Self-organization of reference structure and its effect on decision accuracy

伊藤　真利子

博士（理学）

総合研究大学院大学
先導科学研究科
生命共生体進化学専攻

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Mariko Ito

Doctor of Philosophy

Department of Evolutionary Studies of Biosystems
School of Advanced Sciences
SOKENDAI (The Graduate University for Advanced Studies)

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Abstract

It has been revealed by empirical and theoretical works that humans incorporate others' opinions (social information) when they make their decision in many circumstances. Such use of social information can yield an advantage of *collective intelligence*. For example, the majority-rule voting based on independent opinions for a binary choice can result in higher accuracy than when decided by a single individual or expert. However, it has also been shown that the correlation between opinions can undermine collective intelligence. In addition, sequential decision-making, in which each individual makes decision using earlier opinions by other individuals, is known to sometimes lead to situations in which most individuals fail to give correct answers (*incorrect information cascade*). These facts suggest that, although the use of social information would be advantageous to individuals' decision making, once some individuals start using it and their correlated opinions become a part of social information, social information could progressively lose its independency and quality so that no one eventually dares to use it. To my knowledge, the reference structure among people has been given artificially in most of existing experimental and theoretical works for collective intelligence and decision accuracy of humans. However, some studies showed that whom to follow in the reference structure affects the decision accuracy of individuals. Therefore I ask how the reference structure self-organizes, when each individual tries to use social information to secure the accuracy of his/her decision-making. I also evaluate the decision accuracy in the self-organized reference structure.

I try to answer these questions theoretically. To model the reference structure between individuals, I consider a directed network in which each node represents an individual and each directed link represents reference. Individuals are assumed to make a decision sequentially on a given problem with the majority-rule voting among its own and his or her neighbors' opinions. Since each agent makes decision with majority vote, his or her probability to find a correct answer by oneself, which I call his/her "ability", is different from his or her actual

probability of finding a correct answer by referring to others, which I call "performance". I also assume that individuals vary in their ability. It should be natural to assume that each individual assesses the credibility of the referents and decides to either keep or stop following them accordingly. Thus, I assumed the rewiring rule as follows; each individual monitors his or her neighbors' performance and breaks the link if the neighbor's performance becomes worse than a preset threshold. I therefore consider the mutually affecting changes of reference link structure and each agent's opinion accuracy. Through this interaction the network structure is self-organized. This idea is related to *adaptive network models*, in which feedback loops between node dynamics and network topology are considered. I conducted extensive computer simulations on this adaptive network model. I also developed an analytical theory to explain the results obtained in the simulations.

My analysis shows the following results. (A) The distribution of the number of followers in the self-organized network significantly differed from the initial Poisson distribution for the random network. In fact, the distribution of the number of followers in the self-organized network was close to exponential distribution. This suggested that there were a few nodes that had much larger number of followers than the mean. (B) The mean number of followers increased approximately exponentially, i.e., more than linearly, with agents' ability. Therefore small difference in ability can lead to large difference in the number of followers in the self-organized network. (C) The mean performance of an agent increased linearly with his/her own ability. I defined group performance as the proportion of agents who stated correct answers in the population. The mean performance of each agent and the mean group performance was the lowest when the agent made decisions independently of others, which is improved by collective intelligence when agents can refer to randomly assigned referents in the initial random network and further improved by adaptive rewiring in the self-organized network. The group performance temporally fluctuates by stochasticity in the self-organized network. The temporal standard deviation (SD) of group performance also increased in the same order as the mean group performance, i.e., the group performance temporally fluctuated more when the mean group performance became higher. (D) The threshold for rewiring affected the strength of heterogeneity in the number of followers in the self-organized network. When I set the threshold lower, the heterogeneity in the number of followers became larger. At the same time, the dependence of an agent's mean number of followers on his/her ability was more exaggerated, i.e., agents refer more to higher ability agents in the self-organized network. This leads to a higher mean performance of each agent compared with when the threshold was larger. However, the SD of the group performance, i.e., the fluctuation of the group performance, was also higher for a lower threshold.

To understand the source of centralization of reference links, I decomposed the causal relationship between mean number of followers of each agent and his/her ability into three components: the relationship between the ability and the mean performance, the relationship between the mean performance and the mean duration of keeping a follower, and the relationship between the mean duration of keeping a follower and the mean number of followers. I explained analytically the simulation result of each relationship of these three, by using a theory of stochastic process and some approximation methods related to the network structure. Among these three relationships, only the relationship between the mean performance and the mean duration of keeping a follower is nonlinear and the other relationships are linear. Therefore, I conclude that the nonlinear dependence of the number of followers on agent's ability originates from the non-linear dependence of the mean duration of keeping a follower on the mean performance of that agent. This relationship between the mean performance and the mean duration corresponds to the performance-monitoring process assumed in my model.

To sum up, in the self-organized reference structure, I observed the strong centralization of reference in which the number of one's followers increases more than linearly with his/her ability. The mean performance of each agent was higher compared with a random network or the case of independent decision-making. However, the group performance fluctuated more in the self-organized network. There was a counter-intuitive relationship between the degree of generosity to referents in a society and the mean performance of the society. When I set the rewiring threshold lower (i.e. when individuals are more generous to their referents), individuals refer to higher ability agents in the self-organized reference structure than when I set it higher, leading to the higher mean performance of the society. To my knowledge, there is no study on the decision accuracy in groups showing such a counter-intuitive phenomenon. This result would be testable by empirical studies. In my study, I also found a trade-off between accuracy and stability in the self-organized network. The higher mean performance and more stability (suppression of fluctuation) of performance are incompatible. This trade-off was observed when I compare the performance in the case of independent decision, random references and the high-ability-agent-oriented self-organized networks. It was also observed when I compare the performance in a high-threshold case with that in a low-threshold one.

As future perspectives, I suggest that the following two points are important to be considered in the study of the self-organization of humans' reference structure. Firstly, it may be possible to consider the situation in which humans choose not only reference partners but also the extent to which they depend on social information. As I showed in my study, there is a trade-off between accuracy and stability. If individuals depend more on social information, they may be able to improve their performance on average, but they may be involved in information cascades more frequently. Secondly, it may also be possible to consider that humans conform to others not only to make their decision more accurately but also to correspond to others' expectation (*normative social influence*). Reflecting these features of humans' decision-making in the model of self-organization of reference structure should lead to further understanding of humans' collective decision making and its accuracy.

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Contents

Chapter 1

Introduction

1.1 General Introduction

Interdependence of individuals' opinions in human society

In human society, interdependence of individuals' opinions is inevitable in many circumstances. This is because a person is often influenced by others' opinions in one's decision-making as I will introduce in the following.

Firstly, changes in opinions by humans according to others' opinions have been actually observed by experiments (Bahrami et al., 2010; Mahmoodi et al., 2015; Lorenz et al., 2011; Clément et al., 2015; Kurvers et al., 2015; Mori et al., 2012; Deutsch and Gerard, 1955) and empirical data analyses (Booth et al., 2014; Clement and Tse, 2005; Ramnath et al., 2008) in ecology, sociology, psychology, economics, accounting and neuroscience. In experiments, participants answered various types of tasks, such as a perceptual problem, quantity estimations, knowledge quiz and so on. As for the empirical data, a good example is the data of reports made by financial analysts. Financial analysts are experts who provide reports such as forecasts of a firm's forthcoming for investors. They revise their personal earnings forecasts at their own chosen time during the fiscal period. Their reports have been analyzed by researchers in economics and accounting, and have improved our understanding of how analysts revise their opinions. In these studies, opinions of people before and after the interaction with others' opinions were examined, and researchers concluded that many people changed their opinions by purely imitating or taking into account others' opinions. In addition, some other persons changed their opinions intentionally to avoid others', called contrarian behavior (Galam, 2004; Ramnath et al., 2008). The changes in opinion occurred not only after verbal communications with others (Bahrami et al., 2010; Mahmoodi et al., 2015) but also after observations of others' opinions (Lorenz et al., 2011; Clément et al., 2015; Kurvers et al., 2015; Mori et al., 2012; Asch, 1956; Deutsch and Gerard, 1955; Booth et al., 2014; Clement and Tse, 2005; Ramnath et al., 2008). On the observations of others' opinions, people had chances to receive a various type of information in the experiments or the empirical data. For example, people received a set of economic forecasts reported by each analyst (Ramnath et al., 2008; Booth et al., 2014), a distribution or a list of options chosen by other people (Lorenz et al., 2011; Clément et al., 2015; Kurvers et al., 2015; Mori et al., 2012), and the average of choices by other people (Lorenz et al., 2011; Ramnath et al., 2008; Clement and Tse, 2005).

Secondly, the influence of others' opinions on one's decision-making is detected not only by direct observation of opinion changes. If one frequently states one's opinions soon after a particular person makes a statement, or if one frequently states the same opinion as that of a particular person, it is feasible that he/she is influenced by, or "follows", that person. Actually this idea has been used to estimate the underlying influential relationship among people from actual time-series data of who-said-what, in news articles, blog posts, economic forecasts and so on (Cooper et al., 2001; Gomez Rodriguez et al., 2010). On the other hand, citations in academic papers are examples that clearly show us who influenced whom through those papers (Newman, 2003; Ke et al., 2015). Such influence relationship is often represented by a network, in many disciplines such as sociology and physics (Newman, 2003; Albert and Barabási, 2002; Castellano et al., 2009). In many cases, a node in the network represents an individual who makes one's decision, and a link between nodes represents influence relationship such as reference, following, persuasion, and friendship between them. For example, Gomez Rodriguez et al. (2010), which I cited above, suggested the way of reconstructing a network to show the underlying influence relationship among people from the time-series data of who-said-what.

The literature of humans' opinion formation has discussed who or what strongly affects people's decision-making. "Opinion leaders" (or leaders) are sometimes defined as people who have strong influence on others' decision-making by their outstanding knowledge or by a large number of their connections to others. These leaders are present, for example, in communities of experts, communities of people with the same interest and social networking sites (Ramnath et al., 2008; Cooper et al., 2001; Zhou et al., 2011; Kaiser et al., 2013; Watts and Dodds, 2007; King et al., 2009). It is also known that humans tend to follow opinions that are supported by relatively high proportion or relatively large number of people, e.g. more than a half or a certain quorum of all members in the group (Asch, 1956; Granovetter, 1978; Deutsch and Gerard, 1955; Mori et al., 2012; Eguíluz et al., 2015; Bikhchandani et al., 1992, 1998; Clément et al., 2015; Izuma, 2013; Raafat et al., 2009). This phenomenon is called *conformity*. Interestingly, conformity to people who suggest an obviously wrong opinion can occur, when the number of such people is enough (Asch, 1956; Deutsch and Gerard, 1955; Raafat et al., 2009). Granovetter (1978) suggested, by showing some empirical examples such as spreading of rumors and decision-making about migration, that each person has one's threshold for the number of other people who have already adopted a certain option, at which he/she also adopts the option. This model is called a threshold model. The threshold model suggests that some people adopt a certain choice quickly after observing a few people supported it, while other people wait for the supports by more people before they choose it. The later people can be regarded as those with high resistance to the peer pressure.

Furthermore, in some experiments, we can also observe independent decision-makers who make their decision independently even after they observed others' opinions or even if they can wait for receiving others' opinions (Madirolas and de Polavieja, 2015; Kurvers et al., 2015; Mori et al., 2012). The studies also showed that people do not consistently make decision independently from others (Kurvers et al., 2015; Mori et al., 2012). For example, a participant makes one's decision by oneself when he/she knows the correct answer of a given question, and follows others when he/she is not sure about it (Mori et al., 2012). There are various models that explain how a person aggregates one's own opinion and others' opinions or aggregations of others' opinions, in the fields of ecology, computer science, neuroscience, economics, psychology and physics (Mori et al., 2012; Madirolas and de Polavieja, 2015; Eguíluz et al., 2015; Bahrami et al., 2010; Mahmoodi et al., 2015; Guttman, 2010; Trimmer et al., 2011; Behrens et al., 2008; Sasaki et al., 2013). Most of these models suggest that humans aggregate one's own opinion and others' opinions or aggregations of others' opinions, based on their confidence on their own opinion or the strength of their resistance to others' opinions, and on the number of people who have supported a particular option. Many of these models have been compared to empirical data and have shown the credibility. In this dissertation, I will refer to others' opinions or aggregations of others' opinions as "social information" and one's own original opinion as "private information" hereafter.

Collective intelligence in human society and animal groups

With regard to the reason why humans incorporate others' opinions in making a decision, Deutsch and Gerard (1955) suggested two motivations of conformity, *normative social influence* and *informational social influence*, by their experiment in psychology. Normative social influence stands for the influence of conformity that comes from one's hope to respond to others' expectation. On the other hand, informational social influence stands for the influence of conformity that comes from one's hope to obtain the evidence for one's decision.

Informational social influence should be in part explained by the advantage of opinion aggregations, called *collective intelligence*. Collective intelligence, also called swarm intelligence, refers to the ability of a group to perform cognitive tasks and to make decision correctly (Woolley et al., 2010; Krause et al., 2010). Woolley et al. (2010) showed by their psychology experiments that a human group has its intelligence (collective intelligence) in the same manner as an individual has. In many cases, the definition of collective intelligence also includes the condition under which intelligence of the group outperforms that of an individual in the group (Krause et al., 2010; Wolf et al., 2013; Sasaki et al., 2013; Kurvers et al., 2015).

Collective intelligence has been observed not only in groups of humans but also in swarms, flocks or schools of other animals. For example, Argentine ants, *Linepithema humilis*, can find the shortest foraging trail by following chemical pheromone of others and by leaving one's pheromone (Couzin, 2009). Ants *Temnothorax albipennis* and *Temnothorax rugatulus*, and honey bees *Apis mellifera*, can find a preferable new nest in the following way (Conradt and Roper, 2005; Couzin, 2009; Sasaki et al., 2013). Some individuals, called scouts, assess candidates of the new home and each of them brings others to the candidate when she finds that the candidate is good. Once the number of individuals who accept a candidate exceeds a certain threshold, the other individuals move there (to the new nest). Such a decision system, in which the remaining individuals also accept the choice when the number of individuals who accept the choice exceeds a certain quorum, is called quorum rule.

Quorum rule is used by many species from insects, fish, quadrupeds, primates to humans (Wolf et al., 2013; Conradt and Roper, 2003; Couzin, 2009; Sasaki et al., 2013; Conradt and Roper, 2005; Kurvers et al., 2014; Pratt and Sumpter, 2006; Clément et al., 2015). The threshold in quorum rule is related to *speed-accuracy trade-off* of their decision-making (Couzin, 2009). If the threshold is high, the quorum rule leads to more careful assessment and higher accuracy of decision-making, while there is a risk that it is harder for the group to adopt the correct answer. On the other hand, a low threshold leads to rapid decision-making, while the accuracy decreases. It is known that groups in a various species such as ants, fish and humans, can adjust the threshold according to the environment (Pratt and Sumpter, 2006; Couzin, 2009; Conradt and Roper, 2005; Wolf et al., 2013; Kurvers et al., 2014).

A special case of quorum rule is majority-rule, where the threshold is exactly a half. Majority-rule is very popular in human society. For example, in modern societies, humans frequently use the rule in politics such as leader elections and other referendums (Hastie and Kameda, 2005; Conradt and Roper, 2003). It is known that the majority-rule voting based on independent choices for a binary question can result in higher accuracy than when decided by a single individual (Ladha, 1992; Nitzan, 2009; Kao and Couzin, 2014; Krause et al., 2010; Wolf et al., 2013; Laan et al., 2017). A classic argument relating to this phenomenon was shown by Marquis de Condorcet in 1785 (Kao and Couzin, 2014). The Condorcet jury theorem states that, by the law of large numbers, the probability that the majority in a group chooses the correct answer for a binary question approaches unity as the number of individuals in the group approaches infinity, if all individuals have the same probability to choose the correct answer by themselves that is more than 0.5, and if individuals' choices are independent from each other before applying majority-rule voting. The extensions of the theorem have been studied, including works that relax the assumption of independence of individuals' choices, and those which consider difference among individuals' probability to choose the correct answer by themselves (Ladha, 1992; Nitzan, 2009; Grofman et al., 1983; Kao and Couzin, 2014). Some of these researches suggest that the accuracy of the result of majority-rule voting by non-experts for a binary question can be better than that of a highly expertized person, who has higher accuracy in solving the given problem than the others in the group (Grofman et al., 1983; Ladha, 1992; Nitzan, 2009). Their results can be illustrated by the following example. Consider a group with three individuals differing in their probability of giving the correct answer for a binary question when they make a decision alone. Suppose that their accuracy is given by $p_1 = 0.75$, $p_2 = 0.7$, and $p_3 = 0.65$. Then, the accuracy of the majority-rule voting of these three is 0.785 [†]. This means that even the individual with the highest accuracy of the three, $p_1 = 0.75$, can improve the accuracy by following the result of the majority-rule voting among these three.

In quantity estimations, it is also known that a group can outperform an expert by taking the average or the median of a large number of independent estimations (Galton, 1907; Lorenz

[†] $p_1p_2p_3 + (1-p_1)p_2p_3 + p_1(1-p_2)p_3 + p_1p_2(1-p_3) = 0.785 (> 0.75)$. The first term of the left-hand side in this equation is the probability that all three persons chose the correct answer by themselves. The sum of the second to the forth terms is the probability that two persons out of the three chose the correct answer by themselves. Therefore, the left-hand side of the equation is the probability that the majority chose the correct answer by themselves.

et al., 2011; Krause et al., 2010; Madirolas and de Polavieja, 2015; Kao et al., 2018; King et al., 2012). This is often called the wisdom of crowds. For example, in Galton (1907), data of estimations by 787 farmers about the weight of a dressed ox was collected. The median of these estimations was quite close to the true weight—the error was within 1% of the true value. Similarly, in an experiment in which subjects estimated the number of marbles in a jar, the mean of the estimations was within 1.5% of the true value (Krause et al., 2010). These phenomena can be explained by using the laws in statistics such as the law of large numbers. Which statistic, for example the mean or the median, is a good indicator that contributes to the wisdom of crowds depends on the distribution of people's estimations. For example, if an estimation approximately follows the log-normal distribution so that a logarithm of the estimation follows the normal distribution where the mean is the logarithm of the true value, then the median of the people's estimations becomes close to the true value, as the number of estimations becomes large. Actually, when one has to estimate a relatively large number, such as guessing the length of the border of two countries (Lorenz et al., 2011), the distribution of an estimation is known to be close to a log-normal distribution (Lorenz et al., 2011; Madirolas and de Polavieja, 2015). On the other hand, if an estimation follows an almost symmetric distribution where its center is the true value, the mean of estimations is close to the true value (Lorenz et al., 2011; Madirolas and de Polavieja, 2015; Kao et al., 2018).

