\alpha\, s^\beta\, q^\gamma\, s^\delta}{2} \\
&- \frac{s^\alpha\, r^\beta\, q^\gamma\, s^\delta}{2} - \frac{q^\alpha\, r^\beta\, q^\gamma\, s^\delta}{4} - \frac{r^\alpha\, q^\beta\, q^\gamma\, s^\delta}{4} - \frac{q^\alpha\, q^\beta\, q^\gamma\, s^\delta}{6} + s^\alpha\, s^\beta\, s^\gamma\, r^\delta + \frac{q^\alpha\, s^\beta\, s^\gamma\, r^\delta}{2} \\
&+ \frac{r^\alpha\, r^\beta\, s^\gamma\, r^\delta}{2} + \frac{q^\alpha\, r^\beta\, s^\gamma\, r^\delta}{4} + \frac{s^\alpha\, q^\beta\, s^\gamma\, r^\delta}{2} + \frac{r^\alpha\, q^\beta\, s^\gamma\, r^\delta}{4} + \frac{q^\alpha\, q^\beta\, s^\gamma\, r^\delta}{3} + \frac{2\, s^\alpha\, s^\beta\, r^\gamma\, r^\delta}{3} \\
&- \frac{r^\alpha\, s^\beta\, r^\gamma\, r^\delta}{6} + \frac{q^\alpha\, s^\beta\, r^\gamma\, r^\delta}{3} - \frac{s^\alpha\, r^\beta\, r^\gamma\, r^\delta}{6} + \frac{s^\alpha\, q^\beta\, r^\gamma\, r^\delta}{3} + s^\alpha\, s^\beta\, q^\gamma\, r^\delta - \frac{r^\alpha\, s^\beta\, q^\gamma\, r^\delta}{4} \\
&+ \frac{q^\alpha\, s^\beta\, q^\gamma\, r^\delta}{4} - \frac{s^\alpha\, r^\beta\, q^\gamma\, r^\delta}{4} + \frac{s^\alpha\, q^\beta\, q^\gamma\, r^\delta}{4} + s^\alpha\, s^\beta\, s^\gamma\, q^\delta - \frac{r^\alpha\, s^\beta\, s^\gamma\, q^\delta}{2} - \frac{s^\alpha\, r^\beta\, s^\gamma\, q^\delta}{2} \\
&- \frac{q^\alpha\, r^\beta\, s^\gamma\, q^\delta}{4} - \frac{r^\alpha\, q^\beta\, s^\gamma\, q^\delta}{4} - \frac{q^\alpha\, q^\beta\, s^\gamma\, q^\delta}{6} + s^\alpha\, s^\beta\, r^\gamma\, q^\delta - \frac{r^\alpha\, s^\beta\, r^\gamma\, q^\delta}{4} + \frac{q^\alpha\, s^\beta\, r^\gamma\, q^\delta}{4} \\
&- \frac{s^\alpha\, r^\beta\, r^\gamma\, q^\delta}{4} + \frac{s^\alpha\, q^\beta\, r^\gamma\, q^\delta}{4} + 2\, s^\alpha\, s^\beta\, q^\gamma\, q^\delta + \frac{q^\alpha\, s^\beta\, q^\gamma\, q^\delta}{2} + \frac{s^\alpha\, q^\beta\, q^\gamma\, q^\delta}{2} \\
&+ \Bigg(\frac{s^\alpha\, \delta^{\beta\gamma}\, s^\delta}{4} + \frac{r^\alpha\, \delta^{\beta\gamma}\, s^\delta}{8} + \frac{q^\alpha\, \delta^{\beta\gamma}\, s^\delta}{8} + \frac{\delta^{\alpha\gamma}\, s^\beta\, s^\delta}{4} + \frac{\delta^{\alpha\gamma}\, r^\beta\, s^\delta}{8} \\
&+ \frac{\delta^{\alpha\gamma}\, q^\beta\, s^\delta}{8} + \frac{s^\alpha\, \delta^{\beta\gamma}\, r^\delta}{8} + \frac{\delta^{\alpha\gamma}\, s^\beta\, r^\delta}{8} + \frac{s^\alpha\, \delta^{\beta\gamma}\, q^\delta}{8} + \frac{\delta^{\alpha\gamma}\, s^\beta\, q^\delta}{8} \\
&+ \frac{s^\alpha\, \delta^{\beta\delta}\, s^\gamma}{4} + \frac{r^\alpha\, \delta^{\beta\delta}\, s^\gamma}{8} + \frac{q^\alpha\, \delta^{\beta\delta}\, s^\gamma}{8} + \frac{\delta^{\alpha\delta}\, s^\beta\, s^\gamma}{4} + \frac{\delta^{\alpha\delta}\, r^\beta\, s^\gamma}{8} \\
&+ \frac{\delta^{\alpha\delta}\, q^\beta\, s^\gamma}{8} + \frac{s^\alpha\, \delta^{\beta\delta}\, r^\gamma}{8} + \frac{\delta^{\alpha\delta}\, s^\beta\, r^\gamma}{8} + \frac{s^\alpha\, \delta^{\beta\delta}\, q^\gamma}{8} + \frac{\delta^{\alpha\delta}\, s^\beta\, q^\gamma}{8} \Bigg) (q+r+s)^2. \quad \text{(E.7)}
\end{aligned}
$$

$$
S^{(D-4)bt^2}_{G[\phi\phi hh]} = (D-4)bt^2 \frac{1}{(4\pi)^{D/2}} \int \frac{d^D p d^D q d^D r d^D s}{(2\pi)^{4D}} (2\pi)^D \delta^D(p+q+r+s) \phi(p)\phi(s) h_{\alpha\beta}(q) h_{\gamma\delta}(r) \times T^{4\alpha\beta\gamma\delta}(q,r,s), \quad \text{(E.8)}
$$

