Effects of Parasitic Collision Points

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Abstract

In multibunch circular colliders with small bunch spacing we have to deal with parasitic collision points (PCPs) when the opposite bunches interact outside the main interaction point (IP). The interactions in PCPs may change the closed orbits and influence the luminosity. Here we will study some of the coherent effects which may occur due to PCPs. The study of this subject has just started and is far from complete because the usual designs are done to minimize the effects of PCPs as they are considered harmfull for beam dynamics. Due to increasing luminosity in future designs this may change as the currents become larger, bunch spacing smaller and the separation in PCPs decreases. As will be shown here, there are important effects under these conditions which is not neccesserely harmfull but could be even benefecial for beam dynamics. The high non-linearity of the interactions and the large number of bunches in the model inevitably force extensive computer simulations.

Contents

1 Introduction

One of the most important parameters of colliders is the luminosity. It is defined as particles' production rate for a unit reaction cross section. The highest possible value of the luminosity is one of the main goals of the collider physics. For example for B-factories commissioned now at KEK [1] and SLAC [2], the desired value of luminosity is approximately 10^{34} $1/[\mathrm{s\ cm}^2]$. In the simplest case of the head-on collision of two short bunches with a frequency f_b, the luminosity is defined by the following formula:

$$L = f_b \frac{N_1 N_2}{4S}. \tag{1}$$

Here $N_{1,2}$ is the number of particles in the colliding bunches and S is the effective transverse cross section of these bunches in the IP. Formula 1 is derived assuming the lengths of the colliding bunches (σ_s) are smaller than the values of the β-functions at the IP ($\sigma_s \ll \beta$). There are many factors that may limit the luminosity of the collider. Most significant occur due to the electromagnetic interaction of the colliding bunches at the IP. Electromagnetic fields of one bunch deflect the particles of it's counter moving partner, which results in numerous perturbations of their orbits. This is a many fold phenomenon which usually is called the beam-beam interaction. It may result in numerous instabilities of coherent and/or incoherent oscillations of particles/bunches. Comprehensive description of this phenomenon as well as the relevant limitations on the collider luminosity is a very complicated problem. Its solution is far from the completion even now. For this reason, the designers of the modern colliders typically use some simplified but well tested criteria. One is that based on the assumption that the strength of the beam-beam interaction can be described specifying the threshold value of the so-called beam-beam parameter. The last is defined as the small amplitude betatron oscillation tune shift per one IP. For example, for particles from the beam 2 these tune shifts for vertical and horizontal oscillations read:

$$\xi_y = \frac{N_1 r_0 \beta_y}{2\pi\gamma\sigma_y\sigma_x},\ \xi_x = \frac{N_1 r_0 \beta_x}{2\pi\gamma\sigma_x^2},\ \sigma_x \gg \sigma_y. \tag{2}$$

Here $\sigma_{x,y}$ are the horizontal and vertical r.m.s. widths of the first beam, $r_0 = e^2/mc^2$ is the classical radius of the electron, γ is particles' relativistic factor, $\beta_{x,y}$ are the horizontal and vertical β-functions at the IP. From Eq. (2) and assuming that ξ_y is limited ($\xi_y \le \xi_\infty$) we find

$$\frac{N_1}{\pi\sigma_y\sigma_x} = \frac{2\xi\gamma}{r_0\beta_y}, \tag{3}$$

or

$$L = f_b N_2 \frac{\xi_\infty\gamma}{2r_0\beta_y}. \tag{4}$$

This formula clearly indicates that collider luminosity is proportional to the beam current, threshold value ξ_∞ and inversely proportional to the value of β-function at the IP.

According to Eq. (4) the luminosity of a collider can be increased either increasing a single bunch current or increasing the number of bunches in the beam, i.e. increasing f_b. For these reasons, an increase in the total beam current inevitably causes a decrease in the longitudinal bunch to bunch spacing in the beam. Typically, the lattice of the interaction region is designed to separate colliding bunches transversally after the collision. However even for a two-rings collider the sufficient electromagnetic separation can become impossible if the bunch spacing become very small. In this case, one of the problems arouses with parasitic collisions of bunches. Parasitic collisions happen due to the small bunch spacing which is used to increase the full current. In this work we discuss some of the coherent effects of PCPs. This is a rather complicated matter in itself. So we do not discuss here non-coherent effects due to interactions of colliding bunches at PCPs. There can be some multipole oscillations, or changes in a bunch charge distribution, etc.

On the other hand numerous coherent effects provide a wide field for study. In this work we focused on a consequence of PCP interaction: the longitudinal coupling of transverse betatron oscillations. It means that one bunch motion is influenced by other neighboring bunches. This is similar to wake field but due to the configuration of PCP region the transverse excitation of bunches propagates both ways - upstream and downstream from the initially excited bunch. For an enumeration of the studied effects see Sec. 4. Also to limit the time of the numerical calculations to reasonable values only transverse motion was studied. The longitudinal kick was not included as the crossing angles are small in the model studied here. And as shown in

Ref.[5] in this case no significant synchro-betatron resonances can occur. On the other hand, large crossing angles would reduce the longitudinal coupling and destroy the effect studied in the present work.

It was found that many coherent effects due to PCPs may have positive influence on beam dynamics. That is clear because using the barycentric model it is not easy to kick the whole rigid bunch out of the aperture, so we could expect only the luminosity loss due to bunch separation at the IP. Many of the negative sides of PCPs come from incoherent effects such as loss of the particles in "tails" (see Ref. [7]). So one should carefully consider both coherent and incoherent effects in actual design.

2 Review of other works

A few studies have been done concerning PCP interaction. Most of them concerning CESR as the PCPs or LRBBI (long range beam-beam interaction) were found to be the reason of luminosity limitation (Refs. [7]-[10]). Those works contain experimental results and simulations. However simulation methods, that I encountered there, were weak-strong models for stability, life-time and limiting current calculations. Also K. Hirata did some numerical study of the barycentric model for KEKB and found that the design is not critical within the framework of this model (Refs. [3]-[5]). Our results are in general agreement with his study. In our work we would like to provide more understanding of the coherent barycentric strong-strong interaction.

3 Barycentric Model

3.1 The model

So let us study the barycentric motion of bunches. The simple model is as follows: the motion outside the interaction region is described using a linear 4x4 (x, p_x, y, p_y) matrix transformation, we assume that the PCP region is a straight section where bunches collide with an arbitrary crossing angle (typical values are few milliradians). The number of PCPs can be varied, in most calculation it was taken to be 3 PCPs (including IP). To model the beam-beam interaction we use the rigid bunch gaussian model. It means that the bunches are considered as rigid macroparticles with the gaussian charge distribution. In that case the interaction between corresponding bunches is described by the formula [4]:

$$\delta y'_{\pm}+i\delta x'_{\pm}=\pm\frac{N_{\mp}r_e}{\gamma_{\pm}}f\left(x_+-x_-+D_x\left(n\right),y_+-y_-+D_y\left(n\right);\Sigma_x\left(n\right),\Sigma_y\left(n\right)\right), \tag{5}$$

where $D_{x,y}$ is the bunches' separation as a function of n - PCP number and $f\left(x,y,\sigma_x,\sigma_y\right)$is

$$f=\sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}}\left\{w\left(\frac{x+iy}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right)-\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right)w\left(\frac{\frac{\sigma_y}{\sigma_x}x+i\frac{\sigma_x}{\sigma_y}y}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right)\right\} \tag{6}$$

Here w is the complex error function, Σ_x is the effective beam size

$$\Sigma_x = \sqrt{(\sigma_x^+)^2 + (\sigma_x^-)^2}$$

D_x is the orbit separation. This formula was used in numerical tracking for both IP and PCP interactions.

So this model allows to study the coherent effects with dipole oscillations of bunches and works when bunches' sizes are not changed significantly.

3.2 Code

To do the numerical tracking for this model a C++ code was written. It can perform the tracking of bunches through arbitrary rings with linear parameters, the PCP region is a straight section with arbitrary number of PCPs. The base parameters were taken from [3], see Table 1. Some external checks for code consistency were done. For example the spectrum of the betatron oscillation matches the expected tune shift due to beam-beam interactions (Fig. 1). Or comparison shown on Fig. 2.

