Lepton flavor violation in supersymmetric grand unified theories

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Abstract

In this thesis, we present a detailed analysis of $\mu^+ \rightarrow e^+\gamma$ and $\mu^+ \rightarrow e^+e^+e^-$ with polarized muons in supersymmetric grand unified theory (SUSY GUT). In particular, we focus on various P- and T-odd asymmetries which are defined relative to the initial muon polarization. First, we discuss lepton flavor violation (LFV) in SUSY GUT based on the minimal supergravity model. A brief review of the minimal supersymmetric standard model (MSSM) and SUSY GUT is also included. Next, we develop a model-independent framework for analyzing the $\mu^+ \rightarrow e^+\gamma$ and $\mu^+ \rightarrow e^+e^+e^-$ processes with polarized muons. We define various P- and T-odd asymmetries relative to the initial muon polarization. Finally, we present the results of our numerical calculations for these asymmetries in the SU(5) and SO(10) SUSY GUT. As a result of a detailed numerical analysis, we found that the asymmetries and the ratio of two branching fractions are useful to distinguish different models. We show that the P-odd asymmetry of $\mu^+ \rightarrow e^+\gamma$ becomes +100%–−100% in the SO(10) GUT, whereas it becomes 100% in the SU(5) GUT. It is also shown that the T-odd asymmetry of $\mu^+ \rightarrow e^+e^+e^-$ can reach 15% in the SU(5) GUT within various EDM constraints for the SUSY CP violating phases, whereas it is small in the SO(10) GUT. The distribution of differential branching ratios and asymmetries of $\mu^+ \rightarrow e^+e^+e^-$ process are also useful to distinguish different models.

Contents

Chapter 1

Introduction

The Standard Model (SM) [1] has succeeded enormously since its formulation 30 years ago. Almost all results of experiments can be explained within the framework of the SM, except for the recent discovery of neutrino oscillation. The SM is a renormalizable anomaly-free theory and is self-consistent. Although it can be applied to an arbitrary energy scale, we know the theory must be cut off and unified with the theory of gravity at the Planck scale ($\sim 10^{18}$ GeV).

Theoretically, however, there is an important clue to explore the theory beyond the SM. The SM has a scalar field called the Higgs field, which plays an important roll in that it breaks the gauge symmetry $SU(2) \times U(1)$ to $U(1)_{em}$ and gives masses to the weak bosons and to all matter fermions at the electro-weak (EW) scale ($\sim 10^2$ GeV). The mass of the Higgs field is a unique parameter which has a mass dimension in the SM, which determines the EW scale. Generally, the mass parameter of a scalar field is highly unstable under quantum corrections. Its square diverges quadratically with a cut-off of the theory. If the SM describes nature correctly until the Planck scale, this quantum correction would amount to an order of 10^{18} GeV. Extreme fine tunings for parameters at the Planck scale are needed in every step of perturbation to cancel the correction and keep the EW scale. It is hard to imagine that a mechanism to control the quantum correction between the Planck scale to the EW scale with such a high precision is a part of the physics at the Planck scale , which is responsible for the bare parameters of the SM at the Planck scale (hierarchy problem) [2].

To remove such fine tunings from the theory, we must introduce new physics at a scale not much lager than the EW scale. There are three scenarios which realize such a theoretical reasoning. The first one is the supersymmetry (SUSY) scenario[3], the second one is the technicolor scenario[4] and the third one is the extra-dimension scenario[5]. The first one excludes any quadratic divergence of the scalar field by imposing extra symmetry on the theory. The second one removes a fundamental scalar field, itself, from the theory and replaces it with a composite of fermions. The third one utilizes extra dimensions in addition to the ordinary four dimensions of space time to solve the hierarchy problem. Among them, the SUSY scenario is very attractive because a minimal SUSY extension of the SM agrees very well with the hypothesis of grand unification of gauge groups, as well as the result of precision measurements of the gauge coupling constants. It predicts many new particles at the TeV scale. Their spectrum may carry information about an ultra-high energy scale not very far from the Planck scale.

Thus, a direct experimental search for a high-energy frontier at the TeV scale is indispensable. It may open a way to a new era of particle physics. However, such a direct experiment is not easy, because it needs a long time scale, highly specialized manpower and an enormous amount of money , which can hardly be afforded by one nation. It is important to collect any evidence of new physics through low-energy experiments with new ideas, and to clarify the form of the new physics as precisely as possible so as to strengthen the foundation of such a direct experiment.