where

$$
\begin{aligned}
T^{4\alpha\beta\gamma\delta}(q,r,s) = {} & \frac{7\,\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,(q\cdot r)\,s^2}{36}+\frac{7\,\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,(q\cdot r)\,s^2}{36}+\frac{3\,s^\alpha\,\delta^{\beta\gamma}\,s^\delta\,s^2}{8}+\frac{3\,r^\alpha\,\delta^{\beta\gamma}\,s^\delta\,s^2}{16}+\frac{3\,q^\alpha\,\delta^{\beta\gamma}\,s^\delta\,s^2}{16} \\
& +\frac{3\,\delta^{\alpha\gamma}\,s^\beta\,s^\delta\,s^2}{8}+\frac{3\,\delta^{\alpha\gamma}\,r^\beta\,s^\delta\,s^2}{16}+\frac{3\,\delta^{\alpha\gamma}\,q^\beta\,s^\delta\,s^2}{16}+\frac{3\,s^\alpha\,\delta^{\beta\gamma}\,r^\delta\,s^2}{16}+\frac{7\,r^\alpha\,\delta^{\beta\gamma}\,r^\delta\,s^2}{36}+\frac{7\,q^\alpha\,\delta^{\beta\gamma}\,r^\delta\,s^2}{36} \\
& +\frac{3\,\delta^{\alpha\gamma}\,s^\beta\,r^\delta\,s^2}{16}+\frac{7\,\delta^{\alpha\gamma}\,r^\beta\,r^\delta\,s^2}{36}+\frac{7\,\delta^{\alpha\gamma}\,q^\beta\,r^\delta\,s^2}{36}+\frac{3\,s^\alpha\,\delta^{\beta\gamma}\,q^\delta\,s^2}{16}+\frac{7\,q^\alpha\,\delta^{\beta\gamma}\,q^\delta\,s^2}{36}+\frac{3\,\delta^{\alpha\gamma}\,s^\beta\,q^\delta\,s^2}{16} \\
& +\frac{7\,\delta^{\alpha\gamma}\,q^\beta\,q^\delta\,s^2}{36}+\frac{3\,s^\alpha\,\delta^{\beta\delta}\,s^\gamma\,s^2}{8}+\frac{3\,r^\alpha\,\delta^{\beta\delta}\,s^\gamma\,s^2}{16}+\frac{3\,q^\alpha\,\delta^{\beta\delta}\,s^\gamma\,s^2}{16}+\frac{3\,\delta^{\alpha\delta}\,s^\beta\,s^\gamma\,s^2}{8}+\frac{3\,\delta^{\alpha\delta}\,r^\beta\,s^\gamma\,s^2}{16} \\
& +\frac{3\,\delta^{\alpha\delta}\,q^\beta\,s^\gamma\,s^2}{16}+\frac{3\,s^\alpha\,\delta^{\beta\delta}\,r^\gamma\,s^2}{16}+\frac{7\,r^\alpha\,\delta^{\beta\delta}\,r^\gamma\,s^2}{36}+\frac{7\,q^\alpha\,\delta^{\beta\delta}\,r^\gamma\,s^2}{36}+\frac{3\,\delta^{\alpha\delta}\,s^\beta\,r^\gamma\,s^2}{16}+\frac{7\,\delta^{\alpha\delta}\,r^\beta\,r^\gamma\,s^2}{36} \\
& +\frac{7\,\delta^{\alpha\delta}\,q^\beta\,r^\gamma\,s^2}{36}+\frac{3\,s^\alpha\,\delta^{\beta\delta}\,q^\gamma\,s^2}{16}+\frac{7\,q^\alpha\,\delta^{\beta\delta}\,q^\gamma\,s^2}{36}+\frac{3\,\delta^{\alpha\delta}\,s^\beta\,q^\gamma\,s^2}{16}+\frac{7\,\delta^{\alpha\delta}\,q^\beta\,q^\gamma\,s^2}{36}-\frac{\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,(q\cdot s)\,(r\cdot s)}{2} \\
& -\frac{\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,(q\cdot s)\,(r\cdot s)}{2}-\frac{5\,\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,(q\cdot r)\,(r\cdot s)}{36}-\frac{5\,\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,(q\cdot r)\,(r\cdot s)}{36}-\frac{\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,q^2\,(r\cdot s)}{4}-\frac{\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,q^2\,(r\cdot s)}{4} \\
& -\frac{r^\alpha\,\delta^{\beta\gamma}\,s^\delta\,(r\cdot s)}{4}-\frac{q^\alpha\,\delta^{\beta\gamma}\,s^\delta\,(r\cdot s)}{4}-\frac{\delta^{\alpha\gamma}\,r^\beta\,s^\delta\,(r\cdot s)}{4}-\frac{\delta^{\alpha\gamma}\,q^\beta\,s^\delta\,(r\cdot s)}{4}-\frac{s^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(r\cdot s)}{4}-\frac{5\,r^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(r\cdot s)}{36} \\
& -\frac{5\,q^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(r\cdot s)}{36}-\frac{\delta^{\alpha\gamma}\,s^\beta\,r^\delta\,(r\cdot s)}{4}-\frac{5\,\delta^{\alpha\gamma}\,r^\beta\,r^\delta\,(r\cdot s)}{36}-\frac{5\,\delta^{\alpha\gamma}\,q^\beta\,r^\delta\,(r\cdot s)}{36}+\frac{s^\alpha\,\delta^{\beta\gamma}\,q^\delta\,(r\cdot s)}{4}+\frac{q^\alpha\,\delta^{\beta\gamma}\,q^\delta\,(r\cdot s)}{9} \\
& +\frac{\delta^{\alpha\gamma}\,s^\beta\,q^\delta\,(r\cdot s)}{4}+\frac{\delta^{\alpha\gamma}\,q^\beta\,q^\delta\,(r\cdot s)}{9}-\frac{r^\alpha\,\delta^{\beta\delta}\,s^\gamma\,(r\cdot s)}{4}-\frac{q^\alpha\,\delta^{\beta\delta}\,s^\gamma\,(r\cdot s)}{4}-\frac{\delta^{\alpha\delta}\,r^\beta\,s^\gamma\,(r\cdot s)}{4}-\frac{\delta^{\alpha\delta}\,q^\beta\,s^\gamma\,(r\cdot s)}{4} \\
& -\frac{s^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(r\cdot s)}{4}-\frac{5\,r^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(r\cdot s)}{36}-\frac{5\,q^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(r\cdot s)}{36}-\frac{\delta^{\alpha\delta}\,s^\beta\,r^\gamma\,(r\cdot s)}{4}-\frac{5\,\delta^{\alpha\delta}\,r^\beta\,r^\gamma\,(r\cdot s)}{36}-\frac{5\,\delta^{\alpha\delta}\,q^\beta\,r^\gamma\,(r\cdot s)}{36} \\
& +\frac{s^\alpha\,\delta^{\beta\delta}\,q^\gamma\,(r\cdot s)}{4}+\frac{q^\alpha\,\delta^{\beta\delta}\,q^\gamma\,(r\cdot s)}{9}+\frac{\delta^{\alpha\delta}\,s^\beta\,q^\gamma\,(r\cdot s)}{4}+\frac{\delta^{\alpha\delta}\,q^\beta\,q^\gamma\,(r\cdot s)}{9}-\frac{\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,(q\cdot s)\,r^2}{4}-\frac{\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,(q\cdot s)\,r^2}{4} \\
& -\frac{\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,q^2\,r^2}{4}-\frac{\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,q^2\,r^2}{4}+\frac{s^\alpha\,\delta^{\beta\gamma}\,s^\delta\,r^2}{4}+\frac{\delta^{\alpha\gamma}\,s^\beta\,s^\delta\,r^2}{4}+\frac{s^\alpha\,\delta^{\beta\gamma}\,q^\delta\,r^2}{4}+\frac{q^\alpha\,\delta^{\beta\gamma}\,q^\delta\,r^2}{4}+\frac{\delta^{\alpha\gamma}\,s^\beta\,q^\delta\,r^2}{4} \\
& +\frac{\delta^{\alpha\gamma}\,q^\beta\,q^\delta\,r^2}{4}+\frac{s^\alpha\,\delta^{\beta\delta}\,s^\gamma\,r^2}{4}+\frac{\delta^{\alpha\delta}\,s^\beta\,s^\gamma\,r^2}{4}+\frac{s^\alpha\,\delta^{\beta\delta}\,q^\gamma\,r^2}{4}+\frac{q^\alpha\,\delta^{\beta\delta}\,q^\gamma\,r^2}{4}+\frac{\delta^{\alpha\delta}\,s^\beta\,q^\gamma\,r^2}{4}+\frac{\delta^{\alpha\delta}\,q^\beta\,q^\gamma\,r^2}{4} \\
& -\frac{5\,\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,(q\cdot r)\,(q\cdot s)}{36}-\frac{5\,\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,(q\cdot r)\,(q\cdot s)}{36}+\frac{r^\alpha\,\delta^{\beta\gamma}\,s^\delta\,(q\cdot s)}{4}-\frac{q^\alpha\,\delta^{\beta\gamma}\,s^\delta\,(q\cdot s)}{4}+\frac{\delta^{\alpha\gamma}\,r^\beta\,s^\delta\,(q\cdot s)}{4} \\
& -\frac{\delta^{\alpha\gamma}\,q^\beta\,s^\delta\,(q\cdot s)}{4}-\frac{s^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(q\cdot s)}{4}+\frac{r^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(q\cdot s)}{9}-\frac{5\,q^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(q\cdot s)}{36}-\frac{\delta^{\alpha\gamma}\,s^\beta\,r^\delta\,(q\cdot s)}{4}+\frac{\delta^{\alpha\gamma}\,r^\beta\,r^\delta\,(q\cdot s)}{9} \\
& -\frac{5\,\delta^{\alpha\gamma}\,q^\beta\,r^\delta\,(q\cdot s)}{36}-\frac{s^\alpha\,\delta^{\beta\gamma}\,q^\delta\,(q\cdot s)}{4}-\frac{5\,q^\alpha\,\delta^{\beta\gamma}\,q^\delta\,(q\cdot s)}{36}-\frac{\delta^{\alpha\gamma}\,s^\beta\,q^\delta\,(q\cdot s)}{4}-\frac{5\,\delta^{\alpha\gamma}\,q^\beta\,q^\delta\,(q\cdot s)}{36}+\frac{r^\alpha\,\delta^{\beta\delta}\,s^\gamma\,(q\cdot s)}{4} \\
& -\frac{q^\alpha\,\delta^{\beta\delta}\,s^\gamma\,(q\cdot s)}{4}+\frac{\delta^{\alpha\delta}\,r^\beta\,s^\gamma\,(q\cdot s)}{4}-\frac{\delta^{\alpha\delta}\,q^\beta\,s^\gamma\,(q\cdot s)}{4}-\frac{s^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(q\cdot s)}{4}+\frac{r^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(q\cdot s)}{9}-\frac{5\,q^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(q\cdot s)}{36} \\
& -\frac{\delta^{\alpha\delta}\,s^\beta\,r^\gamma\,(q\cdot s)}{4}+\frac{\delta^{\alpha\delta}\,r^\beta\,r^\gamma\,(q\cdot s)}{9}-\frac{5\,\delta^{\alpha\delta}\,q^\beta\,r^\gamma\,(q\cdot s)}{36}-\frac{s^\alpha\,\delta^{\beta\delta}\,q^\gamma\,(q\cdot s)}{4}-\frac{5\,q^\alpha\,\delta^{\beta\delta}\,q^\gamma\,(q\cdot s)}{36}-\frac{\delta^{\alpha\delta}\,s^\beta\,q^\gamma\,(q\cdot s)}{4} \\
& -\frac{5\,\delta^{\alpha\delta}\,q^\beta\,q^\gamma\,(q\cdot s)}{36}+\frac{\delta^{\alpha\gamma}\,\delta^{\beta\delta}\,(q\cdot r)^2}{4}+\frac{\delta^{\alpha\delta}\,\delta^{\beta\gamma}\,(q\cdot r)^2}{4}-\frac{q^\alpha\,\delta^{\beta\gamma}\,s^\delta\,(q\cdot r)}{4}-\frac{\delta^{\alpha\gamma}\,q^\beta\,s^\delta\,(q\cdot r)}{4}-\frac{s^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(q\cdot r)}{4}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{q^\alpha\,\delta^{\beta\gamma}\,r^\delta\,(q\cdot r)}{4}-\frac{\delta^{\alpha\gamma}\,s^\beta\,r^\delta\,(q\cdot r)}{4}-\frac{\delta^{\alpha\gamma}\,q^\beta\,r^\delta\,(q\cdot r)}{4}-\frac{r^\alpha\,\delta^{\beta\gamma}\,q^\delta\,(q\cdot r)}{4}-\frac{\delta^{\alpha\gamma}\,r^\beta\,q^\delta\,(q\cdot r)}{4}-\frac{q^\alpha\,\delta^{\beta\delta}\,s^\gamma\,(q\cdot r)}{4}\\
&-\frac{\delta^{\alpha\delta}\,q^\beta\,s^\gamma\,(q\cdot r)}{4}-\frac{s^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(q\cdot r)}{4}-\frac{q^\alpha\,\delta^{\beta\delta}\,r^\gamma\,(q\cdot r)}{4}-\frac{\delta^{\alpha\delta}\,s^\beta\,r^\gamma\,(q\cdot r)}{4}-\frac{\delta^{\alpha\delta}\,q^\beta\,r^\gamma\,(q\cdot r)}{4}-\frac{r^\alpha\,\delta^{\beta\delta}\,q^\gamma\,(q\cdot r)}{4}\\
&-\frac{\delta^{\alpha\delta}\,r^\beta\,q^\gamma\,(q\cdot r)}{4}+\frac{s^\alpha\,\delta^{\beta\gamma}\,s^\delta\,q^2}{4}+\frac{r^\alpha\,\delta^{\beta\gamma}\,s^\delta\,q^2}{4}+\frac{\delta^{\alpha\gamma}\,s^\beta\,s^\delta\,q^2}{4}+\frac{\delta^{\alpha\gamma}\,r^\beta\,s^\delta\,q^2}{4}+\frac{r^\alpha\,\delta^{\beta\gamma}\,r^\delta\,q^2}{4}+\frac{\delta^{\alpha\gamma}\,r^\beta\,r^\delta\,q^2}{4}\\
&+\frac{s^\alpha\,\delta^{\beta\delta}\,s^\gamma\,q^2}{4}+\frac{r^\alpha\,\delta^{\beta\delta}\,s^\gamma\,q^2}{4}+\frac{\delta^{\alpha\delta}\,s^\beta\,s^\gamma\,q^2}{4}+\frac{\delta^{\alpha\delta}\,r^\beta\,s^\gamma\,q^2}{4}+\frac{r^\alpha\,\delta^{\beta\delta}\,r^\gamma\,q^2}{4}+\frac{\delta^{\alpha\delta}\,r^\beta\,r^\gamma\,q^2}{4}+3\,s^\alpha\,s^\beta\,s^\gamma\,s^\delta\\
&+\frac{3\,r^\alpha\,s^\beta\,s^\gamma\,s^\delta}{2}+\frac{3\,q^\alpha\,s^\beta\,s^\gamma\,s^\delta}{2}+\frac{3\,s^\alpha\,r^\beta\,s^\gamma\,s^\delta}{2}+\frac{5\,r^\alpha\,r^\beta\,s^\gamma\,s^\delta}{2}+\frac{5\,q^\alpha\,r^\beta\,s^\gamma\,s^\delta}{4}+\frac{3\,s^\alpha\,q^\beta\,s^\gamma\,s^\delta}{2}+\frac{5\,r^\alpha\,q^\beta\,s^\gamma\,s^\delta}{4}\\
&+\frac{7\,q^\alpha\,q^\beta\,s^\gamma\,s^\delta}{9}+\frac{3\,s^\alpha\,s^\beta\,r^\gamma\,s^\delta}{2}+\frac{r^\alpha\,s^\beta\,r^\gamma\,s^\delta}{4}+\frac{3\,q^\alpha\,s^\beta\,r^\gamma\,s^\delta}{4}+\frac{s^\alpha\,r^\beta\,r^\gamma\,s^\delta}{4}+\frac{3\,r^\alpha\,r^\beta\,r^\gamma\,s^\delta}{4}+\frac{3\,q^\alpha\,r^\beta\,r^\gamma\,s^\delta}{8}\\
&+\frac{3\,s^\alpha\,q^\beta\,r^\gamma\,s^\delta}{4}+\frac{3\,r^\alpha\,q^\beta\,r^\gamma\,s^\delta}{8}+\frac{7\,q^\alpha\,q^\beta\,r^\gamma\,s^\delta}{18}+\frac{3\,s^\alpha\,s^\beta\,q^\gamma\,s^\delta}{2}-\frac{r^\alpha\,s^\beta\,q^\gamma\,s^\delta}{2}+\frac{q^\alpha\,s^\beta\,q^\gamma\,s^\delta}{4}-\frac{s^\alpha\,r^\beta\,q^\gamma\,s^\delta}{2}\\
&-\frac{q^\alpha\,r^\beta\,q^\gamma\,s^\delta}{4}+\frac{s^\alpha\,q^\beta\,q^\gamma\,s^\delta}{4}-\frac{r^\alpha\,q^\beta\,q^\gamma\,s^\delta}{4}-\frac{5\,q^\alpha\,q^\beta\,q^\gamma\,s^\delta}{18}+\frac{3\,s^\alpha\,s^\beta\,s^\gamma\,r^\delta}{2}+\frac{r^\alpha\,s^\beta\,s^\gamma\,r^\delta}{4}+\frac{3\,q^\alpha\,s^\beta\,s^\gamma\,r^\delta}{4}\\
&+\frac{s^\alpha\,r^\beta\,s^\gamma\,r^\delta}{4}+\frac{3\,r^\alpha\,r^\beta\,s^\gamma\,r^\delta}{4}+\frac{3\,q^\alpha\,r^\beta\,s^\gamma\,r^\delta}{8}+\frac{3\,s^\alpha\,q^\beta\,s^\gamma\,r^\delta}{4}+\frac{3\,r^\alpha\,q^\beta\,s^\gamma\,r^\delta}{8}+\frac{7\,q^\alpha\,q^\beta\,s^\gamma\,r^\delta}{18}+\frac{7\,s^\alpha\,s^\beta\,r^\gamma\,r^\delta}{9}\\
&-\frac{5\,r^\alpha\,s^\beta\,r^\gamma\,r^\delta}{18}+\frac{7\,q^\alpha\,s^\beta\,r^\gamma\,r^\delta}{18}-\frac{5\,s^\alpha\,r^\beta\,r^\gamma\,r^\delta}{18}+\frac{7\,s^\alpha\,q^\beta\,r^\gamma\,r^\delta}{18}+\frac{q^\alpha\,q^\beta\,r^\gamma\,r^\delta}{2}+\frac{5\,s^\alpha\,s^\beta\,q^\gamma\,r^\delta}{4}-\frac{r^\alpha\,s^\beta\,q^\gamma\,r^\delta}{4}\\
&+\frac{3\,q^\alpha\,s^\beta\,q^\gamma\,r^\delta}{8}-\frac{s^\alpha\,r^\beta\,q^\gamma\,r^\delta}{4}-\frac{q^\alpha\,r^\beta\,q^\gamma\,r^\delta}{4}+\frac{3\,s^\alpha\,q^\beta\,q^\gamma\,r^\delta}{8}-\frac{r^\alpha\,q^\beta\,q^\gamma\,r^\delta}{4}+\frac{3\,s^\alpha\,s^\beta\,s^\gamma\,q^\delta}{2}-\frac{r^\alpha\,s^\beta\,s^\gamma\,q^\delta}{2}\\
&+\frac{q^\alpha\,s^\beta\,s^\gamma\,q^\delta}{4}-\frac{s^\alpha\,r^\beta\,s^\gamma\,q^\delta}{2}-\frac{q^\alpha\,r^\beta\,s^\gamma\,q^\delta}{4}+\frac{s^\alpha\,q^\beta\,s^\gamma\,q^\delta}{4}-\frac{r^\alpha\,q^\beta\,s^\gamma\,q^\delta}{4}-\frac{5\,q^\alpha\,q^\beta\,s^\gamma\,q^\delta}{18}+\frac{5\,s^\alpha\,s^\beta\,r^\gamma\,q^\delta}{4}\\
&-\frac{r^\alpha\,s^\beta\,r^\gamma\,q^\delta}{4}+\frac{3\,q^\alpha\,s^\beta\,r^\gamma\,q^\delta}{8}-\frac{s^\alpha\,r^\beta\,r^\gamma\,q^\delta}{4}-\frac{q^\alpha\,r^\beta\,r^\gamma\,q^\delta}{4}+\frac{3\,s^\alpha\,q^\beta\,r^\gamma\,q^\delta}{8}-\frac{r^\alpha\,q^\beta\,r^\gamma\,q^\delta}{4}+\frac{5\,s^\alpha\,s^\beta\,q^\gamma\,q^\delta}{2}\\
&\qquad\qquad+\frac{3\,q^\alpha\,s^\beta\,q^\gamma\,q^\delta}{4}+\frac{r^\alpha\,r^\beta\,q^\gamma\,q^\delta}{2}+\frac{3\,s^\alpha\,q^\beta\,q^\gamma\,q^\delta}{4}\\
&+\Bigg(\frac{3\,s^\alpha\,\delta^{\beta\gamma}\,s^\delta}{8}+\frac{3\,r^\alpha\,\delta^{\beta\gamma}\,s^\delta}{16}+\frac{3\,q^\alpha\,\delta^{\beta\gamma}\,s^\delta}{16}+\frac{3\,\delta^{\alpha\gamma}\,s^\beta\,s^\delta}{8}+\frac{3\,\delta^{\alpha\gamma}\,r^\beta\,s^\delta}{16}+\frac{3\,\delta^{\alpha\gamma}\,q^\beta\,s^\delta}{16}+\frac{3\,s^\alpha\,\delta^{\beta\gamma}\,r^\delta}{16}\\
&+\frac{3\,\delta^{\alpha\gamma}\,s^\beta\,r^\delta}{16}+\frac{3\,s^\alpha\,\delta^{\beta\gamma}\,q^\delta}{16}+\frac{3\,\delta^{\alpha\gamma}\,s^\beta\,q^\delta}{16}+\frac{3\,s^\alpha\,\delta^{\beta\delta}\,s^\gamma}{8}+\frac{3\,r^\alpha\,\delta^{\beta\delta}\,s^\gamma}{16}+\frac{3\,q^\alpha\,\delta^{\beta\delta}\,s^\gamma}{16}+\frac{3\,\delta^{\alpha\delta}\,s^\beta\,s^\gamma}{8}\\
&+\frac{3\,\delta^{\alpha\delta}\,r^\beta\,s^\gamma}{16}+\frac{3\,\delta^{\alpha\delta}\,q^\beta\,s^\gamma}{16}+\frac{3\,s^\alpha\,\delta^{\beta\delta}\,r^\gamma}{16}+\frac{3\,\delta^{\alpha\delta}\,s^\beta\,r^\gamma}{16}+\frac{3\,s^\alpha\,\delta^{\beta\delta}\,q^\gamma}{16}+\frac{3\,\delta^{\alpha\delta}\,s^\beta\,q^\gamma}{16}\Bigg)(q+r+s)^2.
\end{aligned}
\tag{E.9}
$$