Dilemmas in the use of social information

So far, I have introduced how humans incorporate others' opinions into one's decision-making, and how collecting and aggregating opinions contribute to more accurate decision-making. It seems that humans are always able to enjoy the advantage of collective intelligence when they use social information. However, it has also been shown that the correlation between opinions can undermine the effect of collective intelligence (Ladha, 1992; Lorenz et al., 2011; Kao and Couzin, 2014; Madirolas and de Polavieja, 2015; Laan et al., 2017). For example, recall the aforementioned case of three individuals, person#1–#3, with the probability of choosing the correct answer being $p_1 = 0.75$, $p_2 = 0.7$, and $p_3 = 0.65$, respectively. If person #3 mimics the opinion of person #2 before the majority-rule voting, then the accuracy of the result of the majority-rule voting among these three becomes 0.7 [‡], which is now smaller than the accuracy of the expert. In this example, actually, only the opinion by person #2 is effective, because two or three persons (the majority of the three) choose the correct answer when and only when person #2 chooses the correct answer by their influence relationship. In such a way, opinion correlation between voters decreases the effective number of opinions and undermines the effect of collective intelligence.

[‡] $p_1 p_2 + (1 - p_1) p_2 = p_2 = 0.7$. The first term of the left-hand side in this equation is the probability that person #1 and person #2 chose the correct answer by themselves. Note that, in this case, person #3 also chose the correct answer, since person #3 mimics #2. The second term is the probability that person #1 chose the wrong answer and person #2 chose the correct answer by themselves. Similarly to the first term, person #3 chose the correct answer, in this case. Therefore, the left-hand side of the equation is the probability that the majority (two out of the three) chose the correct answer by themselves. However, we can know the accuracy of the result of the majority-rule voting in this case ($p_2 = 0.7$) without calculation, because only the opinion of person #2 is effective as I explain in the main text.

In quantity estimations, social information can damage the advantage of wisdom of crowds, too (Lorenz et al., 2011; Madirolas and de Polavieja, 2015; Laan et al., 2017). In Lorenz et al. (2011), they showed that social information causes a convergence of estimations in their experiment. They also showed that the true value shifts to a peripheral position of the estimations through the convergence of estimations by social influence. In this way, the wisdom of crowds was damaged by social influence (Lorenz et al., 2011).

Sequential decision-making, in which each individual can make decision by observing earlier answers by other individuals, is known to sometimes lead to incorrect *information cascade*. Once some individuals stated an answer, a following decision-maker may be influenced by that answer, as depicted in threshold model in Granovetter (1978). Through this process, a particular answer that happened to be chosen by the first few individuals becomes more likely to be chosen and eventually becomes the dominant opinion, to which it is difficult for individuals to oppose. This phenomenon is called information cascade. Such a propagated answer is not necessarily correct, and in such a case it is called incorrect information cascade. Information cascades, including incorrect information cascade, can be actually seen in human society (Bikhchandani et al., 1992, 1998; Raafat et al., 2009; Granovetter, 1978; Watts, 2002; Buchanan, 2008). Papers in economics such as Bikhchandani et al. (1992) and Bikhchandani et al. (1998), reviewed examples of information cascade. These examples include information cascade in medical practices, scientific theory and in behaviors or ideas related to politics and finance. For example, they described the adoption of a medical practice, called tonsillectomy, by doctors (Bikhchandani et al., 1992). This surgical procedure was often performed routinely without a particular definitive reason. It is pointed out in the review that the doctors did not have enough cutting edge information about the procedure but merely imitated others.

These facts suggest that, although the use of social information would be advantageous to individuals' decision making, once some individuals start using it, their opinions become correlated with the information that they used. When they state their opinions, their correlated opinions become a part of social information again. In this way, social information could progressively lose its independency and quality so that no one eventually dares to use it.

Research question (Self-organization of reference structure)

Following these arguments, some literature investigated how humans can overcome such breakdown of social information by creating the new opinion aggregation rule, by finding the way to extract subgroup that contributes to the wisdom of crowds, and so on (Madirolas and de Polavieja, 2015; Kurvers et al., 2015; King et al., 2012; Prelec et al., 2017; Laan et al., 2017; Kao et al., 2018). In these studies, the improvement of social information was confirmed by experiments and data analyses. One of such studies suggests that humans can enjoy the advantage of collective intelligence by controlling whom to follow, such as by following well-performed individuals in quantity estimation problems (King et al., 2012). Hofstra et al. (2015) showed that an individual's performance depends on how one is connected to others in individuals' influence network, by simulations and an experiment, for the multi-armed bandit problem. In this multi-armed bandit problem, subjects have to choose an option among some choices. The distribution of payoffs assigned to each choice was different from those assigned to the other choices. The subjects did not know which choice corresponds to which distribution. The network structure, such as how it is centralized to some individuals, also affects the group performance. These studies suggest that the structure of reference relationship affects the quality of decision accuracy in a population. However, in most of experimental and theoretical works on social information use and decision accuracy, the reference structure has been given artificially, such as a sequential decision-making by people where the order to state their opinions is fixed. Here I ask how the reference structure self-organizes, when each individual tries to use social information to secure the accuracy of his/her decision-making. I also ask if social information use is still advantageous for each individual in the self-organized reference structure. I try to answer these questions with a mathematical model in my dissertation.

In Section 1.2 in Chapter 1, I will explain the network approach to understanding the self-organization of reference structure. The network model that I constructed is introduced in Chapter 2. In Chapter 3, I show the results about the emergence of the heterogeneous reference structure. I also evaluate the decision accuracy of each individual in the self-organized reference structure. I developed an analytical theory to explain the results obtained in the simulation. The discussion is in Chapter 4. The contents in Chapter 2 and 3, and Section 4.1 in Chapter 4 are published in Ito et al. (2018).

1.2 An adaptive network approach to understanding the self-organization of reference structure

One example of reference relationship in human society is found in the lead–follow relationship between financial analysts as I mentioned in General Introduction (Cooper et al., 2001; Booth et al., 2014; Ramnath et al., 2008; Clement and Tse, 2005; Hirshleifer and Hong Teoh, 2003; Kim et al., 2011). Financial analysts synthesize much of information and provide reports such as forecasts of a firm's forthcoming earnings for investors. They revise their personal earnings forecasts at their own chosen time during the fiscal period. It has been suggested in the literature that less informative or less experienced analysts follow information developed by analysts called lead analysts, who announced their forecasts earlier, in order to make their decisions more accurately (Cooper et al., 2001; Ramnath et al., 2008; Kim et al., 2011; Guttman, 2010; Zhao et al., 2014).

Such lead-follow behavior is not necessarily limited to financial analysts; we expect to see it more generally in our society when each of us can refer to earlier opinions to make our decisions. In this study, I consider the process of decision-making in a population in which individuals are mutually connected by reference links. Individuals are assumed to base their opinions on the majority of earlier opinions made by the referred individuals, i.e., I assume a directed network in which each node represents an individual making his/her decision, and each directed link refers to a reference relationship. Thus, the accuracy of an agent's opinion depends on who the agent refers to. Note that the majority-rule voting assumed here is an aggregation rule that is related to collective intelligence as I mentioned in General introduction; it is known that the majority-rule voting based on various (independent) opinions can result in higher accuracy than the one decided by a single agent.

It should be natural to assume that each individual assesses the credibility of the referents and decides to either keep or stop following them accordingly. Thus, a reference link is rewired according to the accuracy of the referred agent, whose accuracy depends on who he/she refers to. Therefore, we need to consider the interaction between the change in opinion caused by network topology and the change of network topology induced by the opinion accuracy of the nodes. This idea is related to *adaptive* or *coevolutionary* networks, in which feedback loops between node dynamics and network topology are considered (Gross and Blasius, 2008; Castellano et al., 2009). There are a number of adaptive network models under various link-rewiring rules including ones that assume game interactions between nodes, such as the prisoner's dilemma game and the minority game, in which a link represents a game interaction or reference. These links are discarded and rewired when the linked game partners are not preferable or when the linked advisers are not reliable (Benczik et al., 2009; Gross and Blasius, 2008; Castellano et al., 2009; Perc and Szolnoki, 2010; Li et al., 2007; Anghel et al., 2004; Garlaschelli et al., 2007; Zhou et al., 2011; Colman and Rodgers, 2014; Holme and Newman, 2006). Some of these models show the emergence of heterogeneous structure in the self-organized network, such as the scale-free degree distribution, in which a small number of individuals acquire a large number of degrees after repeated events of rewiring, even when starting from a homogeneous initial state (Gross and Blasius, 2008; Perc and Szolnoki, 2010;

Li et al., 2007; Anghel et al., 2004; Garlaschelli et al., 2007; Zhou et al., 2011). Some of these models also highlight macroscopic quantities, such as the ratio of cooperators in the population and the quality of propagated information (and "performance" in my model), that change through adaptive rewiring (Gross and Blasius, 2008; Perc and Szolnoki, 2010; Li et al., 2007; Zhou et al., 2011). They showed, in some cases, that the self-organized network with heterogeneous structure has better performance than the initial homogeneous network.

In my model, I focus on the accuracy of decision-making of each individual, which I call "performance", in the self-organized network. It is not clear whether the self-organized network shows good performance. If the self-organized network has high heterogeneity, so that some individuals, called leaders, receive a far larger number of reference edges than the others, then the opinions of agents in the network should be highly correlated. This enhanced correlation between opinions may harm the population performance in the long run, as in the examples of deteriorated social information that I discussed in General introduction. The opposite may be the case because the network structure, which is biased toward referring to more accurate agents, may improve the population performance. The performance in the self-organized network should depend on the network structure.

Therefore, from the viewpoint of network theory, my research questions can be summarized to these two: (1) what property is generated in the self-organized network, for example, how strong is the heterogeneity in the self-organized network, and (2) whether the reference relationship in the self-organized network leads to higher performance than the initial random network. To answer these questions, I conducted computer simulations and developed an analytical theory to explain the results obtained in the simulations.

Chapter 2

Methods

2.1 Model

I consider a directed network made up of N nodes, each of which represents an agent who makes a decision. In this network, a directed link from node i to node j means that agent i refers to agent j when he/she makes a decision. If there is a directed link from i to j, I call agent i a *follower* of agent j, and agent j a *referent* of agent i. For each agent, the number of reference links from him/her is fixed to M. Let a_{ij} be the number of reference links from agent i to agent j—(a_{ij}) is the adjacency matrix of the network (Table 2.1). By definition, $\sum_{j=1}^{N} a_{ij} = M$ and $\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} = NM$. Here I allow for both self-loops and the overlap of links, i.e., a_{ii} is not necessarily 0 and a_{ij} can be more than 1. Each agent in my model repeatedly makes his/her decision while updating his/her referents by the rule explained later (Fig 2.1).

Table 2.1: **Definition of symbols**

Symbol	Descriptions
N	Number of agents
M	Number of reference links from each agent
a_{ij}	Number of reference links from agent i to agent j
p_i	Ability of agent i
Π_i	Probability that agent i gives a correct answer
y_t^{ij}	Evaluated performance of agent j by agent i at time t
y_0	Initial value of y_t^{ij} when agent i newly rewires to agent j
I_t^i	The random variable whose value is 1 (0) if agent i succeeded (failed) in giving a correct answer
θ	Rewiring threshold
α	The extent to which people attach importance to the current result against the history so far in the performance evaluation
$\overline{T_i}$	The mean duration that the agent i keeps his/her follower
$\bar{k}(p)$	The mean in-degree of an agent with ability p

In the initial condition, each agent refers to randomly selected M referents. In other words, a directed random regular graph with M out-degrees is used as the initial state of my model.

In my model, the same binary choice question is given to all agents. One of the choices is correct, and the other is wrong. I assume that agents vary in their probability of solving a problem correctly by themselves (i.e., without referring to others' opinions). I call the probability *ability*. The ability of agent i is denoted by p_i. For example, a question is given to financial analysts, such as whether the earning of a company in the next quarter increases, and all will know the correct answer at the end of the next quarter. After many trials, the analyst's (say agent i's) ability is calculated as the probability that he/she forecasted the

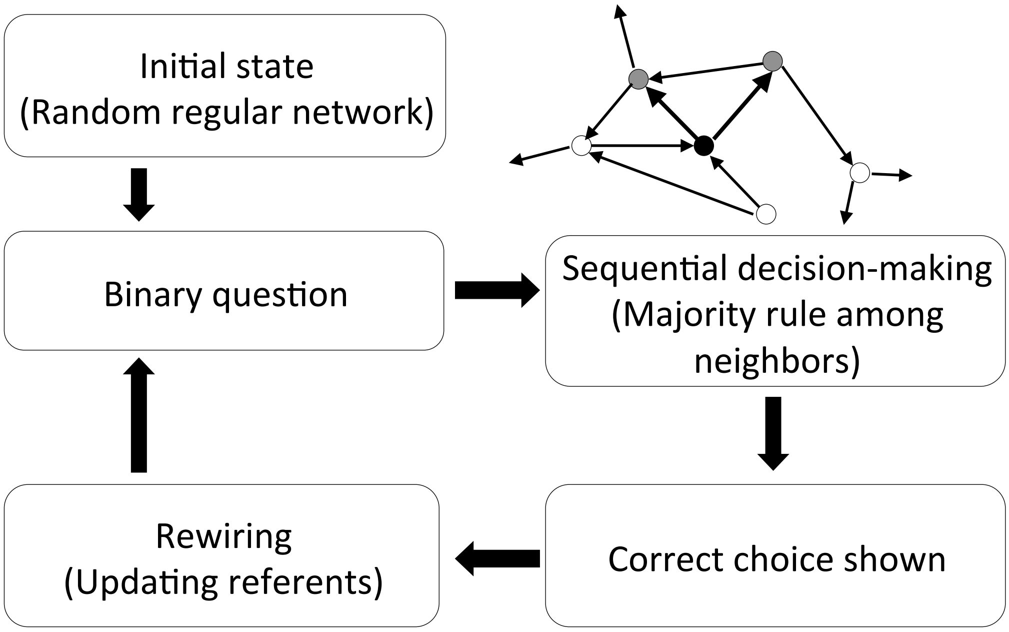

Figure 2.1: **Procedure to update the network.** The initial network is a random regular network. Starting from this initial network, I iterate sequential decision-making and rewiring of the reference links. Illustrated in the top-right part is a sample network, in which the closed dot represents the focal agent, the gray dots represent the agents referred to by the focal agents, and the arrows are reference links.

correct outcome, that is represented by p_i. For each question, all agents state their answers sequentially in a randomly determined order. Hereafter, I call specifically an actual stated choice an *answer*. Now, I explain how agent i states his/her answer. When agent i's turn comes, he/she first sets his/her own choice for the given question without referring to those by his/her referents. The probability that this choice is correct is given by the agent's ability, p_i. In the next step, agent i puts his/her own choice together with the answers of referents that have already been stated and makes a final choice among those choice/answers according to the simple majority-rule. Agent i then states a final choice as his/her answer. For example, if agent i refers to agents i_1, i_2, i_3, i_4 and i_5, and agents i_1 and i_2 have already stated their answers, agent i collects the answer from i_1 and i_2's along with his/her choice and states the majority among these three. In the case of a tie in applying the majority-rule, agent i tosses a coin to decide which choice he/she states. If $a_{ii} \geq 1$ or $a_{ij} \geq 2$, the majority voting becomes weighted majority voting in my model. More specifically, in the case where $a_{ii} \geq 1$, I regard that agent i's choice has a weight $a_{ii} + 1$ in i's majority-rule voting. Similarly, when $a_{ij} \geq 2(i \neq j)$, I regard that agent j's answer has a weight a_{ij} in i's majority-rule voting. Since agent i incorporates other agents' answers, it is clear that agent i's ability p_i is different from the probability with which agent i actually states a correct answer, Π_i. I denote Π_i as the *performance* of agent i.

After all the agents state their answers, the correct answer to the question is given. I assume that each agent monitors and evaluates the performance of his/her referents, as explained later. Each agent breaks the link to the referent if his/her evaluated performance falls below a certain threshold and rewires it to a randomly selected agent excluding referents that are kicked off in this step. The assumption that the newly selected agents are determined randomly is based on the idea that we cannot know the performance of strangers a priori.

I iterate decision-making and rewiring as explained above. In each iteration step t, the *evaluated performance* of referent j by agent i, denoted as y_t^{ij}, is updated as follows. When agent i newly acquires referent j at time t_s, his/her initial level of estimated performance, $y_{t_s}^{ij}$, is set to y_0 for any pair of i and j. Let I_t^j be a variable whose value of 0 represents the failure of agent j to give a correct answer, and a value of 1 represents the agent's success at the iteration time t. I assume that the evaluated performance of agent j by agent i at iteration time t, y_t^{ij}, is updated recursively by

$$y_t^{ij} = (1-\alpha)y_{t-1}^{ij} + \alpha I_t^j. \tag{2.1}$$

Here, α describes the extent to which people in the society attach importance to the current result as compared to the referent's past. I also assume that all agents adopt the same rewiring threshold to kick-off referents, θ. The threshold represents severity of assessment in the society. For example, if the threshold is low, people in the society are generous when they evaluate their referents.