At a low-energy scale, the effects of new physics appear as non-trivial relations among renormalizable coupling constants, or the existence of higher dimensional operators which can not be renormalized. Precision measurements of gauge couplings and a determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements belong to the former case. In order to explore small effects of higher dimensional operators, processes which are forbidden, or are very suppressed, within the SM are important. There are a variety of such processes, including lepton-number violation, baryon-number violation, lepton-flavor violation (LFV) , CP violation, and flavor-changing neutral currents (FCNC).

Theoretically, LFV is a touchstone of new models. In the SM, all possible renormalizable interactions appear and lepton flavor conservation is automatic as a result of its matter content and gauge symmetry. However, an extension of its matter content easily violates lepton- flavor conservation. In the case of a supersymmetric extension of the SM, scalar partners of ordinary leptons become a source of LFV. It imposes severe constraints on the mechanisms of SUSY-breaking. There are many

SUSY-breaking scenarios which escape such constraints. The minimal supergravity model (minimal SUGRA) is the most conventional one.

In recent years, LFV processes have received much attention because it was pointed out that, in supersymmetric grand unified theory (SUSY GUT), the branching ratios for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ and the μ-e conversion rate in a nucleus can reach just below the present experimental values, even if the minimal SUGRA is assumed [6]-[9]. The present experimental upper bounds of these LFV processes are $B(\mu^+ \to e^+\gamma) \leq 1.2 \times 10^{-11}$ [10], $B(\mu^+ \to e^+e^+e^-) \leq 1.0 \times 10^{-12}$[11] and $\sigma(\mu^-\mathrm{Ti}\to e^-\mathrm{Ti})/\sigma(\mu^-\mathrm{Ti}\to \mathit{capture}) \leq 6.1 \times 10^{-13}$ [12]. It is possible that future experiments will improve the sensitivity by two or three orders of magnitude below the current bounds [13, 14].

In this thesis we discuss the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes in SUSY GUT. In particular, we focus on various asymmetries defined with the help of initial muon polarization. Experimentally, polarized positive muons are available by the surface muon method because muons emitted from π^+'s stopped at the target surface are 100% polarized in a direction opposite to the muon momentum [15]. Previously, it has not been positively utilized in rare decay experiments. It is shown in Ref.[16], however, that the muon polarization is useful to suppress the background processes in the $\mu^+ \to e^+\gamma$ search. As for the signal distribution of $\mu^+ \to e^+\gamma$, the angular distribution with respect to the muon polarization can distinguish between $\mu^+ \to e^+_L\gamma$ and $\mu^+ \to e^+_R\gamma$. In the case of $\mu^+ \to e^+e^+e^-$, the distribution in the Dalitz plot and various asymmetries defined with help of the muon polarization carry information on chirality and the Lorentz structure of LFV couplings. In particular, we can define T-odd asymmetry which is sensitive to CP violation in LFV interactions [17, 18]. The purpose of this paper is to give a model-independent framework for analyzing the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes with polarized muons and to investigate specific features of the SU(5) and SO(10) SUSY GUT while focusing on the P- and T-odd asymmetries. A detailed comparison of the T-odd asymmetry with the electron, neutron and Hg electric dipole moments (EDM) is also done by introducing SUSY CP violating phases within the minimal SUGRA model [19].

In chapter 2 and 3, we review SUSY GUT based on the minimal SUGRA model * in the context of flavor physics. In chapter 4, we first present the general

*More comprehensive review of this subject is found in reference [20].

effective Lagrangian for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes. Then, we explain how LFV coupling constants in this Lagrangian emerges in the SUSY GUT. We discuss the model-specific features of these processes in the SU(5) and SO(10) SUSY GUT. In chapter 5, we present a model-independent framework for analyzing the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes with polarized muons. We fix the kinematics for the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes and define various P- and T-odd asymmetries relative to the muon polarization in the center-of-mass system of the muon. In chapter 6, we present the results of our numerical calculations for these asymmetries in the SU(5) and SO(10) SUSY GUT.

Chapter 2

Minimal supersymmetric standard model (MSSM)

I N=1 SUSY Lagrangian

As discussed in the previous chapter, considering the hierarchy problem, SUSY is a very attractive solution because it reduces the degree of divergence of the theory. SUSY is a natural extension of the Poincaré group, which includes a fermionic generator. Consequently, it relates bosons and fermions, which are unified in the same multiplet. In the minimal case of N=1 SUSY, it is known that only wave-function renormalization must be taken into account in the quantum corrections (non-renormalization theorem) [21]. As a result, quadratic divergence does not appear in the theory.