Appendix F

Loop integrals

We encounter the momentum integral when we calculate Feynman diagrams. So, it is convenience to evaluate them previously. We will present their expressions here. For convenience, we use $\bar{D}$ defined as

$$\frac{2}{\bar{D}-4} = \frac{2}{D-4} + \gamma - \ln 4\pi \tag{F.1}$$

in what follows.

First, we evaluate the following integrals which emerge in one loop calculation of the anomalous dimension:

$$\begin{aligned} I^n_\alpha &= \int \frac{d^D k}{(2\pi)^D} \frac{(k^2)^\alpha (k\cdot\ell)^n}{(k^2+z^2)^2[(k-\ell)^2+z^2]^2} \\ &= \Gamma(4)\int_0^1 dx x(1-x) \int \frac{d^D p}{(2\pi)^D} \frac{(p+x\ell)^{2\alpha}(p\cdot\ell + x\ell^2)^n}{(p^2+K)^4}, \end{aligned} \tag{F.2}$$

where K is defined as

$$K = z^2 + x(1-x)\ell^2. \tag{F.3}$$

We also need the following integral:

$$\begin{aligned} J^n_\beta &= \int \frac{d^D k}{(2\pi)^D} \frac{(k\cdot\ell)^n}{(k^2+z^2)^2[(k-\ell)^2+z^2]^2 (k^2)^\beta} \\ &= \frac{\Gamma(\beta+4)}{\Gamma(\beta)} \int_0^1 dx x(1-x)^{\beta+1} \int_0^1 dy y(1-y)^{\beta-1} \int \frac{d^D p}{(2\pi)^D} \frac{(p\cdot\ell + x\ell^2)^n}{(p^2+L)^{\beta+4}}, \end{aligned} \tag{F.4}$$

where L is defined as

$$L = (x+y-xy)z^2 + x(1-x)\ell^2. \tag{F.5}$$

Here, we can not define $J^{(0)}_2$ because we can not take care the IR divergence in this integral using a cutoff z. However, it is no problem because that integral does not appear in our calculations.