2.2 Simulation conditions

I conducted agent-based simulations using parameters $N = 100$, $M = 5$, and $\alpha = 0.1$. I assume that the ability of agents, p_i's, are uniformly distributed in the range of 0.5 to 0.75 by setting $p_i = 0.5 + 0.25i/N$ for $i = 0, 1, \ldots, N-1$. Note that the ability of each agent does not change throughout a simulation run. The initial evaluated performance y_0 is set to 0.625, which is nearly equivalent to the mean ability in the population. I limit the range of the rewiring threshold θ in $0.5 \leq \theta < y_0 = 0.625$. The lower limit for θ, 0.5, is only the accuracy of a coin-toss. The upper limit y_0 is set for the following reason—if we set the threshold greater than this upper limit, the initial performance of a new referent is always evaluated lower than the threshold.

At the initial state, the network is a directed random regular network with out-degree $M = 5$, so the in-degree distribution of each agent is expected to obey the binomial distribution with parameters NM (the total number of reference links in the population) and $1/N$ (the probability that a particular individual is chosen as a referent) and is approximated by the Poisson distribution with mean M, because $N \gg 1$ (Fig 2.2).

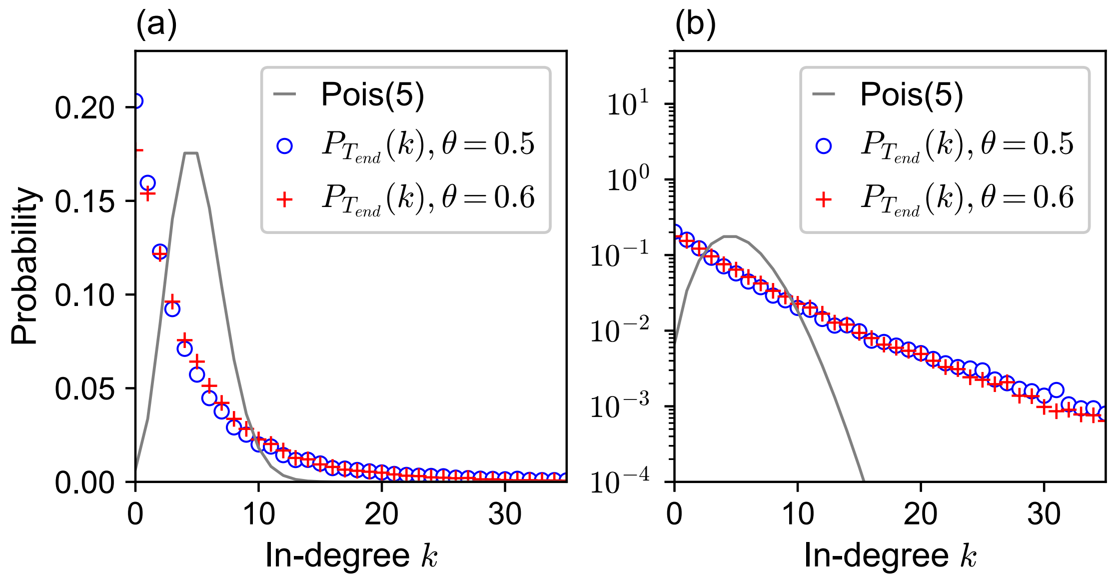

Figure 2.2: **The distribution of the number of followers (in-degree) in the initial and self-organized networks.** (a) The solid line is the initial Poisson distribution with a mean of 5. The markers (the circle and +) denote the in-degree distributions of the self-organized network with rewiring thresholds $\theta = 0.5$ and 0.6, respectively, obtained over 500 independent runs of my simulation. The in-degree distributions in the self-organized networks are significantly different from the initial condition, showing much higher heterogeneity in in-degrees. (b) The same as (a) except that the vertical axis is logarithmically scaled. I can observe the approximately exponential tails in the self-organized networks.

In my simulation, a set of sequential decision-makings of agents, followed by the rewiring of reference links, constituted the events in a unit of time, which repeats itself until the "self-organized network" at $t = T_{\text{end}} = 20,000$ was reached. For each parameter set, the simulations were repeated 500 times. I calculated the frequency of agents having in-degree k at each time step t and averaged them over 500 independent runs. $P_t(k)$ denotes the averaged frequency of agents having in-degree k at time t, which must depend on the rewiring threshold θ. I regard $P_{T_{\mathrm{end}}}(k)$ as the in-degree distribution in the self-organized network. I evaluated the performance of each agent for each simulation run in the self-organized network by averaging the number of correct answers stated in the last $T = 100$ time steps (i.e., the performance of agent i is the average of I_t^i over $T_{\mathrm{end}} - T + 1 \leq t \leq T_{\mathrm{end}}$), at which point I assumed that the network has reached an equilibrium state. The average over 500 independent runs was then calculated and regarded as the mean performance of agent i, Π_i. Therefore, the definition of Π_i is $\Pi_i = \overline{\sum_{t=T_{\mathrm{end}}-T+1}^{T_{\mathrm{end}}} I_t^i / T}$, where the overline represents the average over 500 independent runs. I also calculated the mean group performance and its standard deviation. For each single run, I regarded $\sum_{i=1}^{N} I_t^i / N$ as the group performance at time t and calculated the mean and the standard deviation (SD) of $\left\{\sum_{i=1}^{N} I_t^i / N\right\}_{T_{\mathrm{end}}-T+1 \leq t \leq T_{\mathrm{end}}}$, that is, for the last $T = 100$ time steps. Then I took their average over 500 independent runs to evaluate the group performance and its fluctuation.

Chapter 3

Results

3.1 The heterogeneity in the in-degree distribution

The in-degree distribution $P_{T_{\mathrm{end}}}(k)$ in the self-organized network significantly differed from the initial Poisson distribution for the random network (Fig 2.2). As the in-degrees in the self-organized network were distributed approximately exponentially, there were a few nodes that had much larger in-degrees than the mean. In other words, high heterogeneity in the number of followers evolved through the adaptive rewiring process. The agents attracting many followers can be interpreted as "opinion leaders" in my model.

The mean in-degree $\bar{k}(p_i) = \overline{\sum_{j=1}^{N} a_{ij}}$ of agent i with ability p_i, that is the mean number of followers of agent i, increased exponentially with p_i for each rewiring threshold θ (Fig 3.1). I was able to obtain an approximation equation which $\bar{k}(p_i)$ satisfies in the equilibrium state. Its derivation, which I will describe in detail in Sections 3.2 to 3.5, is illustrated as follows

(Fig 3.2). Suppose we know $\bar{k}(p_i)$. In the equilibrium state, the probability that a randomly sampled reference link from the population is referring to agent i should be proportional to the mean number of followers of agent i and is expressed as

$$\text{Pr.[agent } i \text{ is being referred]} = \frac{\bar{k}(p_i)}{\sum_{j=1}^{N} \bar{k}(p_j)} = \frac{\bar{k}(p_i)}{NM}. \tag{3.1}$$

Based on these probabilities, I can derive the approximation of the mean performance Π_i of agent i, who has ability p_i (see Section 3.2). Given Π_i, the mean duration $\overline{T_i}$ that the agent is kept linked by a follower is calculated in Section 3.3. Finally, given $\overline{T_i}$, the mean number of followers $\bar{k}(p_i)$ of agent i is derived in Section 3.4. Therefore, I obtained an implicit equation of $\bar{k}(p_i)$ in the self-organized network. This equation of $\bar{k}(p_i)$ explains why agents with higher ability have acceleratingly more followers. Since I cannot solve this equation for $\bar{k}(p_i)$, I performed an iterative approximation method to numerically obtain $\bar{k}(p_i)$ against p_i. This procedure is explained in Section 3.5. The results obtained by this numerical calculation agree well with the simulation results.

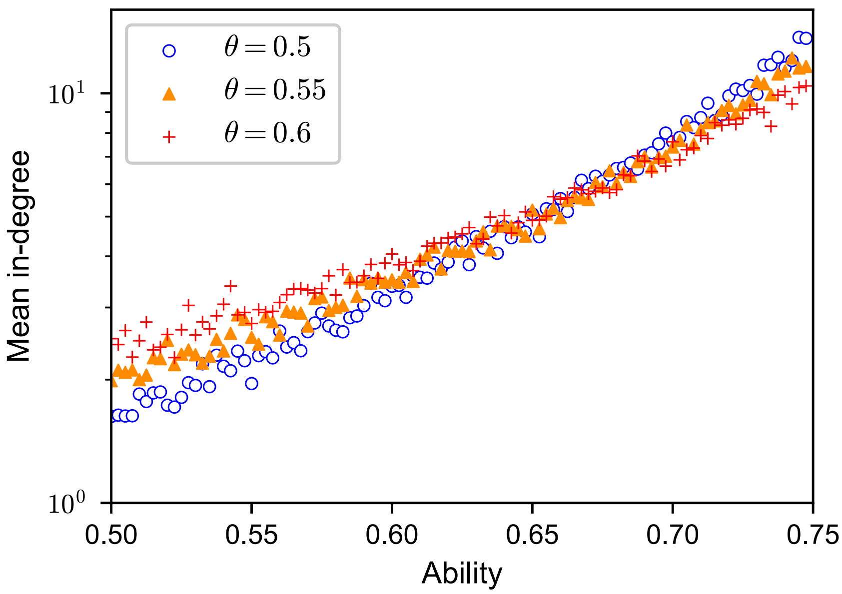

Figure 3.1: **Semi-log plot of the mean in-degree of an agent versus his/her ability in the self-organized network obtained by simulations.** Different symbols represent the results for varying θ. The mean in-degree increases approximately exponentially with ability, and slopes are steeper when I set the threshold lower. The vertical axis is scaled logarithmically.

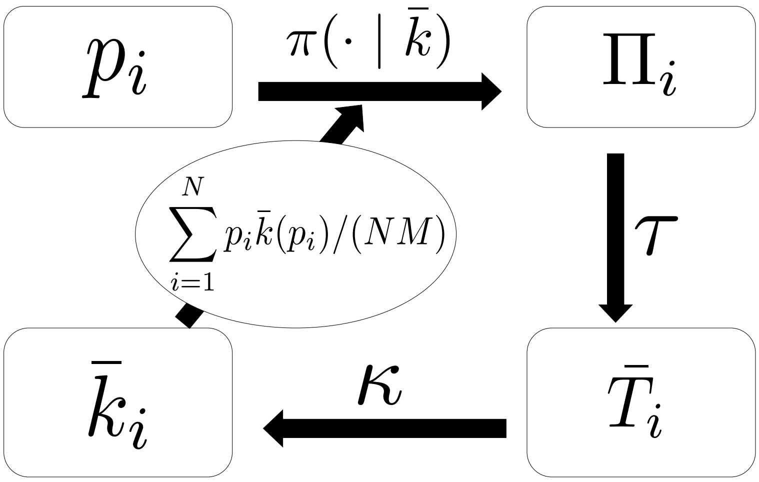

Figure 3.2: **The schematic diagram for the relationship between the ability of agent i, p_i, his/her mean performance Π_i, the mean duration that he/she is kept linked by a follower $\bar{T}_i$, and his/her mean in-degree $\bar{k}_i$ in the equilibrium state.** The mean performance can be derived approximately by the ability p_i and the mean in-degree function $\bar{k}(\cdot)$ through the function $\pi(\cdot \mid \bar{k})$. The mean duration that the agent is kept linked by a follower is obtained by the mean performance (the function τ). Then, the mean in-degree is obtained by the mean duration the agent is kept linked by a follower (the function κ).

3.2 The relationship between the mean performance of an agent and his/her ability

I calculated the mean performance of agent i in the self-organized network, $\Pi_i = \overline{\sum_{t=T_{\mathrm{end}}-T+1}^{T_{\mathrm{end}}} I_t^i/T}$, as explained in Section 2.2. For each threshold, the mean performance of an agent increased linearly with his/her own ability (Fig 3.3(a)). This is because the probability that agent i with ability p_i gives a correct answer is given by the sum of two terms,

$$\begin{aligned}\Pi_i = (1-p_i)\mathrm{Pr.}[&(\text{the number of referents who gave correct answers}) \geq s/2+1] \\ &+p_i\mathrm{Pr.}[(\text{the number of referents who gave correct answers}) \geq s/2],\end{aligned} \tag{3.2}$$

when the number of referents of agent i who have stated their answers before agent i stated his/her own, which I denote by s, is even. Note that here, I neglect self-loops or overlaps in the reference links to simplify my approximation. Also note that the expression of Π_i becomes a slightly complicated when s is odd (Section A in Appendix) since there are $s+1$ answers/choice including his/her own and I have to consider the tie of the number of answers/choice when the majority-rule is applied. However, Π_i can again be described as a linear function of the ability p_i in both cases where s is odd and where there are self-loops or overlaps in the links (Section A in Appendix).

Note that each term $\Pr.[\,\cdot\,]$ in the equations above depends on the ability and the performance of agents who the focal agent refers to; therefore, $\Pr.[\,\cdot\,]$ depends on the distribution of the *ability of referents*, which means the ability of those who are referred to by others, not on the ability of random agents.

I can derive an approximate formula for the slope and the intercept of the linear dependence of Π_i on p_i (Section A in Appendix), which agrees well with the simulation results (Fig 3.3(a)). For later use, let me formally denote this relation as $\Pi_i = \pi(p_i \mid \bar{k})$, where $\pi(\cdot \mid \bar{k})$ maps p_i to Π_i, which itself depends on $\bar{k}$. The approximated slope and intercept depend on the mean ability of referents $\sum_{i=1}^{N} p_i \bar{k}(p_i)/(NM)$, which is a value that represents the distribution of referents' ability.

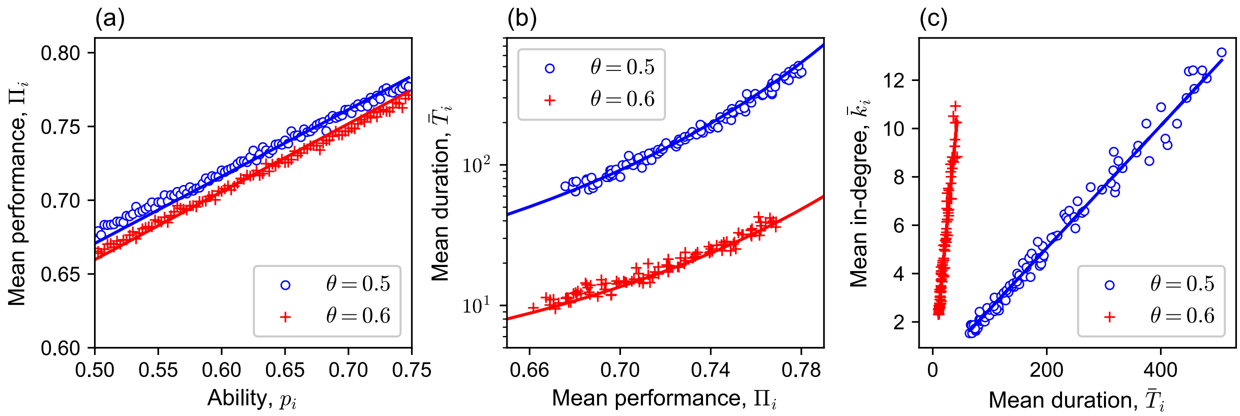

Figure 3.3: **The mean performance Π_i of an agent versus his/her ability p, the mean duration $\bar{T}_i$ that the agent is kept by a follower versus the agent's mean performance Π_i, and the mean in-degree $\bar{k}_i$ of an agent versus the mean duration $\bar{T}_i$ that the agent is kept by a follower in the self-organized network.** (a) The mean performance Π_i of an agent versus his/her ability p in the self-organized network for each threshold θ (the circle and + for thresholds 0.5 and 0.6). The solid lines show the analytical results. The mean performance increases linearly with ability. (b) The semi-log plot of the mean duration $\bar{T}_i$ that the agent is kept by a follower versus the agent's mean performance Π_i for each threshold in the self-organized network (the circle and + for thresholds 0.5 and 0.6). The solid lines show the analytical results. The mean duration increases nearly exponentially with the mean performance. (c) The mean in-degree $\bar{k}_i$ of an agent versus the mean duration $\bar{T}_i$ that the agent is kept by a follower for each threshold in the self-organized network (the circle and + for thresholds 0.5 and 0.6). The solid lines show the analytical results. The mean in-degree is proportional to the mean duration.

3.3 The relationship between the mean duration for which a referent is kept linked by a follower and his/her performance

In my model, each agent monitors the performance of his/her referents and stops referring to them when the evaluated performance falls below a rewiring threshold. Thus, the higher his/her referent's performance is, the longer duration that he/she keeps his/her follower. I herein examine how the duration that an agent is kept referred by a follower is related to the agent's performance.

The mean duration that an agent is kept referred by a follower in the self-organized networks increased approximately exponentially with his/her performance (Fig 3.3(b)): $\overline{T_i} \propto \exp(\beta \Pi_i)$, where $\overline{T_i}$ is the expected duration that agent i keeps his/her follower, Π_i is the performance of agent i, and β is a positive constant.