The number of bunches in beams was chosen to be 200 which is much smaller than in some colliders design. For KEKB it is desired to be few thousands bunches. For the numerical calculations this number is too high and is not necessary for the effects studied here for several reasons. Firstly the description of the effects here is more qualitative then quantative, so they are present in systems with arbitrary number of bunches. Secondly, these effects will not change noticeably if we increase the number of bunches because the strength of longitudinal coupling, which is responsible for them, falls in geometrical progression with the number of bunches between the coupled bunches. This happens because the number of PCPs is much smaller than the total number of bunches in the beam. So when we are looking on the effects observed here, the bunches, that are far in the train from the bunch of interest can not influence the effects. This statement is supported by the results of test simulations, which indicated that there is no correlation of transverse bunch's coordinates between bunches with large longitudinal separation in bunch train (except for the case of resonance trapping Sec. 4.7). So the characteristic length of longitudinal coupling due to PCPs interactions is of order of number of PCPs.

Parameter	value
bunch separation l	0.6m
emittances $\left(\varepsilon_x^{\pm}, \varepsilon_y^{\pm}\right)$	$(1.8 \cdot 10^{-8}, 1.8 \cdot 10^{-10})\, rad \cdot m$
beta-functions $\left(\beta_x^{\pm}, \beta_y^{\pm}\right)$	(1, 0.01)m
half-crossing angle $\alpha_{x,y}$	(7, 0) mrad
orbit separation at IP $D^0_{x,y}$	0m
betatron tunes $\left(\nu_x^{\pm}, \nu_y^{\pm}\right)$	(0.2, 0.15)
damping time T	500 turns
number of bunches N	200
coherent b-b parameter $\Xi_y^{\pm}$	0.025 ($\xi = 0.05$)

Table 1: Base tracking parameters

The damping time of transverse oscillations shown in the Table 1 seems unrealistically short. We should remind the reader that we discuss the development of the coherent oscillations of bunches. In high luminosity colliders the multibunch performance of the beam is provided by the system of feedback damping of dipole oscillations. In this case the corresponding decrements of coherent oscillations of bunches significantly exceed the decrements of incoherent oscillations due to the synchrotron radiation damping. For example, the fast feedback system at KEKB [1] will provide the damping times of coherent oscillations in a range of 1-10ms. The revolution period in KEKB is $10 \mu s$. These values correspond to the damping of coherent oscillations during 100-1000 turns. That indicates that the values of damping rates used in simulation (Table 1) are quite realistic.

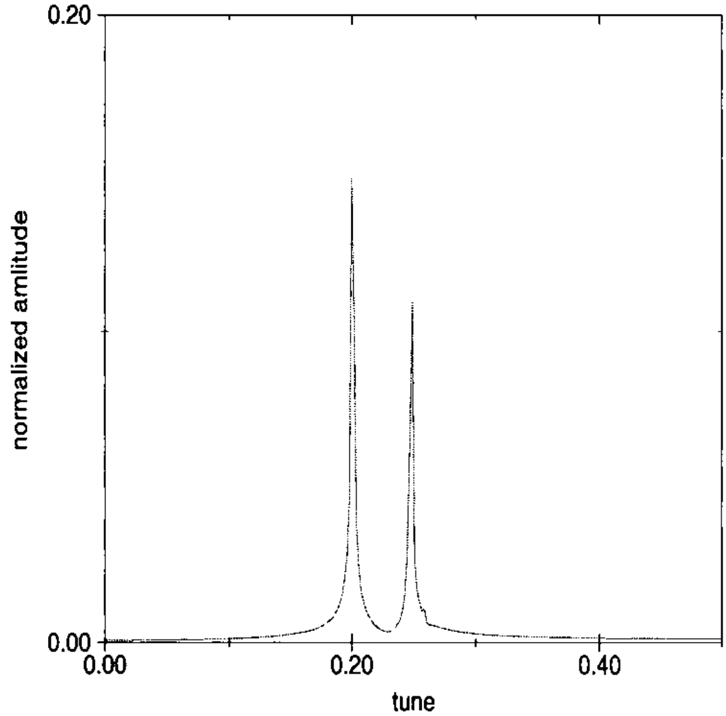

Figure 1: Spectra of X betatron oscillations.

The stable orbit distortion was evaluated for a range of betatron tune and compared with analytical results (8) on Fig. 2. As the tune approaches the integer resonance the orbit distortion grows as expected from (8): $\overrightarrow{O_{P_1}} \sim \frac{1}{\nu}$. We can see also that there is point out of range when $\nu_x = 0.45$ that happens because of half-integer resonance ($\nu_x + \xi_x = 0.5$) causes bunches separation at IP and it has little to do with interaction at PCPs. Anyway in the usable range the distortion is very small because the distance between opposite bunches in the PCPs is the tens of the bunch size so the kick is too weak.

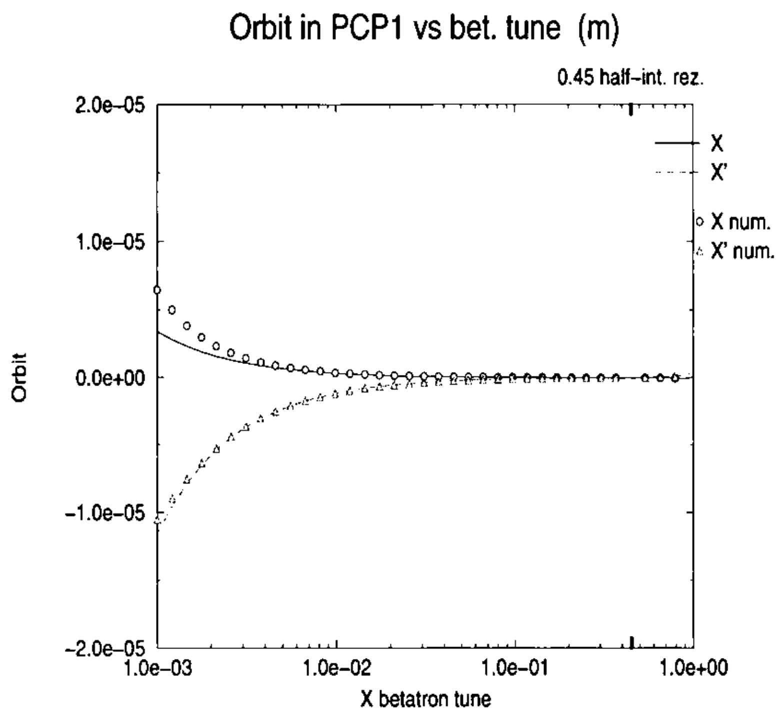

Figure 2: X orbit distortion at PCP 1, comparison of analytical formula (8) and numerical tracking.

4 Effects of PCPs

The interaction in PCPs introduces some effects in beam dynamics. We can divide them first on weak and strong. The "strength" of interaction is defined by the kick in PCPs compared with the kick at the main IP. If the kick (or tune shift) due to PCPs is much smaller than the main IP kick, then the coupling between bunches due to PCPs is too weak to produce noticeable effects. Weak effects happen with moderate beam currents and large enough bunch separation in PCPs. They produce a small stable orbit distortions which is insignificant as will be shown. It is also discussed in Refs. [3]-[6]. In that case the longitudinal coupling is too small to have noticeable dynamic effects. When the kicks at PCPs become large enough some interesting effects may happen. The transverse betatron oscillations then are coupled longitudinally between the successive bunches. That leads to a few phenomena:

- Longitudinal waves of transverse oscillations, which spread from some initial distortion and then damped completely because of synchrotron damping.
- Reduced damping time of a single excited bunch.
- Bunches near the gap (missing bunches in beam) have larger betatron amplitudes.
- Betatron resonances trapping occurs in more ordered fashion due to

longitudinal coupling.

- Diffusion due to stochastic properties of nonlinear forces in IP combined with longitudinal coupling. It leads to a growth of the bunches' betatron amplitude. This is the so-called Arnold diffusion.