In order to evaluate (F.2) by expanding the numerator, we need following integral:.

$$
\begin{aligned}
F_{n,m} &= (4\pi)^2\Gamma(4)\int \frac{d^Dp}{(2\pi)^D}\frac{(p^2)^n(p\cdot\ell)^{2m}}{(p^2+K)^4} \\
&= (\ell^2)^{\frac{D}{2}+n+2m-4}\bar{F}_{n,m},
\end{aligned} \tag{F.6}
$$

where $\bar{F}_{n,m}$ is expressed as

$$
\bar{F}_{n,m} = \frac{1}{(4\pi)^{\frac{D}{2}-2}}\frac{\Gamma\left(m+\frac{1}{2}\right)\Gamma\left(n+m+\frac{D}{2}\right)\Gamma\left(4-n-m-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(m+\frac{D}{2}\right)}\bar{K}^{\frac{D}{2}+n+m-4} \tag{F.7}
$$

and $\bar{K}$ is defined as

$$
\bar{K} = \frac{K}{\ell^2} = w^2 + x(1-x), \quad w^2 = \frac{z^2}{\ell^2}. \tag{F.8}
$$

Since $\bar{F}_{n,m}$ is simplified as

$$
\bar{F}_{n-k,k} = \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma\left(k+\frac{D}{2}\right)\Gamma\left(\frac{1}{2}\right)}\bar{F}_{n,0}, \tag{F.9}
$$

it is enough to calculate $\bar{F}_{n,0}$.

Next, in order to evaluate (F.4) by expanding the numerator, we need the following integral:

$$
\begin{aligned}
R_{m;\beta} &= (4\pi)^2\frac{\Gamma(\beta+4)}{\Gamma(\beta)}\int \frac{d^Dp}{(2\pi)^D}\frac{(p\cdot\ell)^{2m}}{(p^2+L)^{\beta+4}} \\
&= (\ell^2)^{\frac{D}{2}+2m-\beta-4}\bar{R}_{m;\beta},
\end{aligned} \tag{F.10}
$$

where $\bar{R}_{m;\beta}$ is defined as

$$
\bar{R}_{m;\beta} = \frac{1}{(4\pi)^{\frac{D}{2}-2}}\frac{\Gamma\left(m+\frac{1}{2}\right)\Gamma\left(\beta+4-m-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma(\beta)}\bar{L}^{\frac{D}{2}+m-\beta-4}, \tag{F.11}
$$

and $\bar{L}$ is defined as

$$
\bar{L} = \frac{L}{\ell^2} = (x+y-xy)w^2 + x(1-x). \tag{F.12}
$$

Then the integrals (F.2) and (F.4) are respectively expressed in a linear combinations of the following integrals:

$$
\begin{aligned}
\left[x^a\bar{F}_{n,m}\right] &= \int_0^1 dx x^{a+1}(1-x)\bar{F}_{n,m}, \\
\left[x^a\bar{R}_{m;\beta}\right] &= \int_0^1 dx x^{a+1}(1-x)^{\beta+1}\int_0^1 dy y(1-y)^{\beta-1}\bar{R}_{m;\beta}.
\end{aligned} \tag{F.13}
$$

We present the results of these integrals, which we need in our calculations in what follows. We first give the results including only UV divergences, which are

$$\begin{aligned}
&\big[\bar{F}_{4,0}\big] = -\frac{6}{7(\bar{D}-4)} + \frac{729}{980}, \quad \big[\bar{F}_{3,0}\big] = \frac{8}{5(\bar{D}-4)} - \frac{89}{75},\\
&\big[x\bar{F}_{3,0}\big] = \frac{4}{5(\bar{D}-4)} - \frac{89}{150}, \quad \big[x^2\bar{F}_{3,0}\big] = \frac{16}{35(\bar{D}-4)} - \frac{1276}{3675}, \quad \big[\bar{F}_{2,0}\big] = -\frac{2}{\bar{D}-4} + \frac{5}{6},\\
&\big[x\bar{F}_{2,0}\big] = -\frac{1}{\bar{D}-4} + \frac{5}{12}, \quad \big[x^2\bar{F}_{2,0}\big] = -\frac{3}{5(\bar{D}-4)} + \frac{27}{100}, \quad \big[x^3\bar{F}_{2,0}\big] = -\frac{2}{5(\bar{D}-4)} + \frac{59}{300},\\
&\big[x^4\bar{F}_{2,0}\big] = -\frac{2}{7(\bar{D}-4)} + \frac{449}{2940}, \quad \big[\bar{F}_{1,0}\big] = 2, \quad \big[x\bar{F}_{1,0}\big] = 1, \quad \big[x^2\bar{F}_{1,0}\big] = \frac{2}{3},\\
&\big[x^3\bar{F}_{1,0}\big] = \frac{1}{2}, \quad \big[x^4\bar{F}_{1,0}\big] = \frac{2}{5}, \quad \big[x^5\bar{F}_{1,0}\big] = \frac{1}{3}, \quad \big[x^6\bar{F}_{1,0}\big] = \frac{2}{7}.
\end{aligned} \tag{F.14}$$

Besides, we give the results including only IR divergence as follows:

$$\begin{aligned}
&\big[\bar{F}_{0,0}\big] = -2\ln w^2 - 2, \quad \big[x\bar{F}_{0,0}\big] = -\ln w^2 - 1, \quad \big[x^2\bar{F}_{0,0}\big] = -\ln w^2 - 2,\\
&\big[x^3\bar{F}_{0,0}\big] = -\ln w^2 - \frac{5}{2}, \quad \big[x^4\bar{F}_{0,0}\big] = -\ln w^2 - \frac{17}{6}, \quad \big[x^5\bar{F}_{0,0}\big] = -\ln w^2 - \frac{37}{12},\\
&\big[x^6\bar{F}_{0,0}\big] = -\ln w^2 - \frac{197}{60}, \quad \big[x^7\bar{F}_{0,0}\big] = -\ln w^2 - \frac{69}{20}, \quad \big[x^8\bar{F}_{0,0}\big] = -\ln w^2 - \frac{503}{140},
\end{aligned} \tag{F.15}$$

which are calculated by setting $D = 4$ and $w^2 \ll 1$.

Next, we give the results of integrals $[x^a\bar{R}_{m;\beta}]$. The results in the case of $\beta = 1$ are

$$\begin{aligned}
&\big[\bar{R}_{0;1}\big] = \frac{1}{w^2} - 2\ln w^2 - \frac{11}{2}, \quad \big[x\bar{R}_{0;1}\big] = -2\ln w^2 - \frac{5}{2}, \quad \big[x^2\bar{R}_{0;1}\big] = -\ln w^2 - \frac{3}{2},\\
&\big[x^3\bar{R}_{0;1}\big] = -\ln w^2 - \frac{5}{2}, \quad \big[x^4\bar{R}_{0;1}\big] = -\ln w^2 - 3, \quad \big[x^5\bar{R}_{0;1}\big] = -\ln w^2 - \frac{10}{3},\\
&\big[x^6\bar{R}_{0;1}\big] = -\ln w^2 - \frac{43}{12}, \quad \big[\bar{R}_{1;1}\big] = -\frac{1}{4}\ln w^2 - \frac{1}{8}, \quad \big[x\bar{R}_{1;1}\big] = \frac{1}{4}, \quad \big[x^2\bar{R}_{1;1}\big] = \frac{1}{8},\\
&\big[x^3\bar{R}_{1;1}\big] = \frac{1}{12}, \quad \big[x^4\bar{R}_{1;1}\big] = \frac{1}{16}, \quad \big[\bar{R}_{2;1}\big] = \frac{3}{16}, \quad \big[x\bar{R}_{2;1}\big] = \frac{1}{16}, \quad \big[x^2\bar{R}_{2;1}\big] = \frac{1}{32},\\
&\big[\bar{R}_{3;1}\big] = -\frac{5}{32(\bar{D}-4)} + \frac{25}{192}, \quad \big[x\bar{R}_{3;1}\big] = -\frac{1}{16(\bar{D}-4)} + \frac{47}{960}.
\end{aligned} \tag{F.16}$$

The results in the case of $\beta = 2$ are

$$\begin{aligned}
&\big[x\bar{R}_{0;2}\big] = \frac{1}{w^2} - 2\ln w^2 - \frac{41}{6}, \quad \big[x^2\bar{R}_{0;2}\big] = -2\ln w^2 - \frac{17}{6}, \quad \big[x^3\bar{R}_{0;2}\big] = -\ln w^2 - \frac{11}{6},\\
&\big[x^4\bar{R}_{0;2}\big] = -\ln w^2 - \frac{17}{6}, \quad \big[x^5\bar{R}_{0;2}\big] = -\ln w^2 - \frac{10}{3}, \quad \big[x^6\bar{R}_{0;2}\big] = -\ln w^2 - \frac{11}{3},\\
&\big[\bar{R}_{1;2}\big] = \frac{1}{4}\ln w^2 - \frac{2}{3}, \quad \big[x\bar{R}_{1;2}\big] = -\frac{1}{6}\ln w^2 - \frac{1}{9}, \quad \big[x^2\bar{R}_{1;2}\big] = \frac{1}{6}, \quad \big[x^3\bar{R}_{1;2}\big] = \frac{1}{12},\\
&\big[x^4\bar{R}_{1;2}\big] = \frac{1}{18}, \quad \big[\bar{R}_{2;2}\big] = -\frac{1}{8}\ln w^2 - \frac{7}{48}, \quad \big[x\bar{R}_{2;2}\big] = \frac{1}{16}, \quad \big[x^2\bar{R}_{2;2}\big] = \frac{1}{48},\\
&\big[\bar{R}_{3;2}\big] = \frac{5}{48}.
\end{aligned} \tag{F.17}$$