This relationship between a referent's performance and the mean duration for which the referent is kept linked by a follower is derived analytically. As explained in the Method section, an agent's evaluation Y_t of the performance of his/her referent is updated depending on whether the referent's t-th answer was correct ($I_t = 1$) or not ($I_t = 0$), as follows:

$$Y_t = (1 - \alpha) Y_{t-1} + \alpha I_t, \quad t = 1, 2, \ldots, \tag{3.3}$$

where the initial evaluation was set to $Y_0 \equiv y_0$. If the actual performance of the referent is Π, which its follower does not know, I_t $(t = 1, 2, \dots)$ are mutually independent random variables each of which takes a value of 1 with a probability of Π, and a value of 0 with a probability of $1 - \Pi$. The sequence $\{Y_t \mid Y_0 = y_0\}$ then forms a stochastic process. Given $Y_0 = y$, I defined the expected time duration to the time when the evaluated performance hit θ for the first time, $T_\Pi(y)$, as $T_\Pi(y) \equiv \mathrm{E}[\min\{t \mid Y_t \leq \theta\} \mid Y_0 = y]$. $T_\Pi(y)$ is the expected first hitting time to the threshold θ of the stochastic process $\{Y_t \mid Y_0 = y\}$. Then $T_\Pi(y)$ satisfies the recurrence equation:

$$\begin{aligned} T_\Pi(y) &= 1 + \Pi T_\Pi\big(\alpha + (1-\alpha)y\big) \\ &\quad +(1-\Pi)T_\Pi\big((1-\alpha)y\big), \quad y > \theta, \end{aligned} \tag{3.4}$$

and

$$T_\Pi(y) \equiv 0, \quad y \leq \theta. \tag{3.5}$$

Equation (3.4) is derived as follows: if $y > \theta$, the referent is kept linked to the next time step; hence, the addition of 1 in the first term on the right-hand side of Eq (3.4). In the case where the referent gave the correct answer with probability Π, the evaluated performance changes from y to $(1-\alpha)y + \alpha$, and the expected duration after the transition is $T_\Pi\big((1-\alpha)y + \alpha\big)$. The last term is similarly derived for the case of failure. Eq (3.5) simply states that $T_\Pi(y)$ equals 0 if the evaluation y is already less than or equal to the threshold.

The mean duration that agent i with performance Π_i is linked from a follower, $\overline{T_i}$, is then defined by $T_\Pi(y)$ as follows:

$$\overline{T_i} = T_{\Pi_i}(y_0)(\equiv \tau(\Pi_i)), \tag{3.6}$$

The symbol τ in Eq (3.6) denotes the function that maps Π_i to $\overline{T_i}$ (Fig 3.2). Note that $\overline{T_i}$ is greater than 0 since $\theta < y_0$. I solved recurrence Eqs (3.4) and (3.5) numerically and obtained the mean time $T_\Pi(y)$ until which the evaluated performance of a referent with the initial evaluated performance y and the actual performance Π hits the threshold θ for the first time (see Section D in Appendix for the numerical procedure to obtain the mean hitting time). This then led to the mean duration of reference $\overline{T_i}$ defined in Eq (3.6). The analytical formulas (3.4)-(3.6) agree well with the simulation results (Fig 3.3(b)).

3.4 The relationship between an agent's mean number of followers and his/her ability

Here, I derive the mean number of followers or the in-degree $\bar{k}(p_i)$ of agent i as a function of his/her ability p_i.

The probability that an agent is chosen as a new referent for each rewiring event is the same as those for all the others', because each agent rewires its link to a randomly selected agent after he/she kicks off a referent. Thus, the expected number of reference links that agent i receives in the self-organized network is proportional to the mean lifetime of a link to agent i, $\overline{T_i}$. Therefore, the mean in-degree $\bar{k}(p_i)$ of agent i can be expressed as a function of $\overline{T_i}$ as follows:

$$\bar{k}(p_i) = \frac{\overline{T_i}}{\sum_{j=1}^{N} \overline{T_j}} NM (\equiv \kappa(\overline{T_i})). \tag{3.7}$$

The symbol κ in equation (3.7) denotes the function that maps $\overline{T_i}$ to $\bar{k}(p_i)$. This expression of $\bar{k}(p_i)$ agrees well with the simulation data (Fig 3.3(c)). The reason why $\bar{k}$ depends (only) on the ability p_i of agent i is that $\tau(\Pi_i)$ is a function of the agent's performance, Π_i, and $\pi(p_i \mid \bar{k})$ is a function of p_i. See Section C in Appendix for a more formal derivation of $\bar{k}(p_i)$ from a master equation for the probability distribution of the performance of a referred agent.

As noted in the last section and shown in Figs 3.3(a) and 3.3(b), the mean duration that the reference to agent i is kept linked by a follower increases roughly exponentially (but actually slightly faster than exponential) with his/her performance Π_i, and Π_i increases linearly with his/her ability p_i, resulting in an roughly exponential relationship between $\overline{T_i}$ and p_i: $\overline{T}_i \propto e^{\beta' p_i}$. Therefore, the mean in-degree of agent i also increases roughly exponentially with his/her ability:

$$\bar{k}(p_i) = \kappa \circ \tau \circ \pi(\cdot \mid \bar{k})(p_i) \approx (\text{const.}) \times e^{\beta' p_i}, \tag{3.8}$$

where $\circ$ in Eq (3.8) is the composition of functions.

As I discussed above, the mean duration of a link targeted to an agent versus his/her performance is key to predicting how many followers an agent with a given ability can obtain. I show the duration versus performance for various thresholds, which I derived analytically, in Fig 3.4.

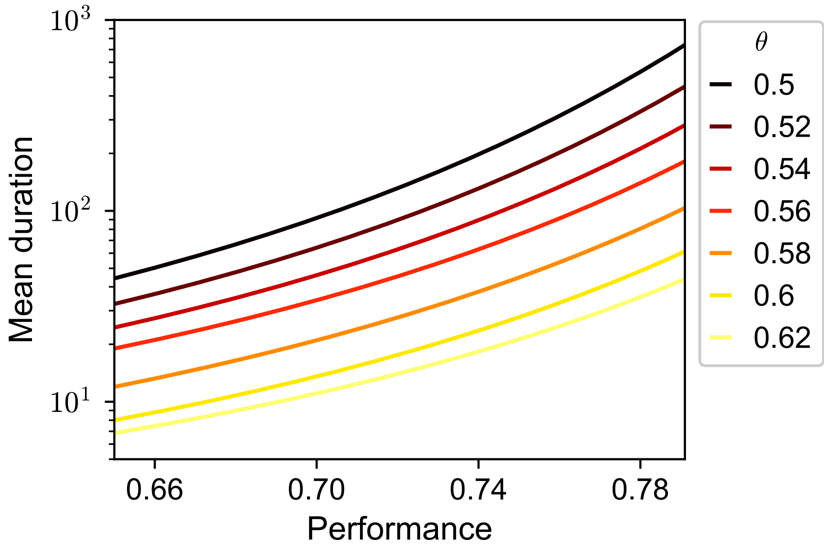

Figure 3.4: **Semi-log plot of the mean duration to be kept by a follower versus performance for each threshold value.** The mean duration is calculated analytically as shown in Section 3.3.The color of the lines ranges from dark to light as the threshold increases.

3.5 Numerical calculation to obtain $\bar{k}(p_i)$

Since Eq (3.8) is implicit in $\bar{k}(p_i)$, I solve it for $\bar{k}(p_i)$ by an iterative approximation method as follows. First, I set $\bar{k}^{(0)}(p_i) = M$ for all p_i as an initial condition (0-th step) of the iterative method. Its n-th iteration counterpart is $\bar{k}^{(n)}(p_i)$. Here is the procedure to obtain $\bar{k}^{(n+1)}(p_i)$ from $\bar{k}^{(n)}(p_i)$. By assuming that a randomly chosen reference link from the population is directed to agent i with a probability of $\bar{k}^{(n)}(p_i)/(NM)$, the mean performance $\Pi_i^{(n)}$ of agent i is obtained as explained in Section 3.2. Given the mean performance $\Pi_i^{(n)}$ of agent i, the mean duration $\overline{T_i}^{(n)}$ that the agent is kept linked by a follower is calculated as explained in Section 3.3. Then $\bar{k}^{(n+1)}(p_i)$ is calculated as

$$\bar{k}^{(n+1)}(p_i) = \frac{\overline{T_i}^{(n)}}{\sum_{j=1}^{N} \overline{T_j}^{(n)}} NM, \tag{3.9}$$

as explained in equation (3.7) in Section 3.4. In other words, I derive $\bar{k}^{(n+1)}(p_i)$ from $\bar{k}^{(n)}(p_i)$, by

$$\bar{k}^{(n+1)}(p_i) = \kappa \circ \tau \circ \pi(\cdot \mid \bar{k}^{(n)})(p_i). \tag{3.10}$$

I repeated this recurrence evaluation until when $\sum_{j=1}^{N}(\bar{k}^{(n+1)}(p_i) - \bar{k}^{(n)}(p_i))^2$ became smaller than 10^{-5} (Fig 3.5(a)). This predicted relationship (solid curves in Fig 3.5(b)) agrees well with the simulation results (markers in Fig 3.5(b)).

(a)

L=Large number

$\bar{k}^{(0)}(p_i) = M$

For $n = 0, 1, \ldots, L$ do:

$$\overline{k}^{(n+1)}(p_i) = \kappa \circ \tau \circ \pi(\cdot \mid \bar{k}^{(n)})(p_i)$$

End

Plot $\bar{k}^{(L)}(p_i)$

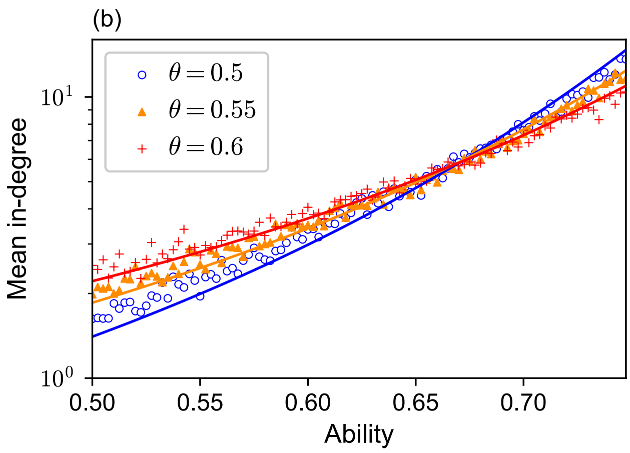

Figure 3.5: **The derivation of the mean in-degree by the iterative approximation method.** (a) The procedure of the iterative approximation method for obtaining the relationship between the mean in-degree and ability. (b) Curves obtained from (a) are shown against plots of simulation data.

3.6 Group performance in the self-organized network

Studying the group performance, $\sum_{i=1}^{N} I_t^i/N$, in the self-organized networks is another objective of my paper. I defined group performance as the proportion of agents who give the correct answer in a sequential decision-making as explained in Section 2.2. The group performance fluctuates temporally (Fig(3.6)). The temporal mean (Fig 3.7(a)) and the temporal standard deviation, SD (Fig 3.7(b)) of the group performance in the self-organized networks were decreasing functions of the rewiring threshold. It is interesting that the stricter the agent's evaluation threshold is for kicking off referents, the worse the long-term group performance is. For comparison, I showed in Figs 3.7(a) and (b) the mean and the SD when all agents choose referents randomly (random reference), as in the initial network state prior to adaptive rewiring (dashed lines in Figs 3.7(a), (b)). I also added those measures in the case where all agents make their decision independently without constructing a network (independent decision; thick horizontal lines in Figs 3.7(a), (b)). The difference between the thick line and the dashed line represents the effect of collective intelligence (decision-making through majority-rule). The difference between the dashed line (random reference network) and the dots (self-organized network after adaptive rewiring) represents the effect of adaptive rewiring of the reference network on group performance, i.e., adaptive rewiring generates a centralized network with preferred connections towards high performance agents. The mean group performance was lowest when agents made decisions by themselves, which is improved by collective intelligence with randomly assigned referents and further improved by adaptive rewiring based on the performance evaluation. Among adaptively rewired networks, those with lower kick-off performance thresholds (i.e., with more generous kick-offs) had higher group performance. I see that the SD of group performance also increased in the same order as the mean group performance in this comparison, i.e., the group performance fluctuated more when the mean group performance became higher.

The mean performance of each agent, $\overline{\sum_{t=T_{\text{end}}-T+1}^{T_{\text{end}}} I_t^i/T}$, against his/her ability in the self-organized networks was compared to both those in the cases of independent decision and of random reference (Fig 3.8). As in the group performance, for a fixed ability value of an agent, the mean performance was the lowest when the agent made decisions independently of others (solid line in Fig 3.8), which is improved by collective intelligence with randomly assigned referents (squares in Fig 3.8) and further improved by adaptive rewiring (circles, triangles and + in Fig 3.8). The effect of the rewiring threshold on the mean performance of each agent was similar to the effect of the threshold on the mean group performance: a looser kick-off threshold led to a higher performance. Fig 3.8 illustrates that the difference between independent decisions and majority voting, either adaptive or not, was reflected in both the slope and the intercept of the performance–ability relationship. However, the differences between the random and adaptive networks and those among different rewiring thresholds were reflected only in their intercepts. This leads to an interesting observation: agents with a lower ability were merited the most in their performance by collective intelligence, and the performance of all agents was improved fairly well by the adaptive rewiring irrespective of their ability.

To summarize, performance in the self-organized network improved compared with the initial random network or the case of independent decision-making. However, the group performance fluctuated more in the self-organized networks, and even more in those networks with higher mean group performance. This implies that a highly "intelligent" population with improved performance, though biased with reference to high-ability agents, can be at risk of a temporal crash in group performance.

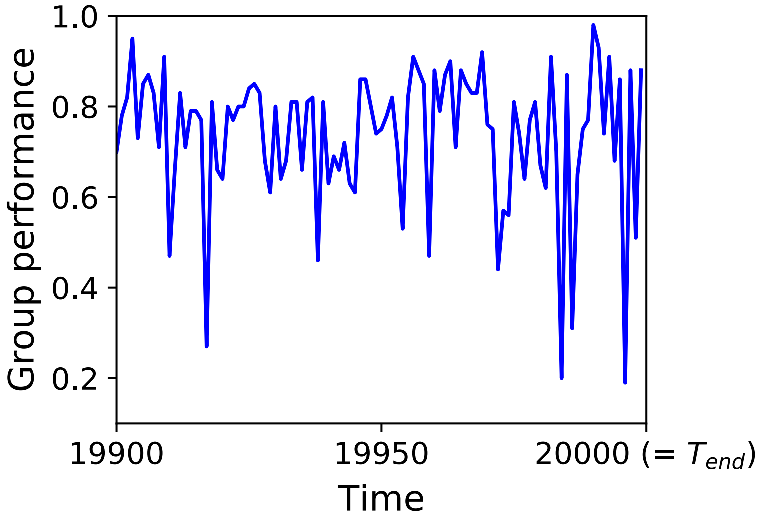

Figure 3.6: **A sample path of group performance.** A sample path of group performance in a period $T_{end} - T + 1 \leq t \leq T_{end}$, i.e. in the self-organized network.

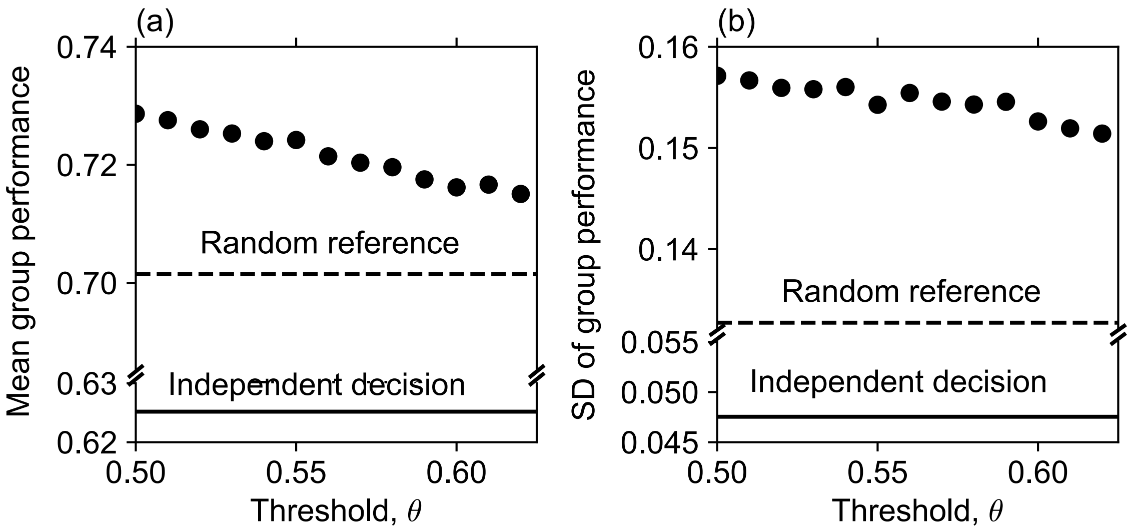

Figure 3.7: **Mean and SD of group performance.** (a) Mean group performance in the self-organized network for each threshold. The dashed line represents the group performance in the initial random network, and the thick horizontal line representss the group performance in the case of independent decision-making. The mean group performance in the self-organized network is higher than that in the random network for all thresholds, and it declines with increasing threshold. (b) The standard deviation SD of group performance versus threshold. The dashed line represents the SD in the random network, and the thick horizontal line represents the SD in the case of independent decision-making. The SD of group performance in the self-organized network is also higher than that in the random network for all thresholds, and it gradually declines with increasing threshold.