4.1 Closed Orbit Distortion

If a closed orbit exists it is obviously different from the designed closed orbit without PCPs. Let us take the configuration with IP at a straight section, the X-plane crossing angle has non-zero value, and the Y-plane crossing angle is zero. Let us consider the X-plane orbit distortion caused by PCPs. First we take 2 PCPs - one at each side of IP at the distance $l/2$, where l is the longitudinal bunch separation. We also take that e^+ and e^- rings have equal parameters. In that case the orbit distortion is symmetric (Fig. 3). So we can easily write the equation and obtain the orbit distortion. We denote the orbit coordinates in corresponding PCP as $\overrightarrow{O_{P_n}} = \begin{bmatrix} x \\ p_x \end{bmatrix}$, kick as a function of distance $\vec{D}$ between bunches: $\vec{K}\left(\vec{D}\right)$ which can be calculated using formula (5), l- bunch separation, α- half of the crossing angle, S_l - matrix of l-length straight section, T_s is the turn matrix of the rings from PCP_3 to PCP_1. Using the symmetry condition ($e^+ \leftrightarrow e^-$), so that kick in

IP is zero we can write:

$$\begin{cases} \overrightarrow{O_{P_2}} = S_{l/2}\left[\overrightarrow{O_{P_1}} + \overrightarrow{K}\left(\begin{bmatrix} l\sin(\alpha) \\ 0 \end{bmatrix} + \overrightarrow{O_{P_1}} - \overrightarrow{O_{P_3}}\right)\right] \\ \overrightarrow{O_{P_3}} = S_{l/2}\overrightarrow{O_{P_2}} \\ \overrightarrow{O_{P_1}} = T_s\left[\overrightarrow{O_{P_3}} + \overrightarrow{K}\left(\begin{bmatrix} -l\sin(\alpha) \\ 0 \end{bmatrix} - \overrightarrow{O_{P_1}} + \overrightarrow{O_{P_3}}\right)\right] \end{cases} \tag{7}$$

As will be shown later with reasonable design parameters the orbit distortion is much smaller than the bunch size which in turn is smaller than the distance between bunches in PCPs ($\sim l\sin(\alpha)$), so we can take the kick at PCPs constant: $\overrightarrow{K}\left(\begin{bmatrix} l\sin(\alpha) \\ 0 \end{bmatrix} + \overrightarrow{O_{P_1}} - \overrightarrow{O_{P_3}}\right) \cong \overrightarrow{K}\left(\begin{bmatrix} l\sin(\alpha) \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ k \end{bmatrix} = const.$ Then easily solving the system (7) we have:

$$\overrightarrow{O_{P_1}} = \frac{lk}{2}\begin{bmatrix} 1 - \frac{l\sin\nu}{2\beta_x(1-\cos\nu)} \\ \frac{\sin\nu}{\beta_x(1-\cos\nu)} \end{bmatrix} \tag{8}$$

There are some interesting dependencies, but as will be shown later this distortion is too small to be important. See comparison of this formula with numerical tracking results on Fig. 2. The similar results were obtained in [6]. So in general the closed orbit distortion is too small to be worried about.

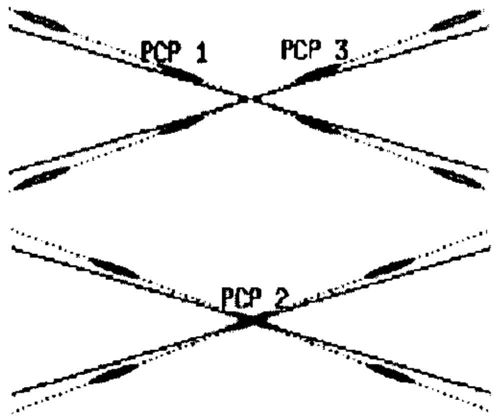

Figure 3: Numbering of PCPs (PCP2 is IP)

4.2 Stability Threshold on ξ_x

Here we try to evaluate analytically the maximum value of the beam-beam parameter ξ_x for which the motion is stable. Here by "stable" we mean that there is no stochastic excitation of the bunches, which can happen when one beam excites the other and vise-versa so the excitation never stops. Without such an excitation they damp to some kind of closed orbits or "fixed points" as will be shown later. So ξ_{max} is the maximum available ξ_x for the given parameters, for which we don't have yet a significant luminosity loss. For optimistic estimates we take a model without resonant behavior. That means that bunches don't have "memory" so their positions in PCPs at each turn are random and depend only on the average amplitude of the betatron oscillations. This assumption is based on the high nonlinearity of IP interaction, which at some time will produce the "decoherency" of successive bunches with different betatron amplitudes. So the bunch at the PCP receives a random kick. The averaged value of this kick depends on the amplitude of the oscillation of the opposite bunch. An instability will occur if one beam excites the other to an amplitude equal or larger then itself; we then have a positive feedback. The beam-beam kick can be linearly approximated:

$$\Delta z' \simeq -\frac{2\pi}{\beta_z}\xi_z \Delta z \tag{9}$$

Then averaging Eq. (9) the threshold of instability for one PCP is when

$$\sigma_{z'_-} = \frac{2\pi}{\beta_z}\xi_z \sigma_{z_+} \tag{10}$$

If we consider N PCPs with zero crossing angle and one-dimensional motion (horizontal plane, we can do it because the vertical beam size is much smaller than horizontal), then all dependences vanish and we have the surprisingly simple relation:

$$\xi_{max} = \frac{1}{2\pi\sqrt{N}} \tag{11}$$

So this formula gives the maximum stability threshold on ξ_x for the flat beam with zero crossing angle. On Fig. 4 is the comparison of this formula with simulation results. In simulations, ξ_{max} values were changed until the luminosity fell a few times. This threshold is very abrupt. The closed orbits for all bunches experience the transition from zero separation at the IP to resonance trapping with opposite bunches separated in diagonal resonance

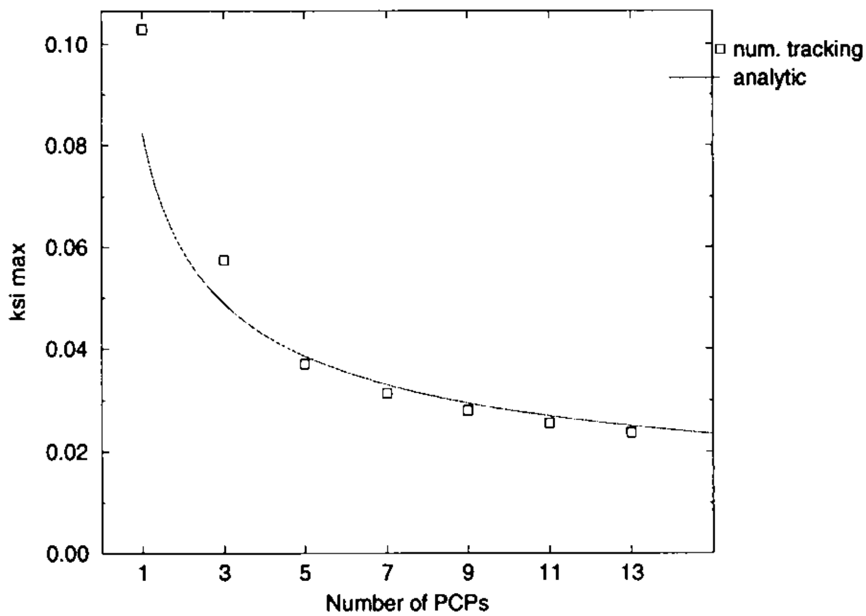

Figure 4: ξ_{max} tracking and using (11)

islands as illustrated in 4.7. To avoid the influence of the resonance effects the tunes were chosen close to zero. For one and three PCPs this model is not very good, as could be expected because the decoherency is weak, but otherwise we can see good agreement. The implication of formula (11) is that the overall luminosity can not be increased by increasing the number of IPs on the ring, it actually remains constant or even falls according to the simulations (Fig. 4). Of course this is true only if the luminosity is limited only by this coherent beam-beam instability and there isn't any artificial feedback damping etc.