The integrals (F.2) and (F.4) are then summarized as follows:

$$
\begin{aligned}
I_0^{(0)} &= \frac{1}{(4\pi)^2}\frac{1}{\ell^4}(2\ln\ell^2 - 2\ln z^2 - 2), \quad I_1^{(0)} = \frac{1}{(4\pi)^2}\frac{1}{\ell^2}(\ln\ell^2 - \ln z^2),\\
I_2^{(0)} &= \frac{1}{(4\pi)^2}\left(-\frac{2}{\bar{D}-4} - \ln z^2\right), \quad I_3^{(0)} = \frac{1}{(4\pi)^2}\ell^2\left(-\frac{2}{\bar{D}-4} - \ln z^2\right),\\
I_4^{(0)} &= \frac{1}{(4\pi)^2}\ell^4\left(-\frac{2}{\bar{D}-4} - \ln z^2\right), \quad I_0^{(1)} = \frac{1}{(4\pi)^2}\frac{1}{\ell^2}(\ln\ell^2 - \ln z^2 - 1),\\
I_1^{(1)} &= \frac{1}{(4\pi)^2}(\ln\ell^2 - \ln z^2 - 1), \quad I_2^{(1)} = \frac{1}{(4\pi)^2}\ell^2\left(-\frac{2}{\bar{D}-4} - \ln z^2\right),\\
I_3^{(1)} &= \frac{1}{(4\pi)^2}\ell^4\left(-\frac{2}{\bar{D}-4} - \ln z^2\right), \quad I_0^{(2)} = \frac{1}{(4\pi)^2}\left(\ln\ell^2 - \ln z^2 - \frac{3}{2}\right),\\
I_1^{(2)} &= \frac{1}{(4\pi)^2}\ell^2\left(-\frac{1}{2(\bar{D}-4)} + \frac{3}{4}\ln\ell^2 - \ln z^2 - 1\right), \quad I_2^{(2)} = \frac{1}{(4\pi)^2}\ell^4\left(-\frac{2}{\bar{D}-4} - \ln z^2\right),\\
I_0^{(3)} &= \frac{1}{(4\pi)^2}\ell^2\left(\ln\ell^2 - \ln z^2 - \frac{7}{4}\right), \quad I_1^{(3)} = \frac{1}{(4\pi)^2}\ell^4\left(-\frac{1}{\bar{D}-4} + \frac{1}{2}\ln\ell^2 - \ln z^2 - \frac{3}{4}\right),\\
I_0^{(4)} &= \frac{1}{(4\pi)^2}\ell^4\left(-\frac{1}{4(\bar{D}-4)} + \frac{7}{8}\ln\ell^2 - \ln z^2 - \frac{13}{8}\right).
\end{aligned}
\tag{F.18}
$$

and

$$
\begin{aligned}
J_1^{(0)} &= \frac{1}{(4\pi)^2}\frac{1}{\ell^6}\left(2\ln\ell^2 - 2\ln z^2 - \frac{11}{2} + \frac{\ell^2}{z^2}\right), \quad J_1^{(1)} = \frac{1}{(4\pi)^2}\frac{1}{\ell^4}\left(2\ln\ell^2 - 2\ln z^2 - \frac{5}{2}\right),\\
J_1^{(2)} &= \frac{1}{(4\pi)^2}\frac{1}{\ell^2}\left(\frac{5}{4}\ln\ell^2 - \frac{5}{4}\ln z^2 - \frac{13}{8}\right), \quad J_1^{(3)} = \frac{1}{(4\pi)^2}\left(\ln\ell^2 - \ln z^2 - \frac{7}{4}\right),\\
J_1^{(4)} &= \frac{1}{(4\pi)^2}\ell^2\left(\ln\ell^2 - \ln z^2 - \frac{33}{16}\right), \quad J_1^{(5)} = \frac{1}{(4\pi)^2}\ell^4\left(\ln\ell^2 - \ln z^2 - \frac{35}{16}\right),\\
J_1^{(6)} &= \frac{1}{(4\pi)^2}\ell^6\left(-\frac{5}{32(\bar{D}-4)} + \frac{59}{64}\ln\ell^2 - \ln z^2 - \frac{131}{64}\right),\\
J_2^{(1)} &= \frac{1}{(4\pi)^2}\frac{1}{\ell^6}\left(2\ln\ell^2 - 2\ln z^2 - \frac{41}{6} + \frac{\ell^2}{z^2}\right), \quad J_2^{(2)} = \frac{1}{(4\pi)^2}\frac{1}{\ell^4}\left(2\ln\ell^2 - 2\ln z^2 - \frac{7}{2} + \frac{\ell^2}{4z^2}\right),\\
J_2^{(3)} &= \frac{1}{(4\pi)^2}\frac{1}{\ell^2}\left(\frac{3}{2}\ln\ell^2 - \frac{3}{2}\ln z^2 - \frac{13}{6}\right), \quad J_2^{(4)} = \frac{1}{(4\pi)^2}\left(\frac{9}{8}\ln\ell^2 - \frac{9}{8}\ln z^2 - \frac{95}{48}\right),\\
J_2^{(5)} &= \frac{1}{(4\pi)^2}\ell^2\left(\ln\ell^2 - \ln z^2 - \frac{35}{16}\right), \quad J_2^{(6)} = \frac{1}{(4\pi)^2}\ell^4\left(\ln\ell^2 - \ln z^2 - \frac{29}{12}\right).
\end{aligned}
\tag{F.19}
$$

Formulae for two loop calculations Next, we present expressions for two loop calculations of Feynman diagrams, which are used to evaluate the anomalous dimensions. We then encounter the integrals in the following form

$$\begin{aligned}\Lambda\left[(\ell^2)^{4-n-\alpha} I_\alpha^{(n)}\right] &= \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{(\ell^2+z^2)^4} (\ell^2)^{4-n-\alpha} I_\alpha^{(n)}, \\ \Lambda\left[(\ell^2)^{4-n+\beta} J_\beta^{(n)}\right] &= \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{(\ell^2+z^2)^4} (\ell^2)^{4-n+\beta} J_\beta^{(n)}.\end{aligned} \tag{F.20}$$

These integrals are respectively expressed in the linear combination of the two loop integrals defined as

$$L\left[x^a \bar{F}_{n,m}\right] = (4\pi)^2 \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{(\ell^2+z^2)^4} (\ell^2)^{\frac{D}{2}} \left[x^a \bar{F}_{n,m}\right], \tag{F.21}$$

$$L\left[x^a \bar{R}_{m;\beta}\right] = (4\pi)^2 \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{(\ell^2+z^2)^4} (\ell^2)^{\frac{D}{2}} \left[x^a \bar{R}_{m;\beta}\right]. \tag{F.22}$$

We first evaluate the integral (F.21). We can easily calculate it when $L[x^a \bar{F}_{n,m}]$ does not include IR divergences. The integral is simplified in this case as

$$\begin{aligned}L\left[x^a \bar{F}_{n,m}\right] &= \left[x^a \bar{F}_{n,m}\right] (4\pi)^2 \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{(\ell^2+z^2)^4} (\ell^2)^{\frac{D}{2}} \\ &= \left[x^a \bar{F}_{n,m}\right] \frac{1}{(4\pi)^{\frac{D}{2}-2}} \frac{\Gamma(D)\Gamma(4-D)}{\Gamma\left(\frac{D}{2}\right)\Gamma(4)} (z^2)^{D-4} \\ &= \left[x^a \bar{F}_{n,m}\right] \left(-\frac{1}{\bar{D}-4} - \frac{4}{3}\right) (z^2)^{D-4}.\end{aligned} \tag{F.23}$$

Using previous calculations, we obtain the results as follows,

$$\begin{aligned}
L\left[\bar{F}_{4,0}\right] &= (z^2)^{D-4}\left(\frac{6}{7(\bar{D}-4)^2}+\frac{391}{980(\bar{D}-4)}\right),\\
L\left[\bar{F}_{3,0}\right] &= (z^2)^{D-4}\left(-\frac{8}{5(\bar{D}-4)^2}-\frac{71}{75(\bar{D}-4)}\right),\\
L\left[x\bar{F}_{3,0}\right] &= (z^2)^{D-4}\left(-\frac{4}{5(\bar{D}-4)^2}-\frac{71}{150(\bar{D}-4)}\right),\\
L\left[x^2\bar{F}_{3,0}\right] &= (z^2)^{D-4}\left(-\frac{16}{35(\bar{D}-4)^2}-\frac{964}{3675(\bar{D}-4)}\right),\\
L\left[\bar{F}_{2,0}\right] &= (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{11}{6(\bar{D}-4)}\right),\\
L\left[x\bar{F}_{2,0}\right] &= (z^2)^{D-4}\left(\frac{1}{(\bar{D}-4)^2}+\frac{11}{12(\bar{D}-4)}\right),\\
L\left[x^2\bar{F}_{2,0}\right] &= (z^2)^{D-4}\left(\frac{3}{5(\bar{D}-4)^2}+\frac{53}{100(\bar{D}-4)}\right),\\
L\left[x^3\bar{F}_{2,0}\right] &= (z^2)^{D-4}\left(\frac{2}{5(\bar{D}-4)^2}+\frac{101}{300(\bar{D}-4)}\right),\\
L\left[x^4\bar{F}_{2,0}\right] &= (z^2)^{D-4}\left(\frac{2}{7(\bar{D}-4)^2}+\frac{671}{2940(\bar{D}-4)}\right),\quad L\left[\bar{F}_{1,0}\right] = -(z^2)^{D-4}\frac{2}{\bar{D}-4},\\
L\left[x\bar{F}_{1,0}\right] &= -(z^2)^{D-4}\frac{1}{\bar{D}-4},\quad L\left[x^2\bar{F}_{1,0}\right] = -(z^2)^{D-4}\frac{2}{3(\bar{D}-4)},\\
L\left[x^3\bar{F}_{1,0}\right] &= -(z^2)^{D-4}\frac{1}{2(\bar{D}-4)},\quad L\left[[x^4\bar{F}_{1,0}\right] = -(z^2)^{D-4}\frac{2}{5(\bar{D}-4)},\\
L\left[x^5\bar{F}_{1,0}\right] &= -(z^2)^{D-4}\frac{1}{3(\bar{D}-4)},\quad L\left[x^6\bar{F}_{1,0}\right] = -(z^2)^{D-4}\frac{2}{7(\bar{D}-4)}.
\end{aligned} \tag{F.24}$$