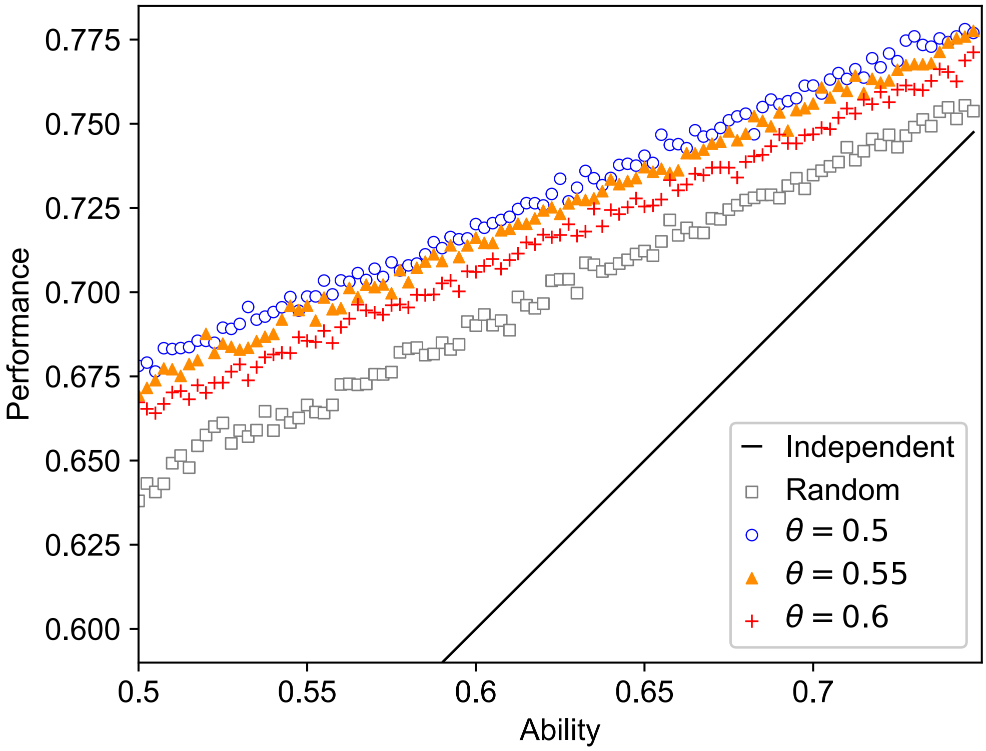

Figure 3.8: **Mean performance of each agent versus his/her ability.** A circle, triangle, and + mark the the mean performance versus ability for thresholds of $\theta = 0.5, 0.55$ and 0.6 in the self-organized network. A black square represents the random network. The solid diagonal line represents the case where performance is equal to ability. Even in the random network, all agents improve their accuracy (the mean performance is higher than the ability for each agent), and low-ability agents can particularly greatly improve it. The lower we set the threshold, the higher the mean performance becomes for each agent.

3.7 The effect of threshold on the unevenness in in-degrees

The threshold θ used for rewiring, which stands for the severity of assessment, affected the self-organized network in the following aspects. First, thresholds affected the strength of heterogeneity in in-degrees among agents. I examined two heterogeneity measures of in-degree distribution at time T_{end}, the Gini coefficient ($G = \sum_{i,j=1}^{N} \mid k_i - k_j \mid / (2N^2\bar{k})$, where k_i and k_j are the in-degrees of agent i and j respectively, and $\bar{k}$ is the mean in-degree of the population (Cowell, 2011)), and the coefficient of variation of in-degrees ($\mathrm{CV} = \sqrt{\mathrm{Var}(k)}/\bar{k}$). They showed substantial dependence on the threshold θ (Fig 3.9). The Gini coefficient and the CV are indices that are originally used to represent the inequality in the distribution of wealth in a society. Here "the number of followers" (or "in-degree") plays a role of "wealth". I measured the inequality in the number of followers by using these indices. Higher values of these indices mean strong heterogeneity in in-degrees. In Fig 3.9, for both indices, the lower is the threshold for rewiring, the higher are the values of these indices. Therefore, both of these two indices show that a lower threshold for rewiring generates stronger inequality in the self-organized in-degree distribution.

The threshold θ also affected the time needed for the system to reach the equilibrium state (Fig 3.10). In Fig 3.10, the lighter colors represent a higher frequency of agents with a given in-degree k (ordinate) against a threshold θ (abscissa) at time t. I can see that the class of individuals with higher k grows faster for higher rewiring thresholds.

The exponential increase of the mean in-degree $\bar{k}(p)$ against ability p is also affected by the threshold θ (Fig 3.1). This nonlinearity in $\bar{k}(p)$ became stronger as the rewiring threshold θ decreased. My analytical formula for the relationship between an agent's mean in-degree and ability (Eqs. (3.7), (3.8)) shows that the strongly biased links towards the agents of high ability is due to the nonlinear dependence of the mean duration that an agent keeps a follower on their performance. We have already seen that the extent to which the mean duration increased with performance was stronger for lower thresholds (Figs 3.3(b) and 3.4). These results can be also seen in Fig 3.11, which shows that the mean ability of referents (averaged over those who are being referred), $\overline{p^*} = \sum_{i=1}^{N} p_i \bar{k}(p_i)/(NM)$, was a decreasing function of the rewiring threshold. This implies that the more the agents seek better referents, the lower is the mean ability of referents. These apparently counterintuitive results are discussed in Section 4.1.

The group performance and the performance of each agent also differed by the threshold. The mean group performance and the performance of each agent became better as the threshold θ decreased (Figs 3.7(a) and 3.8). The SD of the group performance, i.e., the fluctuation of the group performance, also increased as the threshold θ decreased (Fig 3.7(b)).

Therefore, when we set the threshold lower, the heterogeneity in in-degrees became stronger, and reference links were biased more toward higher ability agents. At the same time, I also see that the group performance became better on average, though its temporal fluctuation became greater. I discuss the reason why these results hold in the following Section 4.1.

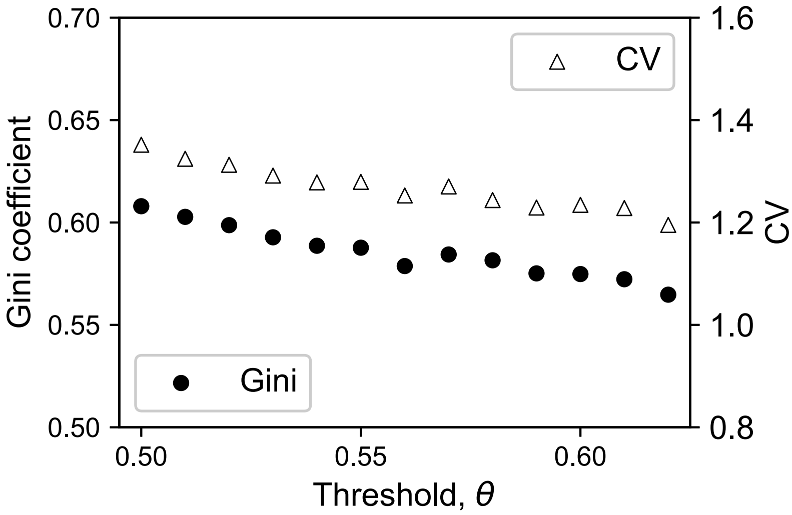

Figure 3.9: **The Gini coefficient and the coefficient of variance (CV) of in-degree distribution.** The Gini coefficient (circle) and the coefficient of variance (triangle) (CV) of in-degree distribution versus threshold. Both of these indices represent the strength of heterogeneity in in-degrees, where higher values mean stronger heterogeneity. Both the Gini coefficient and the CV decline with increasing threshold. Herein, the Gini coefficient G can be calculated as $G = \sum_{i,j=1}^{N} |k_i - k_j|/(2N^2\bar{k})$, where k_i is the in-degree of agent i and $\bar{k}$ is the mean in-degree (Cowell, 2011).

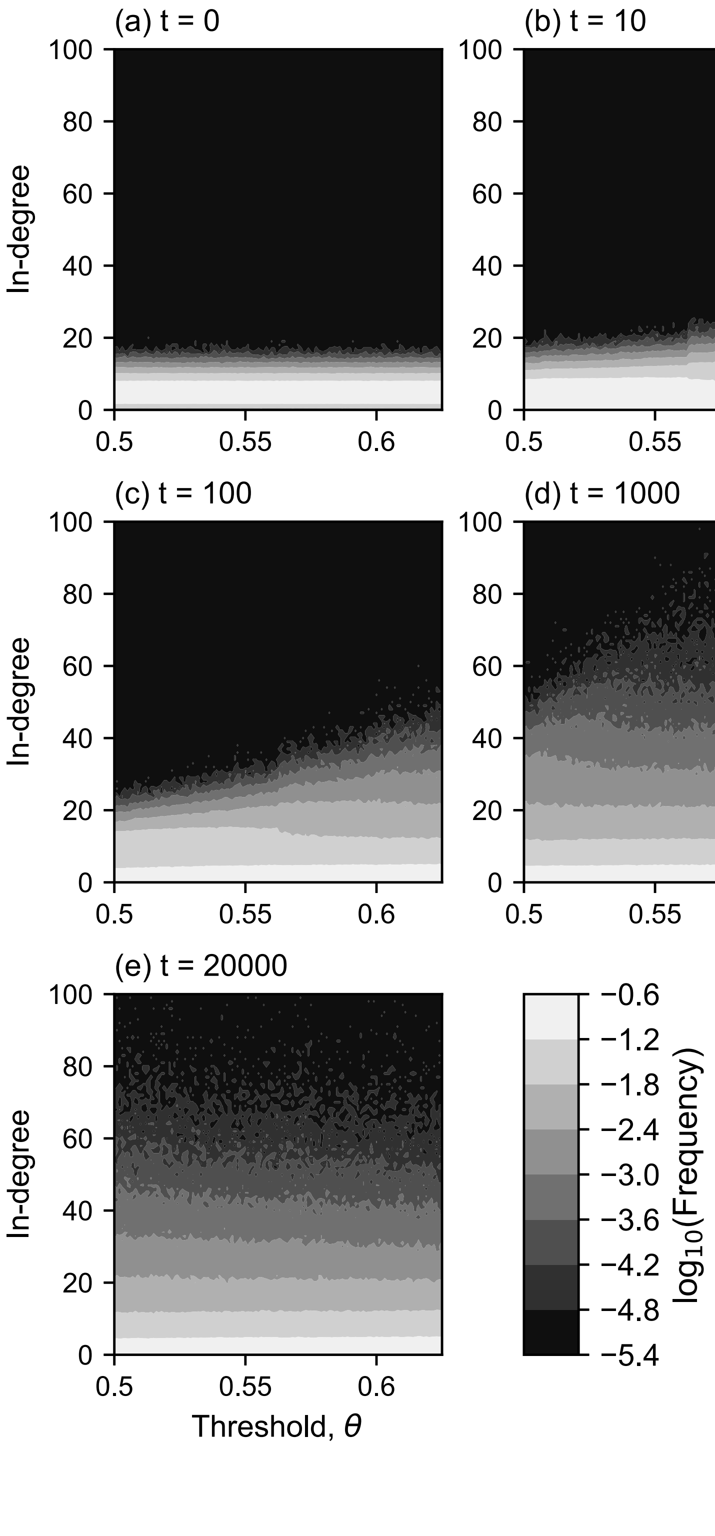

Figure 3.10: **The in-degree distributions for each threshold at the** $t = 0, 10, 1,000$ **and** $20,000(T_{\mathbf{end}})$**.** The in-degree distributions for each threshold at the random network ($t = 0$) and at times 10, 100, 1,000 and 20,000 (T_{end}) are shown in (a), (b), (c), (d), and (e), respectively. For each panel, the horizontal axis corresponds to the threshold, and the vertical axis represents the in-degree. The $\log_{10}(frequency)$ is shown by the gray scale, so when we see a vertical section at a threshold θ, we can see an in-degree distribution for the threshold θ, such as the one shown in Fig 2.2(b).

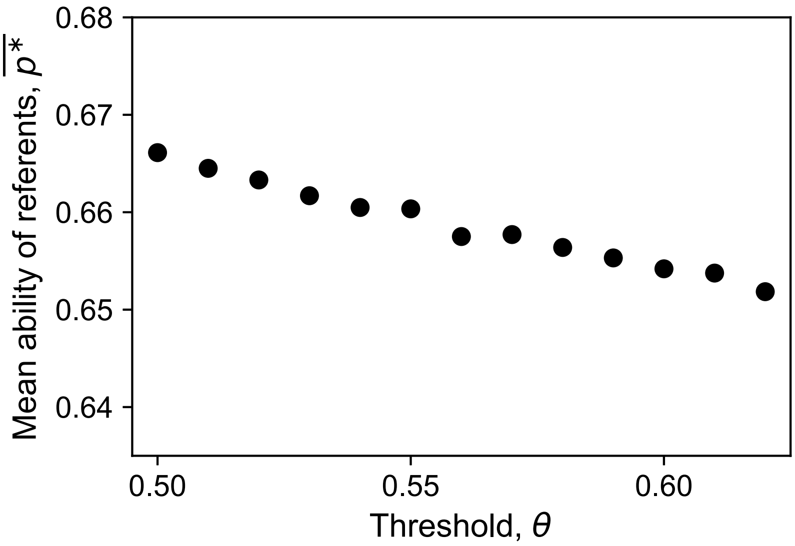

Figure 3.11: **The mean ability of referents in the self-organized network versus threshold.** The lower we set the threshold, the more the mean ability of referents increases.

Chapter 4

Discussion

4.1 Discussion on the self-organized reference structure

In this thesis, I have shown that the reference structure of agents who try to make correct answers by referring to credible agents self-organized into a heterogeneous structure with an exponential in-degree distribution (Albert and Barabási, 2002). The mean in-degree increased exponentially with ability. Therefore small difference in ability can lead to large difference in the number of followers in the self-organized network. My analytical calculation shows that it was the mean duration of an agent to be kept linked by a follower that increased exponentially with his/her performance. The performance-monitoring process in my model generated this nonlinear relationship between performance and mean duration.

I also looked at the performance of each agent and that of the group in the self-organized network and compared them to those in the random network. The mean performance of each agent and the mean group performance improved in the self-organized network through adaptive rewiring compared with the random network. However, the fluctuation of the group performance in the self-organized network was larger than the one in the random network. I discuss this trade-off later in this section.

In addition, I found that the threshold for rewiring, that is the extent of severity, affected the strength of heterogeneity in the in-degrees in the self-organized network. When we set the threshold lower, the heterogeneity in the in-degrees became larger, and at the same time, the dependence of an agent's mean in-degree on his/her ability was more exaggerated, i.e., agents refer more to higher ability agents in the self-organized network, and the mean ability of referents increases. This leads to a higher mean performance of each agent compared with when the threshold was larger, i.e., when the mean ability of referents was lower. Actually, in my derivation of the mean performance explained in Section 3.2 and Section A in Appendix, which predicts the simulation result well, I can show that the mean performance of each agent is an increasing function of the mean ability of referents (Section B in Appendix). However, it is a little against our intuition that agents result in referring to higher ability agents when we set the threshold lower (i.e., when they were more generous to their referents) than when we set it higher (when they were stricter regarding their referents). I interpret this counterintuitive phenomenon as follows. A lower rewiring threshold makes each agent more patient and lowers the desire to kick-off low-ability referents. However, at the same time, a lower threshold contributes to keeping high-ability referents more securely, because a lower rewiring threshold leads to a longer duration for referent-monitoring, leading to a better overall sorting of referent's quality. From my computer simulations, I find that the later effect is stronger. Therefore, in my model, a lower rewiring threshold contributes to generating a more biased reference toward high-ability agents. This result can be tested by an empirical study comparing the generosity of societies and their accuracy in decision-making. For example, we can compare a group in which rewiring occurs easily (that may correspond to a high threshold in my model) such as a group of individuals connected by a social network service, with a group in which rewiring is difficult (that may correspond to a low threshold in my model) such as a group of individuals in a company who are connected tightly, to examine which group can predict the next political leader more accurately.

As I showed so far, how long one can keep a follower greatly affects the structure of the self-organized network. The extent to which people in the society attach importance to the current result as compared to the referent's past is measured by the parameter α. Its reciprocal, $1/\alpha$, gives the mean time an individual remembers a success or a failure of its referent. Indeed, the change in the evaluated performance of the referent, y_t^{ij}, per each time step is proportional to α: $\Delta y_t^{ij} = y_{t+1}^{ij} - y_t^{ij} = \alpha(I_{t+1}^{j} - y_t^{ij})$. In the numerical simulations of this thesis, I set $\alpha = 0.1$. As α becomes larger, the agent's evaluation becomes less dependent on the past and more heavily dependent on the immediate success or failure. This makes the evaluation of followers' performance less reliable. Therefore, a larger α makes it difficult to sort subtle difference in performance between the referents, resulting in weaker centralization of links toward high ability agents and low performance. The effect of the kick-off threshold θ on group performance would also become less pronounced because of the less reliable performance evaluation. Conversely, if α becomes smaller, the evaluation for the performance of referents would become more reliable. However, this raises another problem for a society, because the time required for the referent network to reach an equilibrium, in other words, to acquire high centralization, would become too long. In fact, I confirmed those predictions on the effect of α by computer simulations for several values of α. The results are shown in Section F in Appendix.

There are several trade-offs in my model that affect the understanding of the quality of decision-making by agents who are interacting with one another. First, when we set a lower rewiring threshold, we have to wait longer until the network reaches the equilibrium state where agents have higher mean performance. Thus, we can see a kind of *speed–accuracy trade-off* here. Second, along with stepwise rises of the group performance from independent decision, to random references in the initial state, and then to the high-ability-agent-oriented self-organized networks, the SD of the group performance also increased, i.e., the fluctuation became larger in this order. When we set the threshold lower, we saw again an increase in both the mean and the SD of the group performance in the self-organized network. Therefore, an increase in both the mean and the "stability" (suppression of fluctuation) are difficult to be compatible. High-ability agents collect more followers in the self-organized network than in the initial network; the same is true for the self-organized network of a low threshold compared with that with a high threshold. Adaptive rewiring and a lower kick-off threshold level lead to higher mean performance. However, this is due to a more intense concentration of reference links to high ability agents (Section B in Appendix). This centralization seems to be the reason for the larger fluctuation of the group performance. The agents who attract many followers tend to be the agents with high ability and high performance. However, there are of course cases in which high-ability agents give wrong answers. In such an occasion of failure by agents of high influence, the group performance results in a very low value, which results in the group performance fluctuating wildly.

I have examined only a few types of distribution of agent's ability in the population, which gives the seeds for the generation of a heterogeneous in-degree distribution through adaptive rewiring. Actually, I assumed two types of ability distributions—one is in the current study, the uniform distribution, and the other is shown in Section E in Appendix. The density distribution of ability shown in Section E in Appendix is a linear decreasing function on the interval [0.5, 0.75]. Although both forms of ability distribution yielded exponential in-degree distributions against varying ability, the robustness of the results for the other forms of ability distributions should be tested in the future.