Then we can calculate the optimal tunes regions for 1 PCP (Fig. 5). The ξ_{max} was taken so that luminosity falls by 50%. Actually this instability pushes the bunches out of the head on collision so the separation at the IP occurs and the above model does not work any longer. After this bunches are captured in resonances as discussed in Sec. 4.7. We can see the best choice for tunes, which happens to be well known. For 3 PCP we have Fig. 6. As we can see additional stable regions appear near X quarter-integer tune. It is not significant for current designs, as the crossing angle is far from zero, so traditional tunes should be used.

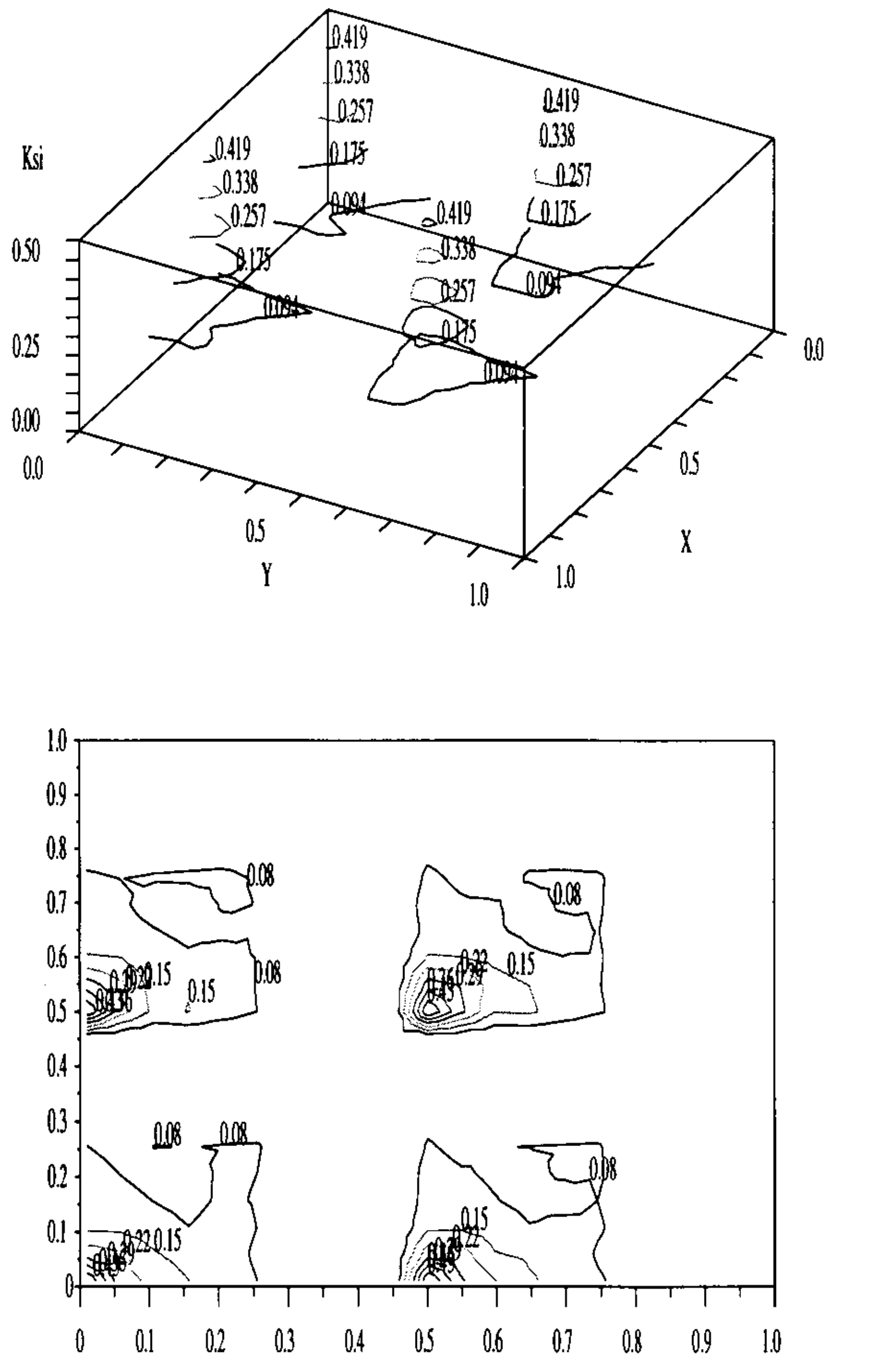

Figure 5: ξ_{max} vs X and Y tunes with 1 PCP, i.e. only IP crossing.

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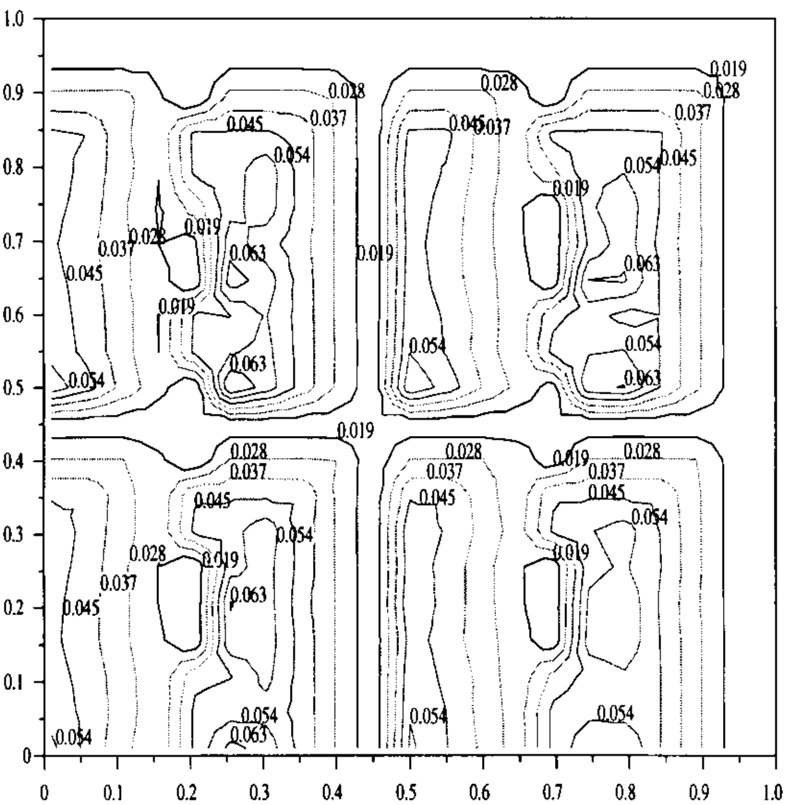


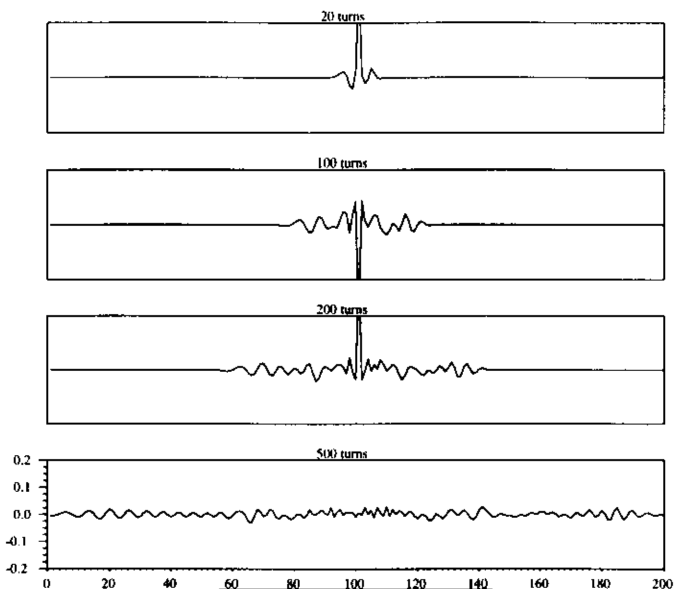
Figure 6: ξ_{max} vs X and Y tunes with 3 PCP

4.3 Longitudinal Waves

Let us see how the longitudinal coupling induced by PCP interaction first reveals itself. The first noticeable phenomenon when we increase the strength of PCP interactions is that some initial distortion of a given bunch spreads longitudinally to the other bunches. So it looks like waves in a pool (Fig. 7). These waves later damp (if PCP interaction is not strong enough - this is discussed later) to the initial undisturbed state due to synchrotron damping. For convenient visualization we will look now only at the horizontal motion.