On the other hand, the calculations of the integral $L[x^a\bar{F}_{0,0}]$ which includes IR divergences are more complicated than that of the integral $L[x^a\bar{F}_{n,m}]$. $L[x^a\bar{F}_{0,0}]$ is now expressed as

$$L\left[x^a\bar{F}_{0,0}\right] = (4\pi)^{4-\frac{D}{2}}\Gamma\left(4-\frac{D}{2}\right)\int_0^1 dx x^{a+1}(1-x)\int\frac{d^D\ell}{(2\pi)^D}\frac{1}{[z^2+x(1-x)\ell^2]^{4-\frac{D}{2}}}\left(\frac{\ell^2}{\ell^2+z^2}\right)^4. \tag{F.25}$$

Here, we expand $\left(\frac{\ell^2}{\ell^2+z^2}\right)^4$ in a power series

$$\left(\frac{\ell^2}{\ell^2+z^2}\right)^4 = \sum_{s=0}^{4}(-1)^s{}_4C_s\left(\frac{z^2}{\ell^2+z^2}\right)^s. \tag{F.26}$$

We find that UV divergences become smaller as s increases in this expression. We pick up the UV divergent terms, while we eliminate the IR divergent terms and finite terms here. In the case of $s=0$,

$$\begin{aligned}
L\left[x^a\bar{F}_{0,0}\right]_{s=0} &= (4\pi)^{4-\frac{D}{2}}\Gamma\left(4-\frac{D}{2}\right)\int_0^1 dx x^{a+1}(1-x)\int\frac{d^D\ell}{(2\pi)^D}\frac{1}{\left[z^2+x(1-x)\ell^2\right]^{4-\frac{D}{2}}}\\
&= (4\pi)^{4-\frac{D}{2}}\Gamma\left(4-\frac{D}{2}\right)\int_0^1 dx x^{a+\frac{D}{2}-3}(1-x)^{\frac{D}{2}-3}\int\frac{d^D\ell}{(2\pi)^D}\frac{1}{\left[\ell^2+\frac{z^2}{x(1-x)}\right]^{4-\frac{D}{2}}}\\
&= (4\pi)^{4-D}\Gamma(4-D)(z^2)^{D-4}\int_0^1 dx x^{a-\frac{D}{2}+1}(1-x)^{-\frac{D}{2}+1}\\
&= (4\pi)^{4-D}(z^2)^{D-4}\Gamma(4-D)\frac{\Gamma\left(a-\frac{D}{2}+2\right)\Gamma\left(2-\frac{D}{2}\right)}{\Gamma\left(a+4-D\right)}.
\end{aligned}\tag{F.27}$$

This calculation leads to the following results:

$$\begin{aligned}
&L\left[\bar{F}_{0,0}\right]=(z^2)^{4-D}\frac{4}{(\bar{D}-4)^2},\quad L\left[x\bar{F}_{0,0}\right]=(z^2)^{4-D}\frac{2}{(\bar{D}-4)^2},\\
&L\left[x^2\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{1}{\bar{D}-4}\right),\quad L\left[x^3\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{3}{2(\bar{D}-4)}\right),\\
&L\left[x^4\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{11}{6(\bar{D}-4)}\right),\quad L\left[x^5\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{25}{12(\bar{D}-4)}\right),\\
&L\left[x^6\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{137}{60(\bar{D}-4)}\right),\quad L\left[x^7\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{49}{20(\bar{D}-4)}\right),\\
&L\left[x^8\bar{F}_{0,0}\right]=(z^2)^{4-D}\left(\frac{2}{(\bar{D}-4)^2}+\frac{363}{140(\bar{D}-4)}\right).
\end{aligned}\tag{F.28}$$

Also, in the case of $s=1$,

$$\begin{aligned}
L\left[x^a\bar{F}_{0,0}\right]_{s=1} &= (4\pi)^{4-\frac{D}{2}}\Gamma\left(4-\frac{D}{2}\right)\int_0^1 dx x^{a+1}(1-x)\int\frac{d^D\ell}{(2\pi)^D}\frac{-4}{\left[z^2+x(1-x)\ell^2\right]^{4-\frac{D}{2}}}\frac{z^2}{\ell^2+z^2}\\
&= -4(4\pi)^{4-\frac{D}{2}}\Gamma\left(5-\frac{D}{2}\right)z^2\int_0^1 dx x^{a+\frac{D}{2}-3}(1-x)^{\frac{D}{2}-3}\int_0^1 dy y^{3-\frac{D}{2}}\\
&\quad\times\int\frac{d^D\ell}{(2\pi)^D}\frac{1}{\left[\ell^2+\left(1-y+\frac{y}{x(1-x)}\right)z^2\right]^{5-\frac{D}{2}}}\\
&= -4(4\pi)^{4-D}\Gamma(5-D)(z^2)^{D-4}\int_0^1 dx x^{a+\frac{D}{2}-3}(1-x)^{\frac{D}{2}-3}\\
&\quad\times\int_0^1 dy y^{3-\frac{D}{2}}\left(1-y+\frac{y}{x(1-x)}\right)^{D-5}.
\end{aligned}\tag{F.29}$$

This parameter integral gives a finite value. Therefore, $L[x^a\bar{F}_{0,0}]$ has no divergences for $s\geq 1$.

Then, we will present the integral (F.22). This is expressed as

$$
\begin{aligned}
L\left[x^a \bar{R}_{m;\beta}\right]_{s=0} =& (4\pi)^{4-\frac{D}{2}} \frac{\Gamma\left(m+\frac{1}{2}\right)\Gamma\left(4+\beta-m-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\beta\right)} \\
&\times \int \frac{d^D\ell}{(2\pi)^D} \frac{(\ell^2)^{\frac{D}{2}}}{(\ell^2+z^2)^4} \int_0^1 dx x^{a+1}(1-x)^{\beta+1} \int_0^1 dy y(1-y)^{\beta-1} \\
&\times \left[(x+y-xy)w^2 + x(1-x)\right]^{\frac{D}{2}+m-\beta-4} \\
=& (4\pi)^{4-\frac{D}{2}} \frac{\Gamma\left(m+\frac{1}{2}\right)\Gamma\left(4+\beta-m-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\beta\right)} \int_0^1 dx x^{a+1}(1-x)^{\beta+1} \int_0^1 dy y(1-y)^{\beta-1} \\
&\times \int \frac{d^D\ell}{(2\pi)^D} \frac{(\ell^2)^{\beta-m}}{\left[(x+y-xy)z^2 + x(1-x)\ell^2\right]^{4+\beta-m-\frac{D}{2}}} \left(\frac{\ell^2}{\ell^2+z^2}\right)^4 . \qquad \text{(F.30)}
\end{aligned}
$$

The integrals $\bar{R}_{m;1}(m=2,3)$ and $\bar{R}_{3,2}$ have no IR divergences, and so we calculate these integrals at $w=0$ at these points. The calculation results are as follows:

$$
\begin{aligned}
L\left[\bar{R}_{2;1}\right] &= -(z^2)^{D-4}\frac{3}{16(\bar{D}-4)}, \quad L\left[x\bar{R}_{2;1}\right] = -(z^2)^{D-4}\frac{1}{16(\bar{D}-4)}, \\
L\left[x^2\bar{R}_{2;1}\right] &= -(z^2)^{D-4}\frac{1}{32(\bar{D}-4)}, \quad L\left[\bar{R}_{3;1}\right] = (z^2)^{D-4}\left(\frac{5}{32(\bar{D}-4)^2} + \frac{5}{64(\bar{D}-4)}\right), \\
L\left[x\bar{R}_{3;1}\right] &= (z^2)^{D-4}\left(\frac{1}{16(\bar{D}-4)^2} + \frac{11}{320(\bar{D}-4)}\right), \\
L\left[\bar{R}_{3;2}\right] &= -(z^2)^{D-4}\frac{5}{48(\bar{D}-4)}.
\end{aligned}
\qquad \text{(F.31)}
$$

Next, let us consider the calculation of integral including IR divergence. We can calculate this using method similar to the calculation of integral (F.21). If $s=0$ in the expansion in power series (F.26),

$$
\begin{aligned}
L\left[x^a \bar{R}_{m;\beta}\right]_{s=0} =& (4\pi)^{4-D}(z^2)^{D-4} \frac{\Gamma\left(m+\frac{1}{2}\right)\Gamma(4-D)\Gamma\left(\frac{D}{2}-m+\beta\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma(\beta)\Gamma\left(\frac{D}{2}\right)} \\
&\times \int_0^1 dx x^{a-\frac{D}{2}+m}(1-x)^{m-\frac{D}{2}+1} \int_0^1 dy y(1-y)^{\beta-1}(x+y-xy)^{D-4} \qquad \text{(F.32)}
\end{aligned}
$$

In case that $\beta=1$ and $m=0$,

$$
L\left[x^a \bar{R}_{0;1}\right]_{s=0} = (4\pi)^{4-D}(z^2)^{D-4}\frac{D}{2}\Gamma(4-D) \int_0^1 dx x^{a-\frac{D}{2}}(1-x)^{1-\frac{D}{2}} \int_0^1 dy y(x+y-xy)^{D-4}. \qquad \text{(F.33)}
$$

Changing variables $x \to 1-u$ and $y \to 1-v$ and integrating u and v using the expansion formula

$$
(1-uv)^{-b} = \sum_{r=0}^{\infty} \frac{\Gamma(b+r)}{\Gamma(b)} \frac{(uv)^r}{r!}, \qquad \text{(F.34)}
$$

this integral is expressed as

$$L\left[x^a\bar{R}_{0;1}\right]_{s=0} = (4\pi)^{4-D}(z^2)^{D-4}\frac{D}{2}\Gamma\left(a-\frac{D}{2}+1\right)\left[\frac{\Gamma(4-D)\Gamma\left(2-\frac{D}{2}\right)}{2\Gamma\left(a+3-D\right)} + \sum_{r=1}^{\infty}\frac{\Gamma(4-D+r)\Gamma\left(2-\frac{D}{2}+r\right)}{\Gamma(r+3)\Gamma(a+3-D+r)}\right]. \quad \text{(F.35)}$$