Just as the Lorentz invariance exhibit enormous power to constrain any possible form of the Lagrangian, SUSY is also very powerful. It is very convenient to use a superfield formalism to treat the SUSY Lagrangian, because SUSY is manifest in this formalism, similar to the Lorentz invariant formalism in the case of the Poincaré group [22]. If we restrict ourselves to fields whose spin is less than one, there are two kinds of multiplets. One is a chiral superfield, Φ, whose physical degrees of freedom are one Weyl fermion field, ψ_α, and one complex scalar field, ϕ, called a sfermion. The other is a vector superfield, V, which describes gauge symmetry, and its physical degrees of freedom are one vector field, v_μ, which represents the gauge boson and one Majorana fermion field, λ_α, called gaugino. In this formalism, renormalizable SUSY Lagrangian is specified by only one holomorphic function of chiral superfields called superpotential $\mathcal{W}(\Phi_i)$. $\mathcal{W}(\Phi_i)$ is a cubic polynomial of chiral superfields, Φ_i, in the case of a renormalizable theory. From this superpotential, an ordinary interaction Lagrangian is derived by the following prescriptions:

$$\mathcal{L}_{int} = \mathcal{L}_f + \mathcal{L}_b, \tag{2.1}$$

$$\mathcal{L}_f = \frac{1}{2}\sum_{ij}\frac{\partial^2 \mathcal{W}}{\partial\phi_i\partial\phi_j}\overline{\psi_i^c}\psi_j + h.c., \tag{2.2}$$

$$\mathcal{L}_b = -\sum_i |\frac{\partial \mathcal{W}}{\partial\phi_i}|^2. \tag{2.3}$$

Note that, in this formula, the fermionic interaction and the bosonic interaction come from the same function, $\mathcal{W}(\Phi_i)$, and their coupling constants are related in a non-trivial manner. In particular, the mass terms of the fermion and the sfermion come from the same bilinear term of the superpotential, and their masses become degenerate.

In N=1 SUSY, in addition to ordinary gauge couplings, which are represented by the covariant derivative $D_\mu = \partial_\mu + igv_\mu^a(T^a)$, two kinds of new gauge couplings appear, as follows:.

$$\begin{aligned}
\mathcal{L}_{gauge} &= (\text{non-SUSY}) - \sqrt{2}g(\overline{\psi_{i\alpha}}(T^a)^\alpha_\beta\lambda^a\phi_i + h.c.) - \frac{1}{2}g^2\sum_a(D^a)^2, \\
D^a &= \sum_i\sum_{\alpha,\beta}(\phi_i^*)^\alpha(T^a)^\beta_\alpha(\phi_i)_\beta,
\end{aligned} \tag{2.4}$$

where g denotes the gauge coupling constant and T^a is a generator of the gauge symmetry. In this formula, the second term is a gaugino-sfermion-fermion coupling and the third term is a sfermion self coupling. Note that in the third term gauge fields do not appear.

II Matter contents and Lagrangian

In the SM, there are five kinds of $SU(3) \times SU(2)_L \times U(1)_Y$ gauge multiplets of a left-handed fermion: q_i, u_i^c, d_i^c, l_i and e_i^c. Here, superscript c denotes the charge conjugation of a right-handed field. They have three sets of copies called generation, and subscript i represents the generation indices ($i = 1 - 3$). In addition to these matter fermions, there is also one scalar multiplet called the Higgs field, h. Their transformation properties and charge assignment under the SM gauge group $SU(3)\times SU(2)_L \times U(1)_Y$ are summarized in Table 2.1.

	q_i	u_i^c	d_i^c	l_i	e_i^c	h
$SU(3)$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
$U(1)_Y$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	1	$-\frac{1}{2}$
B	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0	0
L_i	0	0	0	1	-1	0

Table 2.1: The transformation properties and charge assignment of the SM fields.