Let us see how the speed of propagation of those waves depends on parameters. As would be expected the speed grows with number of PCPs (Fig. 8). The speed is defined by the largest number of bunch (counting from initially excited bunch) which is involved in oscillations, divided by number of turns it takes for the wave to reach it. The definition of speed is not so rigid and measurements are based on figures like Fig. 7.

We can see that the speed increases vary rapidly with the number of PCPs. There is a similar dependence on the beam-beam parameter ξ_y.



Parameter	value
ν_x	0.2
α_x	0
ξ_y	0.1
N_{PCP}	5

Figure 7: Longitudinal waves and the corresponding tracking parameters. X-axis - bunch number, Y- displacement at IP in σ_x units. The initial condition was a displacement of 100th bunch of $0.3\sigma_x$.

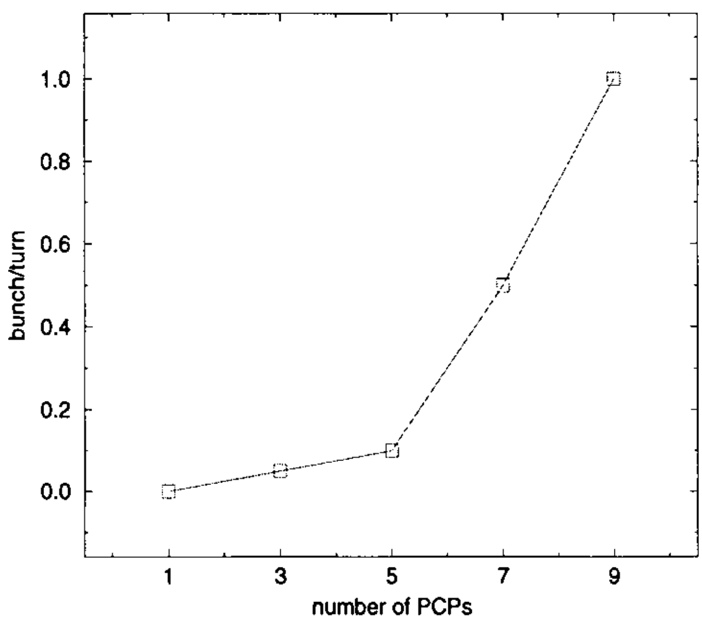
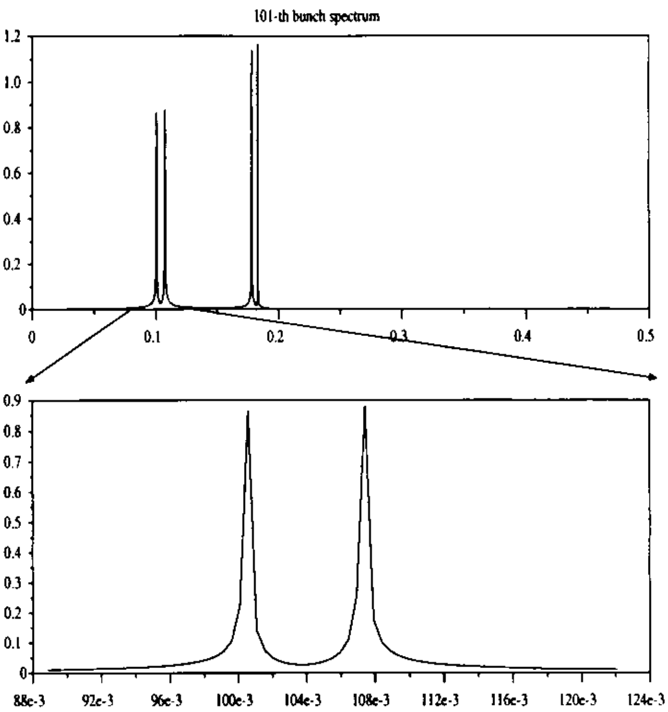


Figure 8: Propagation speed of longitudinal waves vs number of PCPs. The rest parameters the same as on Fig. 7.

4.4 Oscillation modes

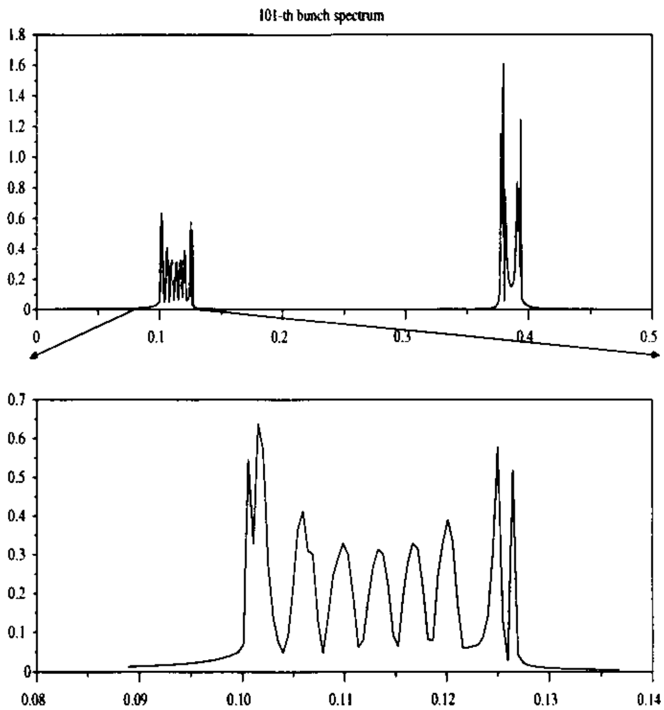
Longitudinal coupling should introduce new oscillation modes. Without PCPs, the tunes split in π and σ modes. Those peaks split further apart due to PCPs (Fig. 9). The distance between the new peaks is roughly proportional to tune shift in PCPs, but the π mode splits a smaller distance.

But as the ξ_y grows more modes appear, as a larger number of bunches become effectively coupled (Fig. 10). The data for spectrums was taken on 2048 turns. If we increase the number of turns the split if spectrum will be filled further with more modes. As can be expected from linear system 200 modes must be jammed there (Fig. 11).



Parameter	value
ν_x	0.1
α_x	0
ξ_y	0.05
N_{PCP}	3

Figure 9: Spectrum of betatron oscillations. Both π and σ tunes split to new modes due to PCPs.



Parameter	value
ν_x	0.1
α_x	0
ξ_y	0.2
N_{PCP}	3

Figure 10: When ξ_y grows even more modes become visible on the spectrum (compare with Fig. 9).

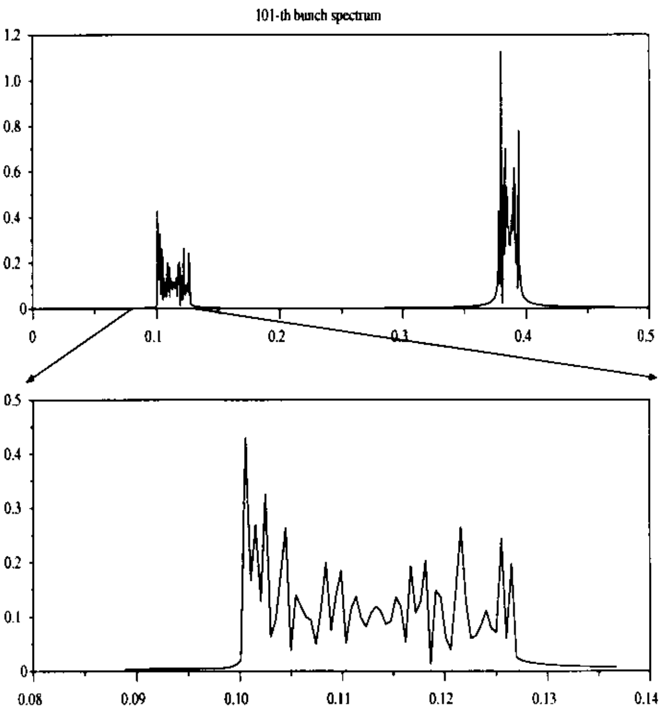


Figure 11: Spectrum of transverse betatron oscillations.