Also, the part of $s=1$ concerning UV divergences can be calculated in the same way as

$$L\left[x^a\bar{R}_{0;1}\right]_{s=1} = -4(4\pi)^{4-D}(z^2)^{D-4}\Gamma\left(2-\frac{D}{2}+a\right)\sum_{r=0}^{\infty}\frac{\Gamma\left(3-\frac{D}{2}+r\right)\Gamma(5-D+r)}{\Gamma(5-D+a+r)\Gamma(r+3)}. \quad \text{(F.36)}$$

The integral in the case of $s \geq 2$ does not contain UV divergence. Therefore, the integral $L[x^a\bar{R}_{0;1}]$ is finally expressed as

$$L\left[x^a\bar{R}_{0;1}\right] = (4\pi)^{4-D}(z^2)^{D-4}\left[\frac{D}{2}\Gamma\left(1-\frac{D}{2}+a\right)\left\{\frac{\Gamma(4-D)\Gamma\left(2-\frac{D}{2}\right)}{2\Gamma(3-D+a)} + \sum_{r=1}^{\infty}\frac{\Gamma(r)\Gamma(r)}{\Gamma(r+3)\Gamma(-1+a+r)}\right\} - 4\Gamma\left(2-\frac{D}{2}+a\right)\sum_{r=0}^{\infty}\frac{\Gamma(1+r)\Gamma(1+r)}{\Gamma(r+3)\Gamma(1+a+r)}\right], \quad \text{(F.37)}$$

where we take $D=4$ for the part that do not contribute to UV divergences. In the same way, integrals $L[x^a\bar{R}_{m;\beta}]$ in other cases are respectively

$$L\left[x^a\bar{R}_{1;1}\right] = (4\pi)^{4-D}(z^2)^{D-4}\frac{\Gamma\left(a+2-\frac{D}{2}\right)}{2}\left[\frac{\Gamma(4-D)\Gamma\left(3-\frac{D}{2}\right)}{2\Gamma(5-D+a)} + \sum_{r=1}^{\infty}\frac{\Gamma(r)\Gamma(r+1)}{\Gamma(r+3)\Gamma(a+r+1)}\right] \quad \text{(F.38)}$$

and

$$L\left[x^a\bar{R}_{0;2}\right] = (4\pi)^{4-D}(z^2)^{D-4}\left[\frac{D(D+2)}{4}\Gamma\left(-\frac{D}{2}+a\right)\left\{\frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma(4-D)}{6\Gamma(2-D+a)} + \sum_{r=1}^{\infty}\frac{\Gamma(r)^2\Gamma(r+2)}{\Gamma(-2+a+r)\Gamma(r+1)\Gamma(r+4)}\right\} - 2D\Gamma\left(1-\frac{D}{2}+a\right)\sum_{r=0}^{\infty}\frac{\Gamma(r+1)\Gamma(r+2)}{\Gamma(r+a)\Gamma(r+4)}\right], \quad \text{(F.39)}$$

$$L\left[x^a\bar{R}_{1;2}\right] = (4\pi)^{4-D}(z^2)^{D-4}\left[\frac{D}{4}\Gamma\left(a+1-\frac{D}{2}\right)\left\{\frac{\Gamma(4-D)\Gamma\left(3-\frac{D}{2}\right)}{6\Gamma(4-D+a)} + \sum_{r=1}^{\infty}\frac{\Gamma(r)\Gamma(r+2)}{\Gamma(r+a)\Gamma(r+4)}\right\} - 2\Gamma\left(a+2-\frac{D}{2}\right)\sum_{r=0}^{\infty}\frac{\Gamma(r+2)^2}{\Gamma(r+4)\Gamma(a+r+2)}\right], \quad \text{(F.40)}$$

$$L\left[x^a \bar{R}_{2;2}\right] = (4\pi)^{4-D}(z^2)^{D-4}\frac{3\Gamma\left(a+2-\frac{D}{2}\right)}{4}\left[\frac{\Gamma(4-D)\Gamma\left(4-\frac{D}{2}\right)}{6\Gamma(a+6-D)} + \sum_{r=1}^{\infty}\frac{\Gamma(r)\Gamma(r+2)^2}{\Gamma(r+1)\Gamma(r+4)\Gamma(a+2+r)}\right]. \quad \text{(F.41)}$$

The calculations are summarized as follows:

$$\begin{aligned}
L[\bar{R}_{0;1}] &= (z^2)^{D-4}\left(\frac{4}{(\bar{D}-4)^2}+\frac{12}{\bar{D}-4}\right), \quad L[x\bar{R}_{0;1}] = (z^2)^{D-4}\frac{4}{(\bar{D}-4)^2},\\
L[x^2\bar{R}_{0;1}] &= (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{1}{2(\bar{D}-4)}\right), \quad L[x^3\bar{R}_{0;1}] = (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{3}{2(\bar{D}-4)}\right),\\
L[x^4\bar{R}_{0;1}] &= (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{2}{\bar{D}-4}\right), \quad L[x^5\bar{R}_{0;1}] = (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{7}{3(\bar{D}-4)}\right),\\
L[x^6\bar{R}_{0;1}] &= (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{31}{12(\bar{D}-4)}\right),\\
L[\bar{R}_{1;1}] &= (z^2)^{D-4}\left(\frac{1}{2(\bar{D}-4)^2}-\frac{1}{4(\bar{D}-4)}\right), \quad L[x\bar{R}_{1;1}] = -(z^2)^{D-4}\frac{1}{4(\bar{D}-4)},\\
L[x^2\bar{R}_{1;1}] &= -(z^2)^{D-4}\frac{1}{8(\bar{D}-4)}, \quad L[x^3\bar{R}_{1;1}] = -(z^2)^{D-4}\frac{1}{12(\bar{D}-4)},\\
L[x^4\bar{R}_{1;1}] &= -(z^2)^{D-4}\frac{1}{16(\bar{D}-4)},\\
L[x\bar{R}_{0;2}] &= (z^2)^{D-4}\left(\frac{4}{(\bar{D}-4)^2}+\frac{14}{\bar{D}-4}\right), \quad L[x^2\bar{R}_{0;2}] = (z^2)^{D-4}\frac{4}{(\bar{D}-4)^2},\\
L[x^3\bar{R}_{0;2}] &= (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{5}{6(\bar{D}-4)}\right), \quad L[x^4\bar{R}_{0;2}] = (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{11}{6(\bar{D}-4)}\right),\\
L[x^5\bar{R}_{0;2}] &= (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{7}{3(\bar{D}-4)}\right), \quad L[x^6\bar{R}_{0;2}] = (z^2)^{D-4}\left(\frac{2}{(\bar{D}-4)^2}+\frac{8}{3(\bar{D}-4)}\right),\\
L[\bar{R}_{1;2}] &= (z^2)^{D-4}\frac{3}{\bar{D}-4}, \quad L[x\bar{R}_{1;2}] = (z^2)^{D-4}\left(\frac{1}{3(\bar{D}-4)^2}-\frac{7}{36(\bar{D}-4)}\right),\\
L[x^2\bar{R}_{1;2}] &= -(z^2)^{D-4}\frac{1}{6(\bar{D}-4)}, \quad L[x^3\bar{R}_{1;2}] = -(z^2)^{D-4}\frac{1}{12(\bar{D}-4)},\\
L[x^4\bar{R}_{1;2}] &= -(z^2)^{D-4}\frac{1}{18(\bar{D}-4)},\\
L[\bar{R}_{2;2}] &= (z^2)^{D-4}\left(\frac{1}{4(\bar{D}-4)^2}-\frac{1}{12(\bar{D}-4)}\right),\\
L[x\bar{R}_{2;2}] &= -(z^2)^{D-4}\frac{1}{16(\bar{D}-4)}, \quad L[x^2\bar{R}_{2;2}] = -(z^2)^{D-4}\frac{1}{48(\bar{D}-4)},\\
L[x^3\bar{R}_{2;2}] &= -(z^2)^{D-4}\frac{1}{96(\bar{D}-4)}.
\end{aligned} \quad \text{(F.42)}$$

Using the above integral formulae, we can calculate UV divergence of the two-loop Λ integral.

The explicit values of the Λ-integral for $I_\alpha^{(n)}$ are given as

$$
\begin{aligned}
\Lambda[\ell^8 I_0^{(0)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\frac{4}{(\bar{D}-4)^2}, \qquad \Lambda[\ell^6 I_1^{(0)}] = \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} - \frac{1}{(\bar{D}-4)}\right),\\
\Lambda[\ell^4 I_2^{(0)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{5}{3(\bar{D}-4)}\right), \qquad \Lambda[\ell^2 I_3^{(0)}] = \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{5}{3(\bar{D}-4)}\right),\\
\Lambda[I_4^{(0)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{5}{3(\bar{D}-4)}\right),\\
\Lambda[\ell^6 I_0^{(1)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\frac{2}{(\bar{D}-4)^2}, \qquad \Lambda[\ell^4 I_1^{(1)}] = \frac{(z^2)^{D-4}}{(4\pi)^4}\frac{2}{(\bar{D}-4)^2},\\
\Lambda[\ell^2 I_2^{(1)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{5}{3(\bar{D}-4)}\right), \qquad \Lambda[I_3^{(1)}] = \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{5}{3(\bar{D}-4)}\right),\\
\Lambda[\ell^4 I_0^{(2)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{1}{2(\bar{D}-4)}\right), \qquad \Lambda[\ell^2 I_1^{(2)}] = \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{5}{2(\bar{D}-4)^2} + \frac{2}{3(\bar{D}-4)}\right),\\
\Lambda[I_2^{(2)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{5}{3(\bar{D}-4)}\right),\\
\Lambda[\ell^2 I_0^{(3)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{3}{4(\bar{D}-4)}\right), \qquad \Lambda[I_1^{(3)}] = \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{3}{(\bar{D}-4)^2} + \frac{13}{12(\bar{D}-4)}\right),\\
\Lambda[I_0^{(4)}] &= \frac{(z^2)^{D-4}}{(4\pi)^4}\left(\frac{9}{4(\bar{D}-4)^2} + \frac{23}{24(\bar{D}-4)}\right).
\end{aligned}
\tag{F.43}
$$