Lastly, I discuss possible modifications of my model. In my model, I assumed that a new link comes randomly regardless of his/her ability value. This was based on the idea that one cannot know the status of strangers —this may be true in some cases in our society. For example, in a population of analysts where a lead-follow relationship (references) exists, a financial analyst may not be able to evaluate the correctness of the analysts whom he/she is not directly following. In such situations, the only thing that an agent can do to improve his/her own performance is to replace an already connected referent who did not give correct answers, with a new referent randomly chosen from the population (Kossinets and Watts, 2006) as I assumed in my model. Actually, an empirical work on a social network in a university (Kossinets and Watts, 2006) shows that such global rewiring is commonly found in a group of individuals sharing the same interaction focus (in my case, making decisions for the same problem). However, it may also be possible to introduce "reputation" into my model; i.e., we may assume that the probability of being newly chosen as a referent depends on one's ability or performance, which is recognized by others in some way such as via reputation. I predict that, under this assumption, we will obtain a scale-free network, which represents strong heterogeneity. This prediction is supported by the following facts. There are a number of studies that explain how scale-free networks are constructed. The "good get richer" mechanism (or fitness model) is one such explanation (Garlaschelli et al., 2007; Zhou et al., 2011; Caldarelli et al., 2002). In the models using the "good get richer" mechanism, each agent is assigned a value, such as *fitness*, and the probability that one can obtain a link is determined based on the *fitness* value. In these models, strong heterogeneity with a power-law degree distribution emerges even if the fitness is not power-law distributed. The *fitness* in such models corresponds to the ability component in my model. Thus, I can predict that we will obtain a scale-free network if the probability of being newly chosen depends directly on one's ability or on one's performance. It is not clear whether a population can achieve high performance under a structure that self-organized in the presence of "reputation" and whether it has high heterogeneity and/or a strong opinion correlation.

In addition, I think that the following issue is worth considering in future. In my study, I assumed that all agents follow the same strategy for decision-making and have the same rewiring threshold. With these simple assumptions, I was able to reveal what the primarily factor leading to the centralization of reference networks is, and to discuss the decision accuracy in the self-organized reference structure. A possible next step would be to analyze the model that allows ability-dependent strategy for each agent, as higher ability agents may have less motivation for referring to others than lower ability agents. If so, the presence of such independent decision makers would improve the efficiency of collective intelligence in the population (Madirolas and de Polavieja, 2015).

4.2 General Discussion

As I explained in General Introduction, the reference structure among people affects the quality of collective intelligence in a group (King et al., 2012; Hofstra et al., 2015). Nonetheless, in many theoretical and experimental studies of collective intelligence, the reference structure has been given artificially. My study has therefore focused on how the reference structure emerges through the mutual interaction between agents who try to improve the accuracy of their decisions. I investigated this problem theoretically by using an adaptive network model. To my knowledge, there is no study that mathematically investigated the self-organization of reference structure among people making use of collective intelligence.

The model revealed that highly centralized reference structure among agents emerges when they try to make correct answers by referring to more accurate agents. In such a network the mean performance is high by virtue of collective intelligence and centralized references towards high ability agents. This means that the opinions of high ability agents are referred more and contributed to the high group performance, but also means that the opinions of agents can be highly correlated by sharing the same referents. The latter should deteriorate the quality of collective intelligence, as its advantage relies on independence between referents' opinions. The net effect of adaptive rewiring was still positive in my model: the agents had high mean performance by the majority-rule voting among their referents than without adaptive rewiring. Even the highest ability agent can improve one's mean performance by the majority-rule voting. However, this result is by no means general — the positive effect of adaptive rewiring on group performance is not always stronger than the negative effect of losing independence between opinions. For example, the group performance in adaptively rewired reference structure can be temporarily worse than under random references, if not worse in the long-term mean. Furthermore, when I replaced the random recruitment of a new referent assumed in previous chapters by the selective recruitment, I observed that the number of followers are power-law distributed in the self-organized network. In this selective recruitment, I assumed that the probability of an agent to be newly chosen as a referent depends on his/her own ability or performance. On this network, the centralization of reference towards high ability agents became much stronger than in the model that I mainly discussed in this dissertation and that assumed the random rewiring, and the mean performance in the self-organized network could be lower than that of the initial random network (data not shown).

My study casts a theoretical light on the reason why we observe incorrect information cascades in our society (Bikhchandani et al., 1992, 1998; Raafat et al., 2009). Humans are strongly influenced by opinions that have already been chosen by many people (Deutsch and Gerard, 1955; Raafat et al., 2009; Asch, 1956; Granovetter, 1978; Bikhchandani et al., 1992, 1998; Mori et al., 2012; Eguíluz et al., 2015). There are many theoretical models of information cascade considering this point (Granovetter, 1978; Watts, 2002; Bikhchandani et al., 1992, 1998; Mori et al., 2012; Eguíluz et al., 2015). In these models, and in my model too, one's decision for a binary question is influenced by the number of individuals who have already chosen each answer. In some of these models (Mori et al., 2012; Eguíluz et al., 2015), the extent to which each individual replaces one's own opinion by that of majority of all the previous statements depends on his/her confidence. Then, if the mean confidence is low, the individual decisions are influenced more by the previous statements, and information cascade occurs more frequently. In my model, whether an agent replaces one's own decision by his/her referents is determined by the majority-rule voting, where each vote by referents is equally weighted. There I also observed that the group performance fluctuates larger when each agent refers to his/her referents than when he/she does not refer anyone.

Information cascade would occurs more strongly when the society has high preference towards a limited number of members. Financial analysts and participants in experiments are also strongly influenced by the opinions stated by expertized people (Ramnath et al., 2008; Cooper et al., 2001; Kaiser et al., 2013; King et al., 2009). I found that the adaptive rewiring in my model brings initial random links without any preference to highly centralized preferences towards credible people (the agents with high abilities) – the individuals are to be strongly influenced by credible people in the evolved network. In this centralized reference structure, the risk of information cascade became stronger than in homogeneous random reference structure.

One may expect a better accuracy of an agent's decision under a stricter monitoring of their referents' quality. However, my model revealed a counter-intuitive relationship between the degree of generosity to referents in a society and the mean performance of the society. When I lower a minimum allowable threshold for the referent's performance before breaking the link (i.e. when individuals are more generous to their referents), individuals have a stronger tendency to refer to high ability agents in the self-organized reference structure than in the case of a higher threshold. Hence, the mean performance of the society is greater when the society is more generous to the referents. To my knowledge, there is no study on the decision accuracy in groups showing such a counter-intuitive phenomenon. To test this result empirically, we may compare a group in which the links are easily rewired with another group in which they are hardly rewired, and ask whether the latter group actually has a more strongly centralized reference structure. We may also ask whether the latter group actually makes more accurate predictions in the same binary questions. In my theoretical study, the lower threshold led to longer and more secure monitoring of referents' performance, and this made the stronger centralization towards high ability agents. I expect that this logical relationship will be tested by laboratory experiments where "*accurate information on relations (network structure) and behaviors (of individuals) can be recorded at every time-point*" (Corten and Buskens, 2010).

Majority-rule voting assumed in my model is only one of many possible decision-making rules that take the collective intelligence into account. Majority-rule voting has been assumed in my study because it has been well studied in the literature of collective intelligence for binary questions (Ladha, 1992; Nitzan, 2009; Grofman et al., 1983; Kao and Couzin, 2014), and assumed in many other models of opinion formation (Castellano et al., 2009; Sîrbu et al., 2017; Benczik et al., 2009). Majority-rule voting is indeed commonly seen in human society, as is exemplified by its ubiquitous adoption in political decision making processes (Hastie and Kameda, 2005). However, there are also other models to explain how humans incorporate others' opinions into one's decision-making for binary questions (Eguíluz et al., 2015; Behrens et al., 2008; Mori et al., 2012; Kurvers et al., 2014; Granovetter, 1978; Trimmer et al., 2011). These models employs a "soften" majority rule in the sense that the probability of adapting a choice is given as a sigmoidally increasing response function of the differential number of individuals who chose either of two answers. In these models, the more does a decision maker depend on oneself, the flatter is the response function and the less is the sensitivity to the majority of others on one's decision. It is an open question that how the degree of centralization and the tendency to cause information cascade will be changed if the model is extended to incorporate such self-confidence of an agent defined, for example, by the accuracy of his/her own answers in the past.

Another contribution of my study to the literature of collective intelligence is to have revealed that there is a trade-off between accuracy and stability in the self- organized reference networks. Which aspect is more preferable should depend on the objective of answering a problem and the demands in the society. It may be possible to consider the situation in which humans choose not only reference partners but also the extent to which they depend on social information – the optimal choice should then depend on the preferred balance between accuracy and stability. If individuals depend more on social information, they may be able to improve their performance on average. However, if individuals want to avoid getting involved in incorrect information cascade, they may have to inhibit too much use of social information. For example, if the group performance determines the growth rate of earnings of the group in a year, group members should have stronger motivation to avoid incorrect information cascade. This is because obtaining very low group performance in a single year thorough incorrect information cascade badly damages the mean growth rate over several years. The mean growth rate over several years is calculated as the geometric mean (not the arithmetic mean) of growth rates of the years, and therefore large fluctuation in the growth rates severely decreases their geometric mean, as has been shown in the classical theories of life history evolution in changing environments (Seger and Brockmann, 1987). On the other hand, restricted dependence on social information started as a fear for information cascade may let the benefit of collective intelligence slip by, further demoting their use. It therefore remains an interesting open question that how the self-organization of reference structure of people evolves under different situations faced by the society.

I mentioned in General Introduction two motivations of conformity proposed so far: normative and informational social influence (Deutsch and Gerard, 1955). In my study, I assumed that people use social information to make their opinions more accurate. I therefore focused on informational social influence, the influence to conformity that stems from one's desire to obtain the evidence for one's decision. On the other hand, normative social influence comes from one's hope to respond to others' expectation, for example, "*in order to avoid being ridiculed, or being negatively evaluated, or even possibly out of a sense of obligation*" (Deutsch and Gerard, 1955). These two motivations are thought to be equally important in human society. Therefore, humans do not always use social information to optimize one's accuracy. For example, Mahmoodi et al. (2015) suggested by their experiment that people weight their own opinions against those of pair-partners nearly equally in their opinion aggregation.

They observed the equality bias even when the equal weight is not optimal and even when the subjects received the feedback about the difference of their own accuracy. The authors pointed out that diffusing the responsibility and avoiding social exclusion can be considered as a part of the reasons of the equality bias. I think it is important to also consider such motivations for the use of social information when we study the self-organization of reference structure and the decision accuracy of people in the structure.

Appendix A

The derivation of the mean performance

Derivation of mean performance

The mean performance of an agent can be described as a linear function of his/her ability as I explained in Section 3.2 in the main text. Here I explain the derivation of the approximate formula for the slope and the intercept of the linear function. In this derivation, I neglected self-loops or overlaps in reference links to simplify my approximation.

In my model, agents make their decisions sequentially, so each agent actually incorporates answers of only referents who have already stated the answer (let me call them "stated referents") among M agents to which he/she links (out-degree$\equiv M$). Therefore, in sequential decision-making, agents are divided into $M+1$ classes: class C_0 of agents who make decisions independently by themselves, class C_1 of agents who put the answer of a referent together with his/her own choice, and so on. Let C_s be the class of agents who put the answers of s referents together with their own choice. There are on average

$$c_s = \sum_{l=s+1}^{N} \binom{M}{s} \left(\frac{l-1}{N}\right)^s \left(1 - \frac{l-1}{N}\right)^{M-s} \tag{A.1}$$

agents in such class as shown below. Suppose that the agent is the l-th earliest to make the decision (l can be chosen from $s+1$ to N with equal probability). Given that the agent is the l-th earliest, there are $l-1$ agents who have already stated their answers. Thus, the number of stated referents of the l-th earliest agent follows the binomial distribution with parameters M and $(l-1)/N$, since agents state their answers in a randomly determined order. This leads to the expression (A.1) for the mean number of agents who belong to class C_s.

To derive an approximation formula for the mean performance, I assumed that all the agents who made their decisions the earliest to the c_0-th earliest belong to class C_0, i.e., they had no stated referent in making their decision; hence, they decided relying only on their own belief. Similarly, all the agents who made the $(c_{s-1}+1)$-th earliest to the c_s-th earliest are assumed to belong to class C_s, $(s = 1, 2, \ldots, M)$, i.e., put the answers of s referents together with their own choices in making their decisions. Here, I further assume that s referents of an agent of class C_s are randomly chosen from the agents of class C_0, C_1, $\ldots$, C_{s-1} in proportion to $c_0 : c_1 : \cdots : c_{s-1}$.

Let $\pi_s(p)$ be the mean performance of an agent who has ability p and belongs to class C_s. Clearly, $\pi_0(p) = p$. Let r_s^* be a random variable that represents the performance of a referent who is being referred by the agents in class C_s. Let me consider an agent who is randomly sampled from the population, say agent i. I approximated the probability that a referent of agent i has ability p as $g(p) \equiv \bar{k}(p)/(NM)$ regardless of the number of followers of the agent, where $\bar{k}(p)$ is the mean in-degree of the agent with ability p, as explained in Section 3.1 in the main text (Eq. (3.1)). Under this assumption, let p^* be a random variable that represents a referent's ability. Thus, I assumed that the ability of a referent p^* follows the distribution g. Using the above simplifying assumptions, r_1^* can be approximated as $r_1^* \approx \pi_0(p^*) = p^*$ since r_1^* is the performance of an agent in class C_0. Furthermore, I approximated the value of $r_1^* = p^*$ by its mean, $\overline{p^*} \equiv \mathrm{E}[p^*] \approx \sum_{i=1}^{N} p_i g(p_i) = \sum_{i=1}^{N} p_i \bar{k}(p_i)/(NM)$.

The performance of an agent in class C_1 can be described as

$$\pi_1(p) = pr_1^* + \frac{1}{2}\left[p(1 - r_1^*) + (1 - p)r_1^*\right] \tag{A.2}$$

$$= \frac{p + r_1^*}{2}. \tag{A.2$'$}$$

The first term on the right-hand side of (A.2) stands for the contribution to the mean performance when both the referent and the agent him/herself gave correct answers. The second term is the contribution when either the referent or the agent him/herself gave a correct answer (the two different opinions tie in this case, and the actual decision is made by tossing a coin; hence, the factor 1/2). Similarly, $\pi_1(p)$ can be approximated as follows:

$$\begin{aligned}\pi_1(p) &= \frac{p + r_1^*}{2} \approx \frac{p + p^*}{2} \\ &\approx \frac{p + \overline{p^*}}{2}.\end{aligned} \tag{A.3}$$

Now, I consider the mean performance of referents who are being referred by the agents in class C_2, $r_2 \equiv \overline{r_2^*}$, as follows. Under my assumption, an agent in class C_2 refers to an agent in class C_0 with a probability of $c_0/(c_0 + c_1)$ and refers to an agent in class C_1 with a probability of $c_1/(c_0 + c_1)$. Thus,

$$r_2^* = \begin{cases} \pi_0(p^*), \text{ with probability } \dfrac{c_0}{c_0 + c_1}, \\ \pi_1(p^*), \text{ with probability } \dfrac{c_1}{c_0 + c_1}. \end{cases} \tag{A.4}$$

Since $\pi_0(p) = p$, and by equation (A.3), functions π_0 and π_1 can be assumed as linear functions—I can see that $\pi_0(p)$ and $\pi_1(p)$ also increase nearly linearly with ability p in my computer simulation (Fig.A.1). Therefore, the mean performance of referents who are being referred by the agents in class C_2, r_2, can be approximated as

$$\begin{aligned} r_2 &\approx \mathrm{E}\left[\frac{c_0}{c_0 + c_1}\pi_0(p^*) + \frac{c_1}{c_0 + c_1}\pi_1(p^*)\right] \\ &= \frac{c_0}{c_0 + c_1}\mathrm{E}[\pi_0(p^*)] + \frac{c_1}{c_0 + c_1}\mathrm{E}[\pi_1(p^*)] \\ &\approx \frac{c_0}{c_0 + c_1}\pi_0(\overline{p^*}) + \frac{c_1}{c_0 + c_1}\pi_1(\overline{p^*}), \end{aligned} \tag{A.5}$$

where $\pi_0(p) = p$ and $\pi_1(p) = (p + \overline{p^*})/2$. Therefore,

$$r_2 \approx \frac{c_0}{c_0 + c_1}\overline{p^*} + \frac{c_1}{c_0 + c_1}\frac{\overline{p^*} + \overline{p^*}}{2} = \overline{p^*}. \tag{A.6}$$

Similarly, $\pi_2(p)$, r_3, $\pi_3(p)$, $\ldots$, r_M, and $\pi_M(p)$ can be derived sequentially as follows: Let $Q(s,j)$ be the probability that j out of s stated referents of an agent in class C_s give correct answers, as

$$Q(s,j) = \binom{s}{j} r_s^j (1 - r_s)^{s-j}. \tag{A.7}$$

When the number of stated referents of an agent is even, there are $s + 1$ answers/choice including his/her own choice. The agent can give a correct answer by majority-rule when more than $s/2$ answers/choice are correct. Therefore, the mean performance of an agent with ability p in class C_s, $\pi_s(p)$, can be described as

$$\begin{aligned}\pi_s(p) &= p \sum_{j+1 \geq s/2+1} Q(s,j) + (1-p) \sum_{j \geq s/2+1} Q(s,j) \\ &= p \sum_{j=s/2}^{s} Q(s,j) + (1-p) \sum_{j=s/2+1}^{s} Q(s,j) \\ &= \sum_{j=s/2+1}^{s} Q(s,j) + pQ\left(s, \frac{s}{2}\right),\end{aligned} \tag{A.8}$$

when s is even. Similarly, when s is odd,

$$\begin{aligned}
\pi_s(p) &= p \sum_{j+1 \geq (s+1)/2+1} Q(s,j) + (1-p) \sum_{j \geq (s+1)/2+1} Q(s,j) \\
&\quad + \frac{1}{2}\left[pQ(s, \frac{s-1}{2}) + (1-p)Q(s, \frac{s+1}{2}) \right] \\
&= \sum_{j=(s+3)/2}^{s} Q(s,j) + \frac{1}{2}Q(s, \frac{s+1}{2}) + \frac{1}{2}\left[Q(s, \frac{s-1}{2}) + Q(s, \frac{s+1}{2}) \right] p. \qquad \text{(A.9)}
\end{aligned}$$

The mean performance of referents who are being referred by the agents in class C_s, r_s, can be derived as

$$\begin{aligned}
r_s &= \frac{\sum_{j=1}^{s-2} c_j}{\sum_{j=1}^{s-1} c_j} r_{s-1} + \frac{c_{s-1}}{\sum_{j=1}^{s-1} c_j} \mathrm{E}[\pi_{s-1}(p^*)] \\
&= \frac{\sum_{j=1}^{s-2} c_j}{\sum_{j=1}^{s-1} c_j} r_{s-1} + \frac{c_{s-1}}{\sum_{j=1}^{s-1} c_j} \pi_{s-1}(\overline{p^*}). \qquad \text{(A.10)}
\end{aligned}$$

In equation (A.10), $\mathrm{E}[\pi_{s-1}(p^*)] = \pi_{s-1}(\mathrm{E}[p^*]) = \pi_{s-1}(\overline{p^*})$ holds because $\pi_{s-1}(p)$ can be inductively assumed to be a linear function of p for any s according to Equations (A.7)–(A.10), and from Fig. A.1—r_s is not a function of p but a function of $\overline{p^*} = \mathrm{E}[p^*] \approx \sum_{i=1}^{N} p_i g(p_i)$. Therefore, the mean performance of agents in class C_s can be represented as

$$\pi_s(p) = A_{g,s} p + B_{g,s}, \qquad \text{(A.11)}$$

where $A_{g,s}$ and $B_{g,s}$ do not depend on p.