4.5 Damping of a single excited bunch

If a single bunch in this system is displaced, it returns to a closed orbit faster than by synchrotron or feedback damping mechanism only. This is reasonable as the excited bunch shares it's oscillation energy with it's neighbors and they are also damped by themselves. Of course after the long enough time after excitation the damping rate will fall back to normal since the neighboring bunches become equally excited. But this time is long in terms of revolution period. This increased damping rate can be quite useful for bunch injection for instance, or for some coherent instability suppression (Fig. 12). Again, this single bunch damping decrement changes in time from initial kick, and eventually will decrease to the value of collective decrement when all bunches become excited. But the time when it will happen is much larger then the time of feedback damping.

The parameter that damps on Figure 12 is $\frac{x^2}{\sigma_x} + \frac{p_x^2}{\sigma_{px}}$. The coefficients for the fits are T_{0pcp}=250 turns, which corresponds to a damping time of T=500 turns used for tracking and T_{3pcp}=130 turns, so the damping time decreases almost by half. On figures 13, 14 the dependence of damping time on ξ_y is shown.

Let us analyze these figures (13, 14). We can see that we can not decrease the damping time by more than 2-fold, if we increase longitudinal coupling further. However the damping time goes to this limit rather rapidly, when we increase ξ_y or decrease the crossing angle. Apparently, the damping time falls when the half-crossing angle is smaller than 2mrad with $\xi_y = 0.1$. So

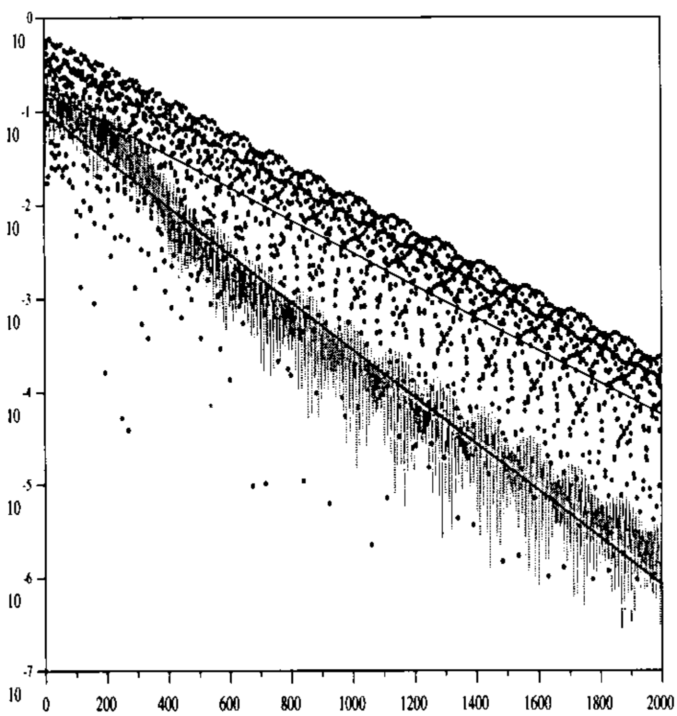


Figure 12: Damping of the excited bunch with 3 PCPs and without. Solid lines - exponential fit of tracking data. The damping times of fits are 250 turns and 170 turns. The axis are action vs turn number. $\xi_y = 0.05$, $\alpha = 0$.

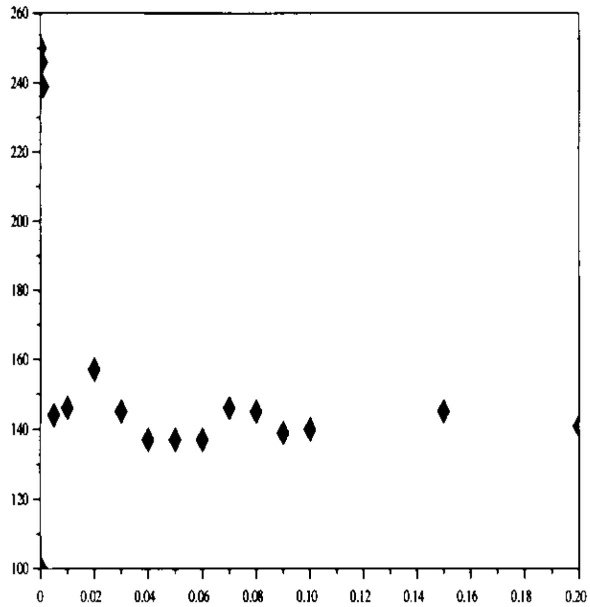


Figure 13: Damping time (in number of turns) of a single excited bunch in turns vs ξ_y. Crossing angle is zero.

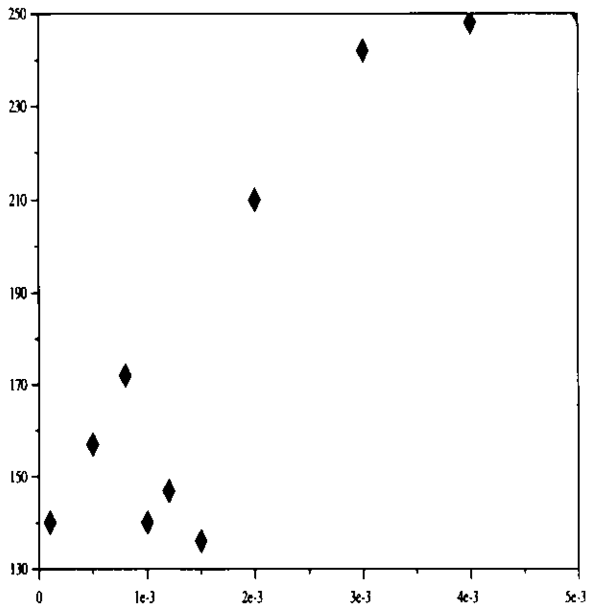


Figure 14: Damping time (in number of turns) of a single excited bunch vs half crossing angle. $\xi_y = 0.1$.

it looks like these effects can be observed and measured on some existing accelerators. Such significant reduction of the single bunch damping time can be quite beneficial to the beam dynamics.

4.6 Missing Bunches

Of course during the injection process the bunch train is not filled completely. Here we investigate what effects can be introduced by missing bunches. A missing bunch produces an obstacle for propagation of longitudinal waves because it reduces the longitudinal coupling abruptly. So let us look at a train's "free end" - the last bunch before the missing one. There can be some danger that the free end bunch will be blown to the large betatron amplitudes as the wave reflects from the missing bunch gap. On Fig. 15 the difference of free end behavior is shown with and without the gap. The initial condition was that of a few randomly excited bunches. The effect is that free end bunch has a slightly larger amplitude in the presence of the gap. This effect is small and seems to introduce no danger for beam dynamics. At least nothing dangerous was found as we scanned parameter space.

It was noticed, that the effect is stronger when the distortion is close to the gap.

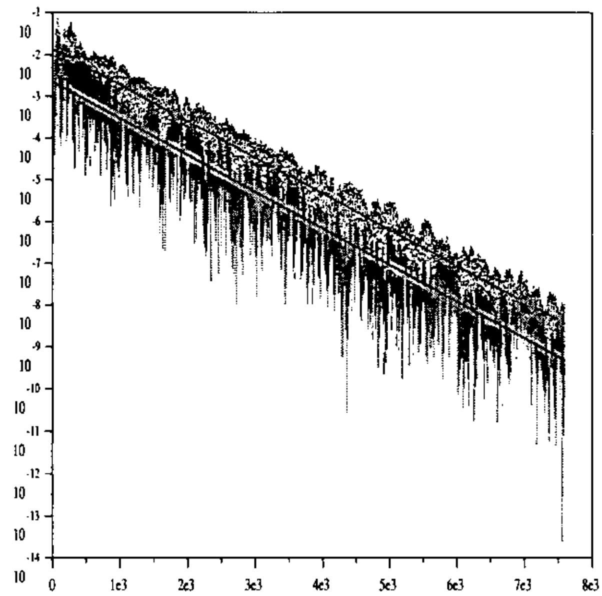


Figure 15: Influence of the gap in the bunch train. The parameter is the same as on Fig. 2.3 vs number of turn.