The interaction Lagrangian of the SM is described as follows:

$$\mathcal{L}_{int} = \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}, \tag{2.5}$$

$$\begin{aligned}\mathcal{L}_{Yukawa} &= -\epsilon^{\alpha\beta} y_{e_{ij}} h_\alpha \overline{e_i} l_{j\beta} - \epsilon^{\alpha\beta} y_{d_{ij}} h_\alpha \overline{d_{i\hat{a}}} q_{j\,\beta}^{\hat{a}} \\ &\quad - y_{u_{ij}} h^{*\alpha} \overline{u_{i\hat{a}}} q_{j\,\beta}^{\hat{a}},\end{aligned} \tag{2.6}$$

$$\mathcal{L}_{Higgs} = -\mu^2 h^{*\alpha} h_\alpha - \frac{\lambda}{2}(h^{*\alpha} h_\alpha)^2, \tag{2.7}$$

where $\hat{a}$ denotes the $SU(3)$ index and α, β denote the $SU(2)$ indices. In this formula the $SU(2)$ invariant tensor, $\epsilon^{\alpha\beta}$, is defined as $\epsilon^{11} = \epsilon^{22} = 0$, $\epsilon^{12} = -\epsilon^{21} = 1$. The square of the Higgs mass, μ, in Eq.(2.7) must be negative, to break $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. Note that all possible renormalizable interactions which are allowed by the gauge symmetry $SU(3)\times SU(2)_L \times U(1)_Y$ appear in the above formula. Whereas the Yukawa interaction Lagrangian, Eq.(2.6), has global symmetries called the baryon number, B, and the lepton number, L. Among them, the lepton number is conserved for each generation (lepton flavor conservation). Their charge assignment is also summarized in table 2.1.

To construct a minimal extension of the SM, we need scalar partners for matter fermions called squarks ; $\widetilde{q}$, $\widetilde{u}^*$, $\widetilde{d}^*$ and sleptons ; $\widetilde{l}$, $\widetilde{e}^*$ and a fermionic partner for Higgs scalar called higgsino, $\widetilde{h}$. We use tilde as a symbol of a SUSY partner. They are unified in a chiral superfield so that $Q\{q, \widetilde{q}\}$, $U^c\{u^c, \widetilde{u}^*\}$, $D^c\{d^c, \widetilde{d}^*\}$, $L\{l, \widetilde{l}\}$,

$E^c\{e^c, \tilde{e}^*\}$ and $H\{\tilde{h}, h\}$. The $SU(3)$, $SU(2)_L$ and $U(1)_Y$ gauge bosons (G_μ, W_μ and B_μ) also have fermionic partners called gluino ($\tilde{G}$), wino ($\widetilde{W}$) and bino ($\tilde{B}$) respectively. In the N=1 SUSY formalism, the Yukawa interaction comes from Eq.(2.2), and the scalar potential comes from Eq.(2.3). Comparing Eq.(2.6), (2.7) and Eq.(2.2), (2.3) the superpotential for the minimal supersymmetric standard model (MSSM) can be written as follows:

$$\begin{aligned} \mathcal{W}_{MSSM} &= \epsilon^{\alpha\beta}(y_e)_{ij}H_{1\alpha}E^c_i L_{j\beta} + \epsilon^{\alpha\beta}(y_d)_{ij}H_{1\alpha}D^c_i Q_{j\beta} \\ &\quad + \epsilon^{\alpha\beta}(y_u)_{ij}H_{2\alpha}U^c_i Q_{j\beta} + \epsilon^{\alpha\beta}\mu H_{1\alpha}H_{2\beta}, \end{aligned} \tag{2.8}$$

where we note that two Higgs doublets, H_1 and H_2, are needed even in the minimal case, because the superpotential must be a holomorphic function of chiral superfields. It is also demanded by a cancellation of the chiral anomaly which we will discuss in the section I of the chapter 3. Note that any self-interaction of the Higgs bosons can not come from a superpotential in the minimal case, because of the gauge invariance. The self-interaction of Higgs bosons alternatively comes from the gauge couplings Eq.(2.4). The square of the mass of the Higgs bosons, $|\mu|^2$, always becomes positive from Eq.(2.3). We can relax this condition in the next section.

In contrast to the SM, the superpotential, Eq.(2.8), is not maximally allowed by the gauge invariance. Because we introduced many scalar partners to the SM, new interactions can be added to the Lagrangian, Eq.(2.5). New contributions to the superpotential can be parameterized as follows:

$$\begin{aligned} \mathcal{W}_{\not{R}} &= \epsilon^{\alpha\beta}\lambda_{ijk}L_{i\alpha}E^c_j L_{k\beta} + \epsilon^{\alpha\beta}\lambda'_{ijk}L_{i\alpha}D^c_j Q_{k\beta} \\ &\quad + \lambda''_{ijk}U^c_i D^c_j D^c_k + \epsilon^{\alpha\beta}\mu_i L_{i\alpha}H_{2\beta}. \end{aligned} \tag{2.9}$$