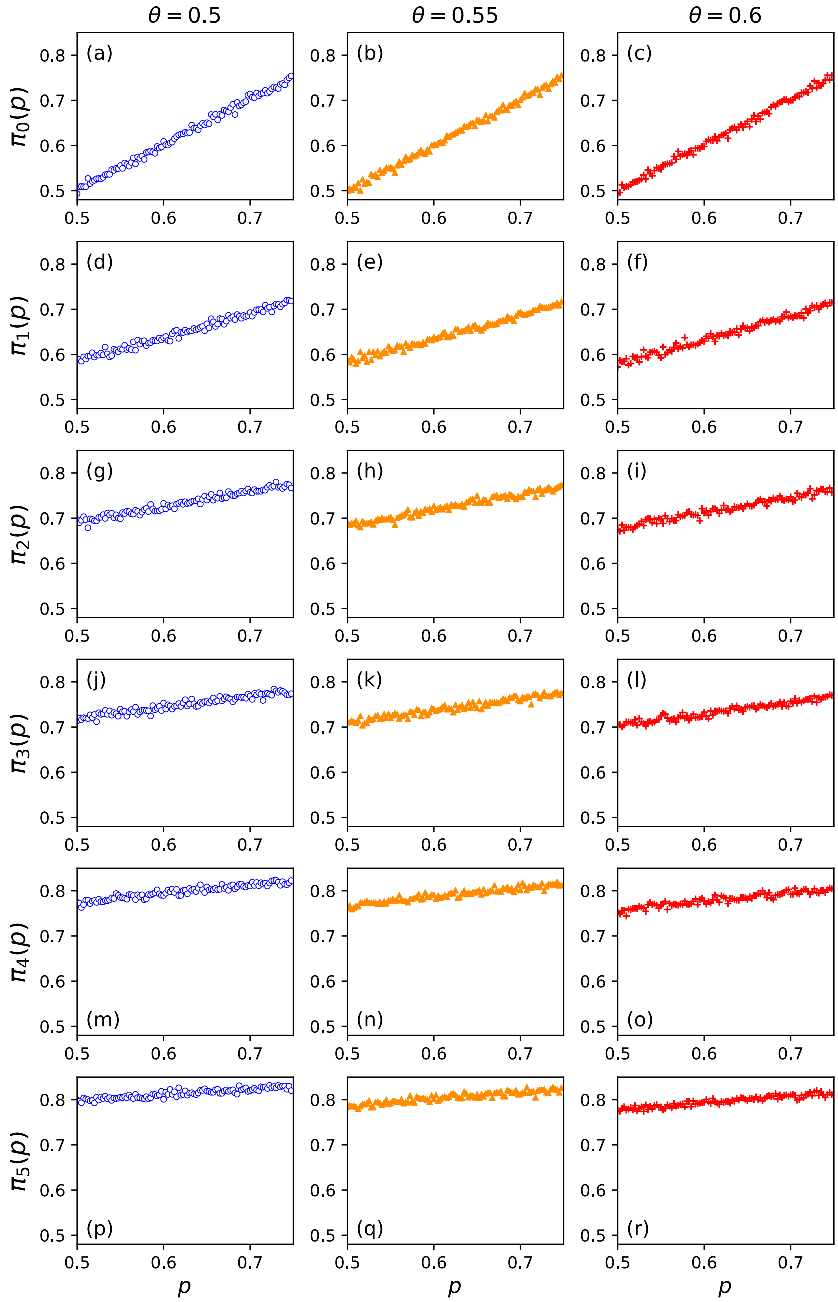

Figure A.1: The mean performance of an agent in class C_s, $\pi_s(p)$ versus his/her ability p are shown for $s = 0, 1, \ldots, 5$ and for rewiring thresholds $\theta = 0.5$, 0.55 and 0.6. Figures in the first, second, third, forth, fifth and sixth rows show the case where the number of stated referents s are 0, 1, 2, 3, 4 and 5, respectively. Figures in the first, second and third columns show the case where the threshold θ are 0.5, 0.55 and 0.6, respectively. In each panel, we can see the linear increase of the mean performance with the ability.

The mean performance depends linearly on the agent's ability.

Finally, the mean performance $\pi(p)$ of an agent with ability p can be approximated as

$$\pi(p) = \frac{\sum_{s=0}^{M} c_s \pi_s(p)}{\sum_{s=0}^{M} c_s}. \tag{A.12}$$

Since $\pi_{s-1}(p)$ is a linear function of p for each s, $\pi(p)$ is also a linear function of p and can be represented as

$$\pi(p) = A_g p + B_g. \tag{A.13}$$

The mean performance calculated by equation (A.13) agrees with the simulation results shown in Fig. 5 (a).

The linearity of mean performance when self-loops and overlaps are allowed

Here, I show that the mean performance can be described as a linear function of the ability even if there are self-loops and overlaps in reference links. First, I consider the case where there are $a_{ii}(\geq 1)$ self-loops of an agent i, where I regard that agent i's choice has a weight of $(a_{ii} + 1)$ to him/herself in i's majority-rule voting. I can calculate the mean performance of agent i as follows. Let s be a_{ii} plus the number of the agent i's stated referents other than agent i. Let $\{d_0, d_1, \ldots, d_{a_{ii}}, \ldots, d_s\}$ be a set of choices and answers, where d_0 represents the choice of agent i him/herself and each of $d_1, \ldots, d_{a_{ii}}$ represents the choice of agent i him/herself relating to the a_{ii} self-loops ($d_1 = d_2 = \cdots = d_{a_{ii}} = d_0$ by definition). Each of $d_{a_{ii}+1}, \ldots, d_s$ represents the answer of each stated referent of agent i. In this case, $(d_0, d_1, \ldots, d_{a_{ii}}) = (1, 1, \ldots, 1)$ with a probability of p_i and $(d_0, d_1, \ldots, d_{a_{ii}}) = (0, 0, \ldots, 0)$ with a probability of $1 - p_i$. From equation (A.8), the probability that agent i makes a correct answer, $\pi_s(p_i)$, is,

$$\begin{aligned} \pi_s(p_i) = \; & p_i \mathrm{Pr.}\Bigg[\left(\frac{(s+1)+1}{2} - (a_{ii}+1)\right) \text{stated referents} \\ & \qquad \text{other than agent } i \text{ gave correct answers}\Bigg] \\ & +(1-p_i)\mathrm{Pr.}\Bigg[\left(\frac{(s+1)+1}{2}\right) \text{stated referents} \\ & \qquad \text{other than agent } i \text{ gave correct answers}\Bigg], \end{aligned} \tag{A.14}$$

when s is even. Note that $\pi_s(p_i)$ shown above is a linear function of p_i. Using the case that s is odd in Equation (A.9), I can again describe $\pi_s(p_i)$ as a linear function of p_i when s is odd. Similarly, I can confirm that the mean performance of an agent can be described as a linear function of the ability even when there are overlaps in reference links.

Appendix B

The relationship between the mean performance and the mean ability of referents

In Section A, I derived the approximation formula for the mean performance $\pi(p)$ of an agent with ability p, which is expressed in terms of the mean ability of referents $\overline{p^*} = \sum_{i=1}^{N} p_i \bar{k}(p_i)/(NM)$.

As discussed in the main text, adaptive rewiring and a lower kick-off threshold lead to both a high mean ability of referents $\overline{p^*}$ (FIG. 12) and a high mean performance of each agent (FIG. 9). In this section, I show that the mean performance of each agent increases with the mean ability of referents according to the formula for the mean performance that I obtained in Section A. We can, therefore, say that adaptive rewiring and a lower kick-off increase the mean ability of referents, and this leads to a high mean performance of each agent.

I will show that $\partial\pi(p)/\partial\overline{p^*} > 0$ for all p and $\overline{p^*}$ with $0 \le p \le 1$ and $0 \le \overline{p^*} \le 1$.

Proof. From the Equation (A.12) in Section A,

$$\frac{\partial\pi(p)}{\partial\overline{p^*}} = \frac{\sum_{i=0}^{M} c_s \partial\pi_s(p)/\partial\overline{p^*}}{\sum_{s=0}^{M} c_s}. \tag{B.1}$$

I show below that $\partial\pi_0(p)/\partial\overline{p^*} = 0$ and $\partial\pi_s(p)/\partial\overline{p^*} > 0$ for $s \ge 1$. Clearly $\partial\pi_0(p)/\partial\overline{p^*} = 0$ as $\pi_0(p) = p$, and $\partial\pi_1(p)/\partial\overline{p^*} = 1/2 > 0$ since $\pi_1(p) = (\overline{p^*} + p)/2$. For $s \ge 2$, I show both $\partial r_s/\partial\overline{p^*} > 0$ and $\partial\pi_s/\partial r_s > 0$ since $\partial\pi_s(p)/\partial\overline{p^*} = (\partial r_s/\partial\overline{p^*})(\partial\pi_s/\partial r_s)$.

First, I show $\partial\pi_s/\partial r_s > 0$. When s is even and $s \ge 2$, by equations (A.8) and (B.1),

$$\frac{\partial\pi(p)}{\partial\overline{p^*}} = \sum_{j=s/2+1}^{s} \frac{\partial Q(s,j)}{\partial r_s} + p\frac{\partial Q\,(s, s/2)}{\partial r_s}, \tag{B.2}$$

where $\partial Q(s,j)/\partial r_s = \binom{s}{j}[jr_s^{j-1}(1-r_s)^{s-j} - (s-j)r_s^j(1-r_s)^{s-j-1}]$. Using the fact that $\binom{m}{n}(m-n) - \binom{m}{n+1}(n+1) = 0$ for any integers m and n with $m > n \ge 0$,

$$\begin{aligned}\frac{\partial\pi(p)}{\partial\overline{p^*}} &= r_s^{s/2-1}(1-r_s)^{s/2-1} \\ &\quad\times\left[\binom{s}{s/2-1}\left(\frac{s}{2}+1\right)r_s + p\binom{s}{s/2}\frac{s}{2}(1-r_s) - p\binom{s}{s/2}\frac{s}{2}r_s\right] \qquad \text{(B.3)}\\ &= r_s^{s/2-1}(1-r_s)^{s/2-1}\frac{s!}{(s/2-1)!(s/2)!}\left[(1-r_s)p + r_s(1-p)\right] > 0. \qquad \text{(B.3}'\text{)}\end{aligned}$$

Similarly, when s is odd and $s \geq 3$,

$$\begin{aligned}\frac{\partial \pi_s(p)}{\partial r_s} &= \frac{1}{2}\binom{s}{(s-1)/2}\frac{s+1}{2}r_s^{(s-1)/2}(1-r_s)^{(s-1)/2} \\ &+ \frac{1}{2}\frac{s!}{[(s-3)/2]![(s+1)/2]!}r_s^{(s-3)/2}(1-r_s)^{(s-3)/2}\left[(r_s-p)^2+p(1-p)\right] > 0. \quad \text{(B.4)}\end{aligned}$$

Therefore, $\partial \pi_s/\partial r_s > 0$ for $s \geq 2$.

Secondly, I show $\partial r_s/\partial \overline{p^*} > 0$ for $s \geq 2$. By equation (A.10) and (A.11),

$$\frac{\partial r_s}{\partial \overline{p^*}} = \frac{\sum_{j=1}^{s-2} c_j}{\sum_{j=1}^{s-1} c_j}\frac{\partial r_{s-1}}{\partial \overline{p^*}} + \frac{c_{s-1}}{\sum_{j=1}^{s-1} c_j}A_{g,s}. \quad \text{(B.5)}$$

Since $r_1 = r_2 = \overline{p^*}$, as explained in Section A, $\partial r_1/\partial \overline{p^*} = \partial r_2/\partial \overline{p^*} = 1 > 0$. Thus, inductively, $\partial r_s/\partial \overline{p^*} > 0$ for $s \geq 2$, according to Equation (B.5). $\square$

Appendix C

The formal derivation of the mean in-degree

Here, I assume that the number of agents N is infinitely large, and p is a continuous value. Let $\tilde{g}(p)$ be the probability density function for the ability of an agent who is being referred to in the self-organized network. In addition, I define $f(\Pi)$ as the probability density function of the performance of the agent who is being referred to in the self-organized network. $\psi(p)$ and $\phi(\Pi)$ respectively denote the unconditional probability density functions of the ability and performance of the agents, who are either being referred to or not. Let $\overline{T_\Pi}$ be the mean duration that the agent with performance Π is kept linked by a follower.

Let $f_t(\Pi)$ be the probability density function for the performance of the agent who is being referred to at iteration time t. The function $f_t(\Pi)$ satisfies the following equation by assuming that a link directing to an agent with performance Π is detached with a probability of $1/\overline{T_\Pi}$ in a unit time interval:

$$f_{t+1}(\Pi) = \left(1 - \frac{1}{\bar{T}_\Pi}\right) f_t(\Pi) + \phi(\Pi) \int_0^1 f_t(\Pi') \frac{1}{\bar{T}_{\Pi'}} d\Pi'. \tag{C.1}$$

The first term on the right-hand side of equation (C.1) corresponds to the probability that a reference link to an agent with performance Π remains without being rewired in a unit time interval. The second term corresponds to the probability that a link is newly rewired to an agent with performance Π after it is discarded. Therefore, in the equilibrium state, the probability density function for the performance of the agent who is being referred to in the self-organized network, $f(\Pi)$, holds:

$$f(\Pi) = \left(1 - \frac{1}{\bar{T}_\Pi}\right) f(\Pi) + \phi(\Pi) \int_0^1 f(\Pi') \frac{1}{\bar{T}_{\Pi'}} d\Pi'. \tag{C.2}$$

Equation (C.2) can be calculated as follows.

$$0 = -\frac{f(\Pi)}{\bar{T}_\Pi} + \phi(\Pi) \int_0^1 \frac{f(\Pi')}{\bar{T}_{\Pi'}} d\Pi',$$

and hence,

$$f(\Pi) = \frac{\bar{T}_\Pi \phi(\Pi)}{\int_0^1 \bar{T}_{\Pi'} \phi(\Pi') d\Pi'}. \tag{C.3}$$

Since $\Pi(p) = A_g p + B_g$ from Equation (A.13) in Section A, $f(\Pi) = \tilde{g}\big((\Pi - B_g)/A_g\big)/A_g$ and $\phi(\Pi) = \psi\big((\Pi - B_g)/A_g\big)/A_g$ are satisfied. Therefore,

$$\tilde{g}(p) = \frac{\bar{T}_{Ap+B}\psi(p)}{\int_0^1 \bar{T}_{Ap'+B}\psi(p')dp'}. \tag{C.4}$$

In the main text, I set $\psi(p)$ as

$$\psi(p) = \begin{cases} 4, & 0.5 \leq p \leq 0.75, \\ 0, & \text{otherwise.} \end{cases} \tag{C.5}$$

i.e., p follows the uniform distribution U(0.5, 0.75). For the finite number of agents N, I can approximate the probability $g(p_i)$ that agent i with ability p_i is being referred from an agent as $g(p_i) \approx \int_{p_i}^{p_{i+1}} \tilde{g}(p')dp'$. Thus the solid lines in FIG. 7 (b) are calculated as

$$\begin{aligned}
\bar{k}(p_i) =& NMg(p_i) \\
\approx& NM \int_{p_i}^{p_{i+1}} \tilde{g}(p')dp' = NM \int_{p_i}^{p_{i+1}} \frac{\bar{T}_{Ap+B}}{\int_0^1 \bar{T}_{Ap'+B}dp'}dp \\
\approx& NM \frac{\bar{T}_{Ap_i+B}}{\sum_{j=1}^{N} \bar{T}_{Ap_j+B}}.
\end{aligned} \tag{C.6}$$

Appendix D

The numerical procedure to obtain the mean duration that an agent keeps a follower

In this section, I explain how I solve the recurrence equations (3.4) and (3.5) in Section 3.3 in the main text,

$$T_{\Pi}(y) = 1 + \Pi T_{\Pi}\big(\alpha + (1-\alpha)y\big) + (1-\Pi)T_{\Pi}\big((1-\alpha)y\big), \quad y > \theta, \tag{D.1}$$

and

$$T_{\Pi}(y) = 0, \quad y \leq \theta, \tag{D.2}$$

for the recurrence of $T_\Pi(y)$. Here $T_\Pi(y)$ represents the mean time until the evaluated performance Y_t of a referent, whose actual performance is Π, hits the threshold θ first time in the stochastic process $\{Y_t \mid Y_0 = y\}$, where t is the time since it is linked by a follower.