4.7 Resonance Trapping

In the case when ξ_y is large enough bunches are trapped in resonance ("islands" on phase plot). With no PCPs, longitudinal distribution of trapped bunches depends entirely on the initial condition. But in the presence of PCPs the longitudinal distribution becomes ordered (Fig. 16, 17) ($\xi_y = 0.11$, crossing angle is zero).

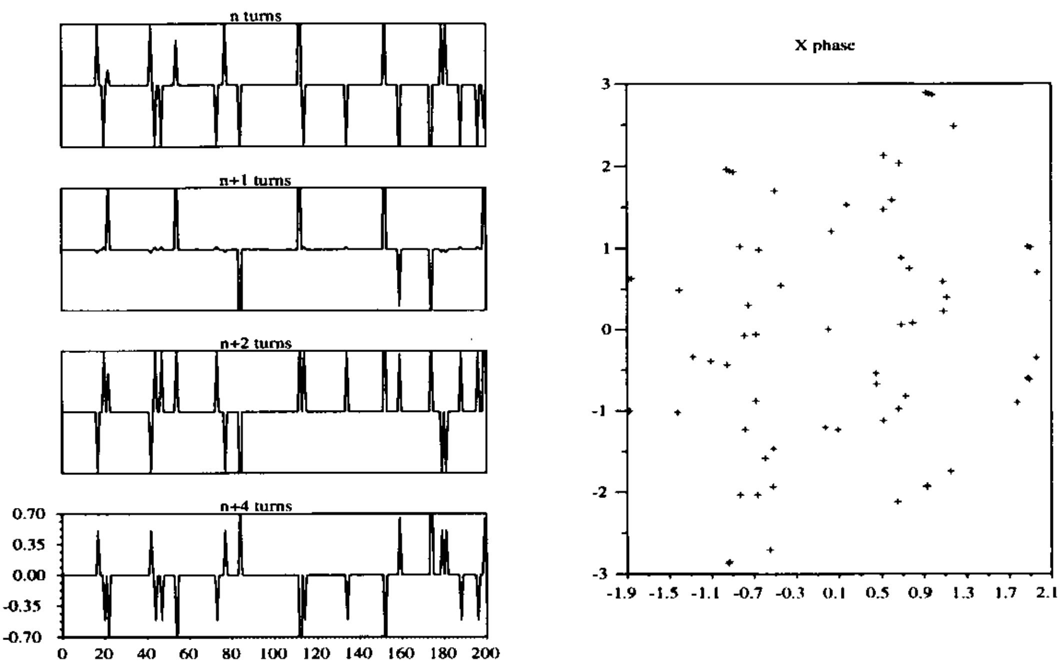


Figure 16: Longitudinal distribution and phase plot of bunches trapped in resonances without PCPs

Comparison of these figures shows that without PCPs bunches are trapped in many different resonances, although most of them fall in the center of coordinates. With 3 PCPs all bunches are trapped in a 4-th order resonance and longitudinal distribution has a period of 4 turns and it is much more ordered

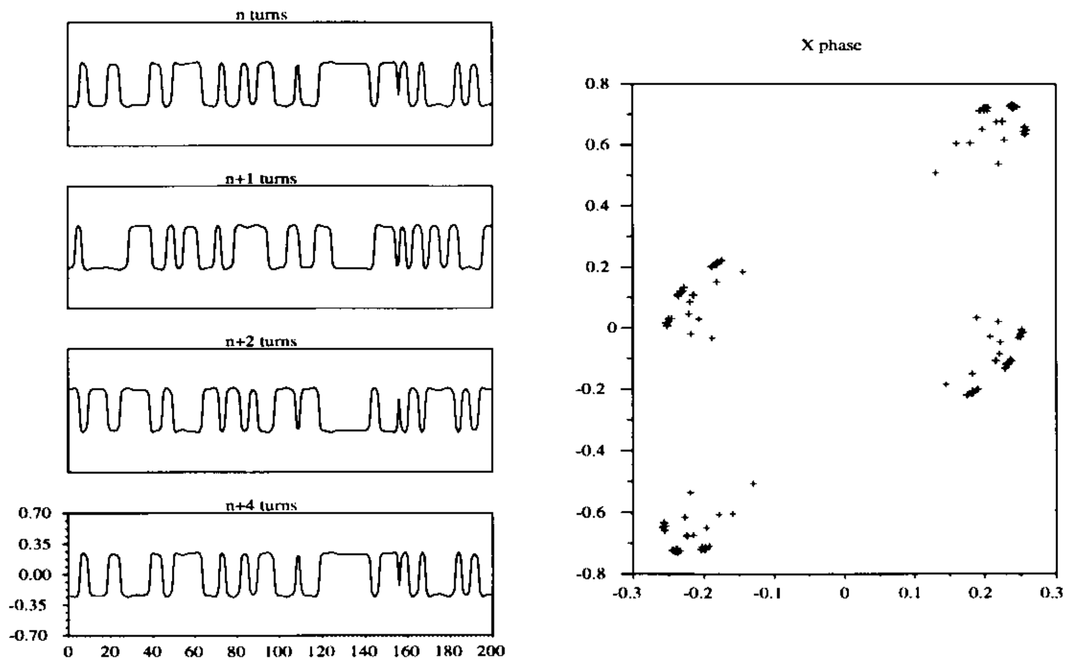


Figure 17: Longitudinal distribution and phase plot of bunches trapped in resonances with 3 PCPs

than without PCPs. The smaller (higher order) resonance islands vanish in the presence of longitudinal coupling and the beam becomes more ordered. This may be an advantage from the point of view of feedback because the longitudinal signal contains less high-frequency harmonics (compare Fig. 16 and 17).

4.8 PCP Diffusion

Here we discuss the effect of beam stochastic heating due to PCPs interaction. When we increase ξ_y we can observe stochastic effects due to nonlinearity of the beam-beam force. Let us switch off the damping in the tracking and look the sum over all bunches of square of the normalized amplitude:

$$\frac{1}{2N}\sum^{2N}\left(\frac{x^2}{\sigma_x}+\frac{p_x^2}{\sigma_{p_x}}\right)$$

In the absence of PCPs (i.e. only IP) we can see (Fig. 18) that the amplitude may grow initially, but then reaches a plateau and keeps approximately a constant value. Quite different is the behavior with 3 PCPs. In that case the square of the amplitude grows linearly. That means that there is a source of noise. This happens because non-linear resonances cause some stochasticity in the bunches' movement, so the position and phase of the successive bunches is uncorrelated, so when a bunch passes through 3 PCPs it experiences a random kick, which produce the observed noise. For electron machines with ordinary parameters it is not significant because the synchrotron damping is much stronger than diffusion, but for protons it should be considered.