The explicit values of the Λ-integrals for $J_\beta^{(n)}$ are given as

$$
\begin{aligned}
\Lambda[\ell^{10} J_1^{(0)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{12}{(D-4)}\right), \qquad \Lambda[\ell^8 J_1^{(1)}] = \frac{(z^2)^{(D-4)}}{(4\pi)^4}\frac{4}{(\bar{D}-4)^2},\\
\Lambda[\ell^6 J_1^{(2)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{5}{2(\bar{D}-4)^2} + \frac{1}{4(\bar{D}-4)}\right), \qquad \Lambda[\ell^4 J_1^{(3)}] = \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{3}{4(\bar{D}-4)}\right),\\
\Lambda[\ell^2 J_1^{(4)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{17}{16(\bar{D}-4)}\right), \qquad \Lambda[J_1^{(5)}] = \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{19}{16(\bar{D}-4)}\right),\\
\Lambda[\ell^{-2} J_1^{(6)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{69}{32(\bar{D}-4)^2} + \frac{241}{192(\bar{D}-4)}\right),\\
\Lambda[\ell^{10} J_2^{(1)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{14}{(\bar{D}-4)}\right), \qquad \Lambda[\ell^8 J_2^{(2)}] = \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{4}{(\bar{D}-4)^2} + \frac{3}{(\bar{D}-4)}\right),\\
\Lambda[\ell^6 J_2^{(3)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{3}{(\bar{D}-4)^2} + \frac{1}{4(\bar{D}-4)}\right), \qquad \Lambda[\ell^4 J_2^{(4)}] = \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{9}{4(\bar{D}-4)^2} + \frac{3}{4(\bar{D}-4)}\right),\\
\Lambda[\ell^2 J_2^{(5)}] &= \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{19}{16(\bar{D}-4)}\right), \qquad \Lambda[J_2^{(6)}] = \frac{(z^2)^{(D-4)}}{(4\pi)^4}\left(\frac{2}{(\bar{D}-4)^2} + \frac{17}{12(\bar{D}-4)}\right).
\end{aligned}
\tag{F.44}
$$

We should note that $\Lambda[\ell^{12} J_2^{(0)}]$ is not defined but it is not necessary in our calculations.

Appendix G

Evaluations of the sum of infinite series

We use the following infinite series to calculate the effective potential:

$$f(x,y) \equiv \sum_{n=3}^{\infty}\sum_{m=0}^{n}\frac{n!}{m!(n-m)!}\frac{(-1)^{n+m-1}}{2n(2n-m-1)(2n-m-2)}x^{2n-m}y^m.$$

The second differential of this function divided by x is easily evaluated as

$$\begin{aligned}
\frac{\partial^2}{\partial x^2}\left(\frac{1}{x}f(x,y)\right) &= \sum_{n=3}^{\infty}\sum_{m=0}^{n}\frac{n!}{m!(n-m)!}\frac{(-1)^{n+m-1}}{2n}x^{2n-m-3}y^m \\
&= \frac{1}{2x^3}\sum_{n=3}^{\infty}\frac{(-1)^{n-1}}{n}\sum_{m=0}^{n}\frac{n!}{m!(n-m)!}x^{2(n-m)}(-xy)^m \\
&= \frac{1}{2x^3}\sum_{n=3}^{\infty}\frac{(-1)^{n-1}}{n}(x^2-xy)^n \\
&= \frac{1}{2x^3}\left[\ln(1+x^2-xy)-x^2+xy+\frac{1}{2}(x^2-xy)^2\right]
\end{aligned}$$

Therefore, we can express $f(x,y)$ as

$$\begin{aligned}
f(x,y) &= x\int_0^x du\int_0^u dv\frac{\partial^2}{\partial v^2}\left(\frac{1}{v}f(v,y)\right) \\
&= \frac{3}{4}x^2+\frac{x^4}{24}+\frac{1}{4}(1-x^2)xy-\frac{3}{8}x^2y^2 \\
&\quad+\left\{\frac{x^2y^2}{8}-\frac{xy}{4}-\frac{x^2}{4}+\frac{1}{4}\right\}\ln(1-xy+x^2) \\
&\quad+\left(\frac{x^2y}{4}-\frac{x}{2}\right)\sqrt{4-y^2}\arctan\left(\frac{x\sqrt{4-y^2}}{2-xy}\right),
\end{aligned}$$

where we use

$$\arctan u \pm \arctan v = \arctan\left(\frac{u \pm v}{1 \mp uv}\right).$$

Next, we define the function $g(x)$ as

$$g(x) \equiv \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)(n+2)} x^{n+2}.$$

Its derivatives are evaluated as follows:

$$\frac{\partial^2 g}{\partial x^2} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n = \ln(1+x).$$

We can then obtain $g(x)$ by integrating the above function as

$$g(x) = \frac{(1+x)^2}{2} \ln(1+x) - \frac{3}{4} x(x+2). \tag{G.1}$$

Similarly, we can also derive the following function

$$h(x) \equiv \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)} x^n$$

as

$$h(x) = \frac{1+x}{x} \left[1 - \ln(1+x)\right]. \tag{G.2}$$

Bibliography

[1] R. Utiyama and B. DeWitt, J. Math. Phys. **3** (1962) 608.

[2] B. DeWitt, in Relativity, Groups and Topology, eds. B. DeWitt and C. DeWitt (Gordon and Breach, New York, 1964); Phys. Rev. **160** (1967) 1113; Phys. Rev. **162** (1967) 1195, 1239.

[3] S. Deser, Proceedings of the Coference on Gauge Theories and Modern Field Theories, edited by R. Arnowitt and P. Nath (MIT Press, Cambridge, 1975).

[4] S. Weinberg, Proceedings of the XVIIth International Conference on High Energy Physics, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, 1974).

[5] S. Weinberg, in General Relativity, an Einstein Centenary Survey, eds. S. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, 1979).

[6] G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincare **XX** (1974) 69; M. Veltman, *Methods in Field Theory*, Les Houches 1975 (North-Holland, Amsterdam, 1976).

[7] K. Stelle, Phys. Rev. **D16** (1977) 953; Gen. Rel. Grav. **9** (1978) 353.

[8] E. Tomboulis, Phys. Lett. **70B** (1977) 361; Phys. Lett. **97B** (1980) 77.

[9] E. Fradkin and A. Tseytlin, Nucl. Phys. **B201** (1982) 469.

[10] I. Avramidy and A. Bravinsky, Phys. Lett. **159B** (1985) 269.

[11] K. Hamada, Prog. Theor. Phys. **108** (2002) 399.

[12] K. Hamada, Found. Phys. **39** (2009) 1356.

[13] K. Hamada, Phys. Rev. D90 (2014) 084038.

[14] K. Hamada and M. Matsuda, Phys. Rev. D **93** (2016) 064051.

[15] R. Riegert, Phys. Lett. **134B** (1984) 56.

[16] I. Antoniadis and E. Mottola, Phys. Rev. D **45** (1992) 2013.

[17] I. Antoniadis, P. Mazur, and E. Mottola, Nucl. Phys. **B388** (1992) 627.

[18] I. Antoniadis, P. Mazur, and E. Mottola, Phys. Rev. D **55** (1997) 4770.

[19] K. Hamada and F. Sugino, Nucl. Phys. **B553** (1999) 283.

[20] K. Hamada and S. Horata, Prog. Theor. Phys. **110** (2003) 1169.

[21] K. Hamada, Int. J. Mod. Phys. **A20** (2005) 5353.

[22] K. Hamada, Phys. Rev. D **85** (2012) 024028; **85** (2012) 124036.

[23] K. Hamada, Phys. Rev. D **86** (2012) 124006.

[24] A. Polyakov, Phys. Lett. **103B** (1981) 207; Mod. Phys. Lett. **A2** (1987) 893.

[25] V. Knizhnik, A. Polyakov and A. Zamolodchikov, Mod. Phys. Lett. **A3** (1988) 819.

[26] J. Distler and H. Kawai, Nucl. Phys. **B321** (1989) 509.

[27] F. David, Mod. Phys. Lett. **A3** (1988) 1651.

[28] N. Seiberg, Prog. Theor. Phys. Suppl. **102** (1990) 319.

[29] J. Teschner, Class. Quant. Grav. **18** (2001) R153.

[30] J. Wess and B. Zumino, Phys. Lett. **37B** (1971) 95.

[31] L. Bonora, P. Cotta-Ramusino and C. Reina, Phys. Lett. **B126** (1983) 305.

[32] D. Capper and M. Duff, Nuovo Cimento **23A** (1974) 173.

[33] S. Deser, M. Duff and C. Isham, Nucl. Phys. **B111** (1976) 45.

[34] M. Duff, Nucl. Phys. **B125** (1977) 334; Class. Quant. Grav. **11** (1994) 1387.

[35] S. Deser and A. Schwimmer, Phys. Lett. **B309** (1993) 279.

[36] L. Brown and J. Collins, Ann. Phys. **130** (1980) 215.

[37] S. Hathrell, Ann. Phys. **139** (1982) 136.

[38] S. Hathrell, Ann. Phys. **142** (1982) 34.

[39] M. Freeman, Ann. Phys. **153** (1984) 339.

[40] K. Hamada, Phys. Rev. D **89** (2014) 104063.

[41] C. Bennett *et al.* (WMAP9 Collaboration), Astrophys. J. Suppl. Ser. **208** (2013) 20.

[42] P. Ade *et al.* (Planck Collaboration), Astron. Astrophys. http://dx.doi.org/10.1051/0004-6361/201321591 (2014).

[43] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[44] K. Hamada and M. Matsuda, in preparation.

[45] L. Abbott, Nucl. Phys. **B185** (1981) 189.

[46] M. Gell-Mann and F. Low, Phys. Rev. **95** (1954) 1300.

[47] K. Symanzik, Comm. Math. Phys. **18** (1970) 227.

[48] C. Callan, Jr., Phys. Rev. **D2** (1970) 1541.

[49] G. 't Hooft, Nucl. Phys. **B61** (1973) 455.

[50] S. Weinberg, Phys. Rev. **D8** (1973) 3497.

[51] K. Hamada and T. Yukawa, Mod. Phys. Lett. **A20** (2005) 509.

[52] K. Hamada, S. Horata, and T. Yukawa, Phys. Rev. **D74** (2006) 123502.

[53] K. Hamada, S. Horata, and T. Yukawa, Phys. Rev. **D81** (2010) 083533.

[54] N. Nielsen, Nucl. Phys. **B101** (1975) 173.

[55] R. Fukuda and T. Kugo, Phys. Rev. **D13** (1976) 3469.

[56] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66** (1979) 1.