Note that the evaluated performance Y_t is always less than 1, because the right side of (1) in the main text represents the internally dividing point of I_t, which is either 0 or 1, and the current value of Y_t. Let b be $(1-\alpha)\theta$, which is the infimum of the realization of Y_t because the evaluated performance is updated to $(1-\alpha)\theta$ when a referent with its evaluated performance θ gives a wrong answer. I discretized the interval $[b,1](\subset \mathbb{R})$ as $B \equiv \{b_0, b_1, \ldots, b_S\}$, where $b_i = b + i\delta$ and $S = [(1-b)/\delta]$, and consider the recurrence equations (D.1) and (D.2) on B, as follows. Here I set δ sufficiently small as $\delta = 0.0001$ in my actual numerical calculation and $[\,\cdot\,]$ represents the Gauss' symbol.

I defined four maps, $\mathcal{I}$, $\mathcal{R}$, $\mathcal{U}$ and $\mathcal{D}$ as follows.

$$\mathcal{I}(y) \equiv [(y-b)/\delta], \tag{D.3}$$

with which $i = \mathcal{I}(b_i)$, representing the discretization of the interval $[b,1]$, and its inverse

$$\mathcal{R}(j) \equiv b + \delta j, \tag{D.4}$$

with which $b_i = \mathcal{R}(i)$,

$$\mathcal{U}(y) \equiv (1-\alpha)y + \alpha, \tag{D.5}$$

and

$$\mathcal{D}(y) \equiv (1-\alpha)y. \tag{D.6}$$

The map $\mathcal{U}$ ($\mathcal{D}$) corresponds to the case that the referent gives a correct (wrong) answer and the evaluated performance Y_t changes better (worse). Therefore, the function $\mathcal{I} \circ \mathcal{U} \circ \mathcal{R}$ ($\mathcal{I} \circ \mathcal{D} \circ \mathcal{R}$) means the change in the discretized evaluated performance when the referent gives a correct (wrong) answer, where $\circ$ means composition of functions.

Let $\boldsymbol{x} = (x_i)$ be a $(S+1)$-dimensional vector, where $x_i = T_\Pi(b_i)$. With this definition, x_i gives the mean first hitting time when $Y_0 = y = b + i\delta$, to the threshold θ of the stochastic process $\{Y_t \mid Y_0 = b + i\delta\}$. The recurrence equations (D.1) and (D.2) for discretized state space in the self-organized network are then expressed as

$$\boldsymbol{x} = A\boldsymbol{x} + \mathbf{1}, \tag{D.7}$$

where $A = (a_{ij})_{i,j=0,\ldots,S}$ is an $(S+1) \times (S+1)$ matrix where the first $\mathcal{I}(\theta) + 1$ rows of A are zero vectors,

$$a_{ij} = 0, \qquad i = 0, 1, ..., \mathcal{I}(\theta); \;\; j = 0, 1, \ldots, S. \tag{D.8}$$

From the $i = \mathcal{I}(\theta) + 1$st to $i = \mathcal{I}\big(\theta/(1-\alpha)\big)$th rows of A are vectors in which only $(i, \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i))$-element is non zero value, Π,

$$a_{ij} = \begin{cases} \Pi, & i = \mathcal{I}(\theta) + 1, \ldots, \mathcal{I}\big(\theta/(1-\alpha)\big), \ \ j = \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i), \\ \\ 0, & i = \mathcal{I}(\theta) + 1, \ldots, \mathcal{I}\big(\theta/(1-\alpha)\big), \ \ j \neq \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i). \end{cases} \tag{D.9}$$

From the $i = \mathcal{I}\big(\theta/(1-\alpha)\big) + 1$st to Sth rows of A are vectors in which only $(i, \mathcal{I} \circ \mathcal{D} \circ \mathcal{R}(i))$-element and $(i, \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i))$-element are non zero values, $1 - \Pi$ and Π, respectively,

$$a_{ij} = \begin{cases} 1 - \Pi, & i = \mathcal{I}(\theta) + 1, \ldots, S, \ \ j = \mathcal{I} \circ \mathcal{D} \circ \mathcal{R}(i), \\ \\ \Pi, & i = \mathcal{I}(\theta) + 1, \ldots, S, \ \ j = \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i), \\ \\ 0, & i = \mathcal{I}(\theta) + 1, \ldots, S, \ \ j \neq \mathcal{I} \circ \mathcal{D} \circ \mathcal{R}(i), \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i). \end{cases} \tag{D.10}$$

and $\mathbf{1} = (1_i)_{i=0,\ldots,S}$ is an $(S+1)$-dimensional vector with

$$1_i = \begin{cases} 0, & i = 0, 1, \ldots, \mathcal{I}(\theta), \\ \\ 1, & i = \mathcal{I}(\theta) + 1, \ldots, S. \end{cases} \tag{D.11}$$

By equation (D.2), the first $\mathcal{I}(\theta)+1$ elements in the vector $\mathbf{1}$ are 0, and the first $\mathcal{I}(\theta)+1$ rows in the matrix A are zero vectors. Since the third term $(1-\Pi)T_\Pi\big((1-\alpha)y\big)(= (1-\Pi)T_\Pi\big(\mathcal{D}(y)\big))$ in the right side of equation (D.1) does not vanish when the initial evaluated performance y satisfies $(1-\alpha)y > \theta$ (i.e. $y > \theta/(1-\alpha)$), the $\big(i,\ \mathcal{I} \circ \mathcal{D} \circ \mathcal{R}(i)\big)$-element of the matrix A is $1-\Pi$

for $i > \mathcal{I}(\theta/(1-\alpha))$. The second term $\Pi T_{\Pi}(\mathcal{U}(y))$ in the right side of equation (D.1) does not vanish when the initial evaluated performance y satisfies $y > \theta$. Thus the $(i,\, \mathcal{I} \circ \mathcal{U} \circ \mathcal{R}(i))$-element of the matrix A is Π for $i > \mathcal{I}(\theta)$.

The equation (D.7) can be solved as

$$\boldsymbol{x} = (E - A)^{-1}\mathbf{1}, \tag{D.12}$$

where E represents the identity matrix.

The $\mathcal{I}(y_0)$-th element in the vector $\boldsymbol{x}$ is the duration that an agent with the performance Π is kept linked from a follower, $\overline{T}$, for the initial evaluated performance $Y_0 = y_0$.

Appendix E

Another distribution of ability

To check the validity of my way to derive $g(p)$, I apply it to another probability density function (p.d.f) of agent's ability. The applied p.d.f. has a saw-toothed shape with the vertical tip at $p = 0.5$, declining linearly with p towards zero at $p = 0.75$:

$$\tilde{\psi}(p) = \begin{cases} 24 - 32p, & 0.5 \le p \le 0.75, \\ 0, & \text{otherwise.} \end{cases} \tag{E.1}$$

I calculated the p.d.f. of the ability of being referred agents $\tilde{g}(p) = \tilde{\psi}(p)\bar{T}_{Ap+B}/\int_0^1 \tilde{\psi}(p')\bar{T}_{Ap'+B}dp'$ as the same way for $\psi(p)$. To compare with the calculation results, I conducted simulation under parameters $N = 100$, $M = 5$, $y_0 = 0.625$ and $\alpha = 0.1$. I set the ability of agents as $p_i = (3 - \sqrt{1 - i/N})/4$, $i = 0, 1, 2, \ldots$, since $a = (3 - \sqrt{1-u})/4$ follows the equation (E.1), where u is a random variable which follows the uniform distribution in $[0, 1]$. The simulation data of the mean in-degree $\bar{k}(p_i)$ of an agent with the ability p_i agrees with $NM \int_{p_i}^{p_{i+1}} \tilde{g}(p')dp'$, see Fig. E.1. Note that I obtained the exponential tailed in-degree distribution again with this p.d.f. $\tilde{\psi}$ (Fig. E.2). As shown in Figure E.1, the mean in-degree also increases exponentially with the ability.

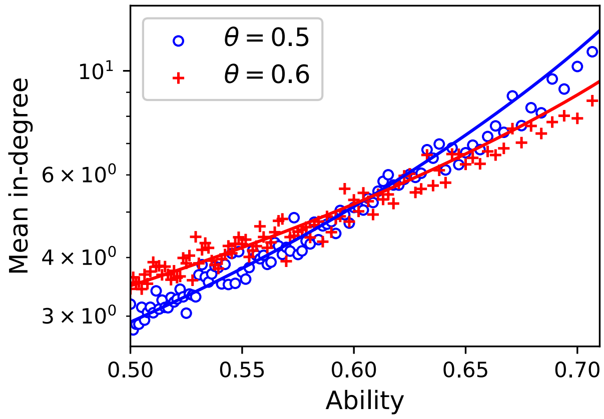

Figure E.1: The semi-log plot of the mean in-degree of an agent versus his/her ability in the self-organized network. Different symbols represent results for 2 thresholds, $\theta = 0.5$ and 0.6. The distribution of the abilities is given by $\tilde{\psi}(p)$. The mean in-degree increases approximately exponentially with ability. The circle and plus are simulation data and the analytical results are shown by solid lines.

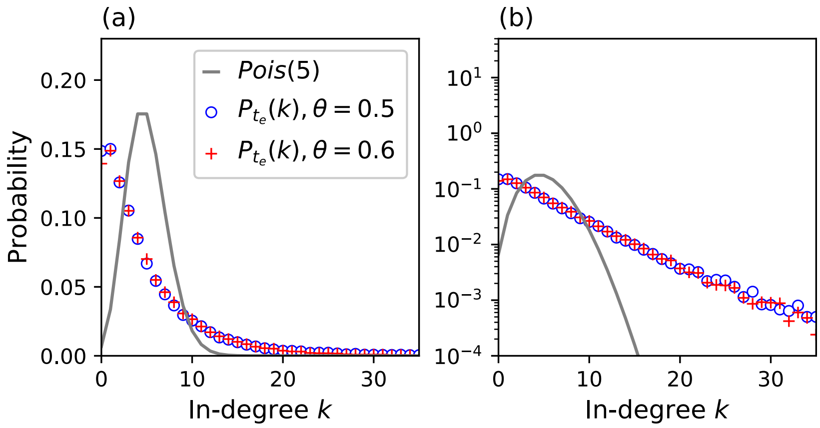

Figure E.2: (a) The solid line is the Poisson distribution with mean 5 representing the initial in-degree distribution. The markers (circle and plus) show the in-degree distributions of the self-organized network with rewiring threshold $\theta = 0.5$ and 0.6, respectively, obtained by 500 independent runs of my simulation. (b) The same as (a) except that the vertical axis is logarithmically scaled. We can see approximately exponential tails in the self-organized networks. In both of these figures, the distribution of the abilities is given by $\tilde{\psi}(p)$.

Appendix F

The effect of the parameter α

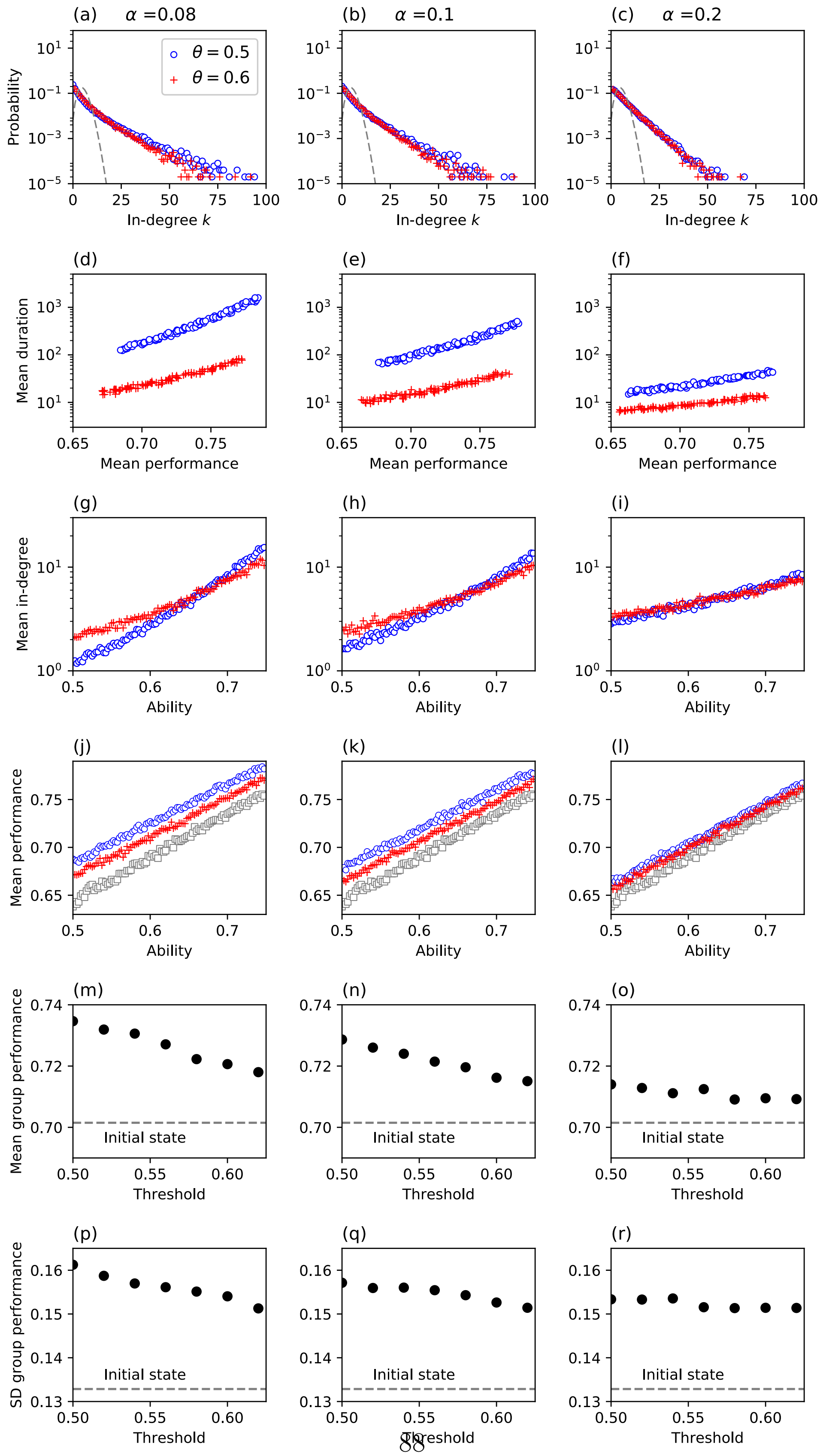
(a) α =0.08
(b) α =0.1
(c) α =0.2
θ = 0.5
θ = 0.6
Probability
In-degree k
(d)
(e)
(f)
Mean duration
Mean performance
(g)
(h)
(i)
Mean in-degree
Ability
(j)
(k)
(l)
Mean performance
Ability
(m)
(n)
(o)
Mean group performance
Initial state
Threshold
(p)
(q)
(r)
SD group performance
Initial state
Threshold

Figure F.1: **The effect of the parameter α that represents the extent to which an agent attaches importance to the immediate past result in evaluating the performance of referents.** Panels in the first to the third columns respectively show the results for $\alpha = 0.08$, 0.1 and 0.2. In all panels, the blue circles represent the results for the rewiring threshold $\theta = 0.5$, and the red crosses are for θ =0.6, in the self-organized networks.
(a)-(c): The in-degree distributions in the initial random network (dashed) and in the self-organized networks with different rewiring thresholds θ =0.5 (blue circles) and θ =0.6 (red crosses). The vertical axis is scaled logarithmically. The initial in-degree distribution follows the Poisson distribution with mean 5. The tails in the in-degree distributions in the self-organized networks are approximately exponential for all cases. The tails of in-degree distributions show steeper declines as α becomes larger.
(d)-(f): The mean duration that an agent is kept by a follower plotted against the agent's mean performance in the self-organized networks with different rewiring thresholds θ =0.5 (blue circles) and θ =0.6 (red crosses). The mean duration increases approximately exponentially (but actually slightly faster than exponentially) with the mean performance for all α values. As α becomes larger, the mean duration of keeping a follower declines. Consequently, both the effects of the mean performance (the slopes of curves) and of the rewiring thresholds (the difference between blue and red points) on the mean duration become less pronounced.
(g)-(i) The mean in-degree of an agent plotted against his/her ability in the self-organized networks with rewiring thresholds 0.5 (blue circles) and 0.6 (red crosses). The vertical axis is scaled logarithmically. The mean in-degree increases approximately exponentially (but actually slightly faster than exponentially) with ability for all values of α. The effect of the ability or the kick-off threshold on the mean in-degree becomes less pronounced as α becomes larger.
(j)-(l): The mean performance of each agent plotted against his/her ability in the initial random networks (the gray boxes) and in the self-organized networks with different rewiring thresholds θ =0.5 (blue circles) and θ =0.6 (red crosses). The mean performance increases linearly with ability for all values of α. As α increases, the effect of the adaptive rewiring (the difference between gray and colored points) and that of rewiring thresholds (the difference between blue and red points) on the mean performance become less pronounced.
The last two rows show the mean, (m)-(o), and the standard deviation (SD), (p)-(r), of group performance in the self-organized networks plotted against the rewiring threshold. The dashed lines represent those in the initial random network. Both mean and standard deviation of group performance are higher than that in the initial random network, and show monotonic decrease with the rewiring threshold for values of α. The effects of the rewiring threshold on the mean and the SD of group performance become less pronounced as α becomes larger.

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