The Fig. 18 leads to a general hypothesis: the diffusion coefficient in average is constant for a given map, it does not depend on betatron amplitudes. Strict linearity supports that hypothesis, as the amplitudes of bunches grow significantly for upper plots on Fig. 18 the diffusion coefficient remains

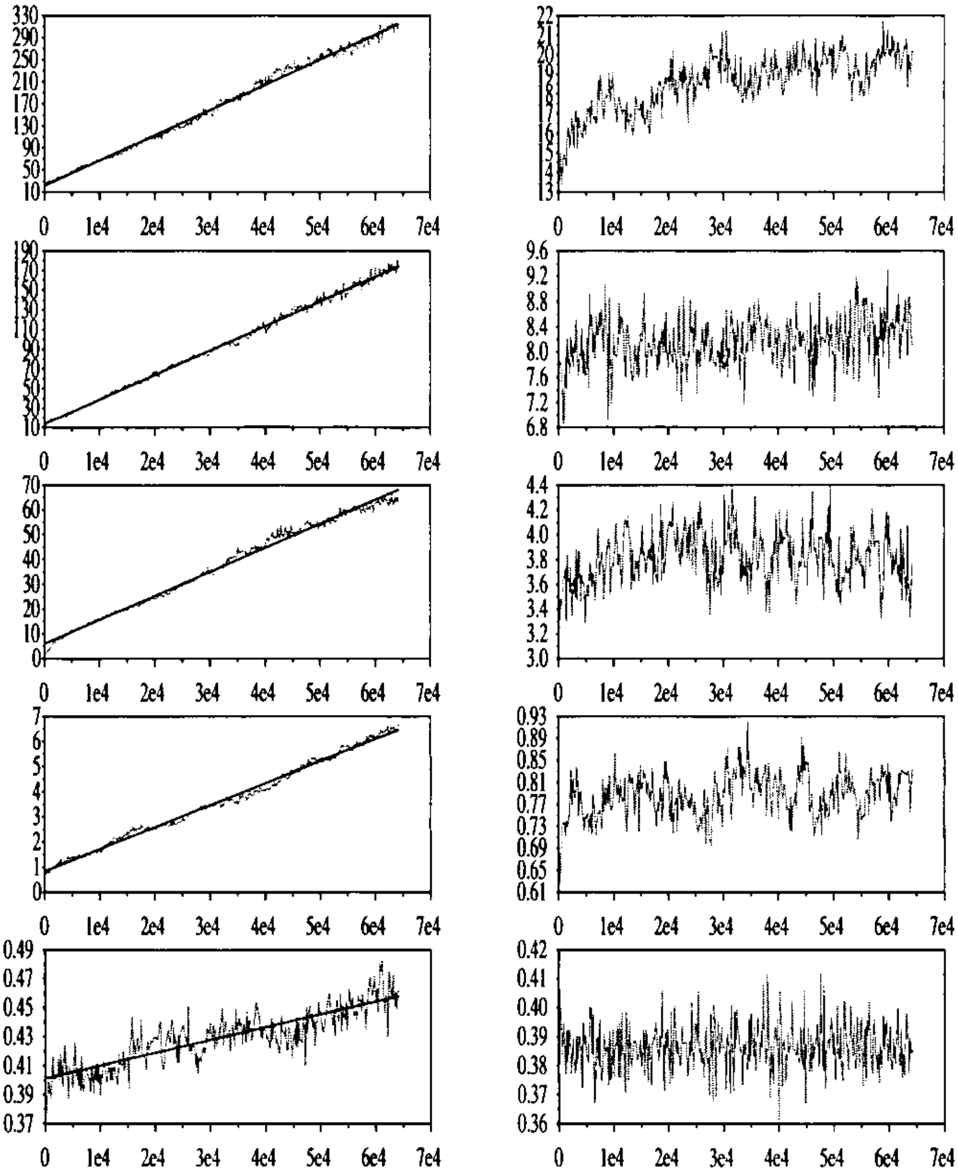


Figure 18: Averaged square of betatron amplitude vs turn number for different values of ξ_y, left half - with PCPs, right - without. The corresponding diffusion coefficients (from linear fit shown as solid line) are (from top to bottom) $4.6 \cdot 10^{-3}$, $2.5 \cdot 10^{-3}$, $1. \cdot 10^{-3}$, $8.8 \cdot 10^{-5}$, $8.8 \cdot 10^{-7}$ and ξ_y is 0.4, 0.3, 0.2, 0.1, 0.05. The initial condition for all plots was the random bunches distribution with $\sigma = 3\sigma_x$.

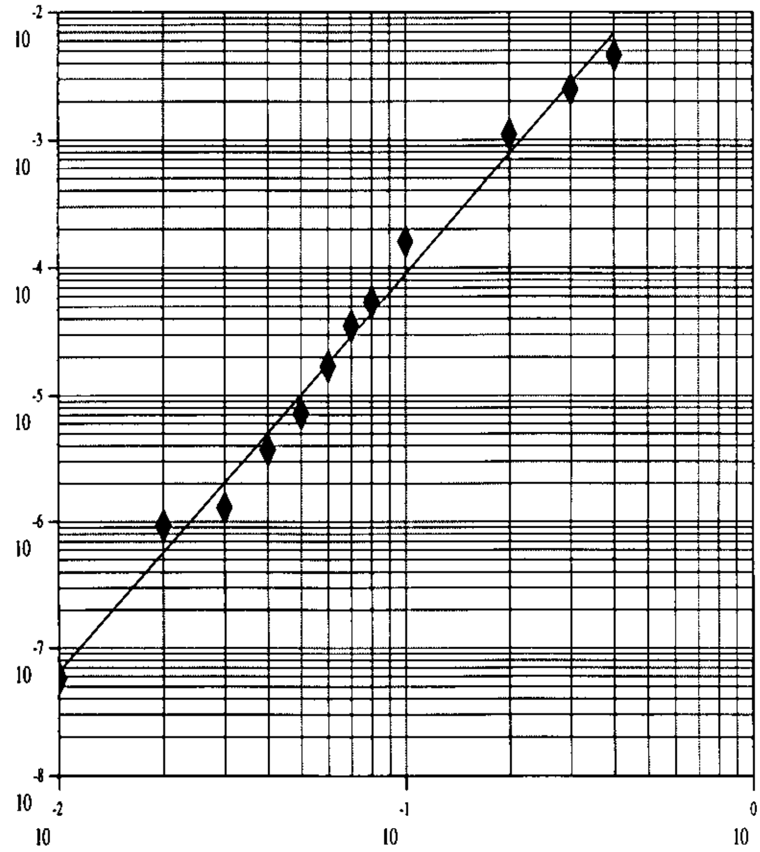


Figure 19: The diffusion coefficient vs ξ_y, the line is a fit.

the same so that the square of the betatron amplitude continues to grow linearly with the same coefficient. Of course the PCPs may be not the only source of longitudinal coupling, wake fields may also contribute. So this effect should be seriously considered for proton machines because in general the longitudinal coupling amplifies strongly the stochastic effects caused by strong non-linearity at the IP. The diffusion coefficient grows rapidly with the increase of ξ_y as shown on Fig. 19. We can see that asymptotic behavior of diffusion coefficient can be described well with the fit:

$$D = 0.12\xi_y^{3.14}$$

This is the so-called Arnold diffusion which happens in multi-dimensional non-integrable systems. And the longitudinal coupling increases the dimension of the system by N - the number of bunches. It is necessary to note that the diffusion appears only when the betatron amplitude of bunches is large enough, so the nonlinearity of the interaction is significant. If we track beams with small initial distortions, the interaction at the IP is almost linear and the diffusion is not observed.

5 Conclusion and Discussion

It was shown that strong PCP interactions introduce some significant coherent effects, a few of them could be useful to exploit for the enhancement of beam dynamics stability. The effects studied here were consequences of longitudinal coupling of transverse motion coming from the barycentric interaction of bunches in PCPs. They are:

- Longitudinal waves and new oscillation modes can be useful for diagnostics, for PCP region analysis and optics tuning.
- Reduced damping time of a single excited bunch. Obviously could be used to improve injection. May help to fight the coherent instabilities caused by wake fields for example.
- Bunches near the gap (missing bunches in beam) have larger betatron amplitudes. Harmful effect but the increase of amplitude was shown to be small for reasonable parameters.
- Betatron resonances trapping occurs in more ordered fashion due to longitudinal coupling. In general this softens the requirements on feedback systems, but this statement should be verified by more simulations.
- Diffusion due to stochastic properties of nonlinear forces in IP combined with longitudinal coupling. It leads to the growth of the bunches betatron amplitude. This was shown to be not critical for electron col-

liders because of the synchrotron damping, but should be concerned for high-luminosity proton machines.

Some of the effects could be detected and would be interesting to observe on modern multibunch colliders. The subject becomes more important with the growing demands on luminosity. There are some experimental indications from PEP-II commissioning that PCP effects improve the transverse bunch stability [11].

There are some other interesting questions for further study within the barycentric model approach. What effects may appear with consideration of transverse betatron coupling, synchrotron motion, feedback system, wake fields, etc? Also the diffusion mechanism requires more study, especially for proton colliders.

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