This superpotential violates the global symmetry of the SM. The first term, the second term and the fourth term violate lepton-number conservation. The third term violates baryon-number conservation. Combinations of these coupling constants are severely constrained from proton-decay experiments. Although the possibilitiy of the existence of these renormalizable coupling constants is a very interesting subject, we do not treat this possibility in this thesis. Alternatively, we simply forbid these coupling constants by imposing a discrete symmetry, called R-parity, on the theory. We assign R-parity + to the Higgs superfields H_1 and H_2 and R-parity - to matter superfields Q_i,U_i^c,D_i^c,L_i and E_i^c. This discrete symmetry forbids a superpotential, Eq.(2.9) *

III SUSY braking and flavor problem in the MSSM

In the previous section, we constructed the minimal SUSY extension of the SM. This exact SUSY model is, however, phenomenologically unacceptable, because the square of the Higgs mass is always positive and can not break the $SU(2)_L \times U(1)_Y$ gauge symmetry; also all scalar partners of the SM fields and gauginos which have not yet been discovered are degenerate with their SUSY partners. In order to exclude the quadratic divergence from the theory, an exact SUSY is not necessary. Some interactions which violate SUSY can be added without quadratic divergence [23]. The first one is a mass term of the sfermion, and the second one is a mass term of gaugino; the third one is a sfermion self-interaction, which can be obtained from a holomorphic function, like the superpotential. They are called soft SUSY breaking terms. In the case of the MSSM, the soft SUSY breaking terms can be parameterized as follows:

$$\begin{aligned}\mathcal{L}_{soft} = & -(m_E^2)_{ij}\widetilde{E}_i^*\widetilde{E}_j - (m_L^2)_{ij}\widetilde{L_i}^*\widetilde{L_j} - (m_D^2)_{ij}\widetilde{D}_i^*\widetilde{D}_j \\ & -(m_U^2)_{ij}\widetilde{U}_i^*\widetilde{U}_j - (m_Q^2)_{ij}\widetilde{Q}_i^*\widetilde{Q}_i - m_{H_1}^2 H_1^\dagger H_1 - m_{H_2}^2 H_2^\dagger H_2 \\ & -[(A_e)_{ij}\epsilon^{\alpha\beta}H_{1\alpha}\widetilde{E}_i^*\widetilde{L}_{j\beta} + (A_d)_{ij}\epsilon^{\alpha\beta}H_{1\alpha}\widetilde{D}_i^*\widetilde{Q}_{j\beta} \\ & +(A_u)_{ij}\epsilon^{\alpha\beta}H_{2\alpha}\widetilde{U}_i^*\widetilde{Q}_{j\beta} + \epsilon^{\alpha\beta}\mu B H_{1\alpha}H_{2\beta} \\ & +\frac{1}{2}M_1\overline{\widetilde{B}_R}\widetilde{B_L} + \frac{1}{2}M_2\overline{\widetilde{W}_R}\widetilde{W_L} + \frac{1}{2}M_3\overline{\widetilde{G}_R}\widetilde{G_L} + h.c.]. \end{aligned} \tag{2.10}$$

Because of the above new interactions, we can break $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. We can also push the masses of the SUSY partners above their experimental bounds. These masses, however, can not become much larger than the EW scale in order to avoid a hierarchy problem.

In the above formula, the mass parameters of the sfermions (m_Q^2, m_U^2, m_D^2, m_L^2 and m_E^2) and trilinear sfermion self-couplings (A_u, A_d and A_e) are 3×3 com-

*R symmetry assigns different charges to SUSY partners. In this case, consequently, the SM fields and the extra Higgs scalar have R-parity + and their SUSY partners have R-parity -.

plex matrices, which generally can not be diagonalized simultaneously with the Yukawa coupling constants in the superpotential, Eq.(2.8), which determine the fermion masses. They can become sources of the FCNC processes and LFV processes. Thus, the form of the soft SUSY breaking terms is constrained from such processes. For example [24], if we assume mixing on the order of the CKM matrix elements, $\epsilon \simeq \sin\theta_C$, the mass difference of the first- and second-generation squarks, $\Delta m_{\tilde{q}}$, is constrained from K^0-$\overline{K^0}$ mixing (See Fig.2.1) , so that

$$\frac{1}{M_2^2}(\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2})^2 \quad < \quad O(10^{-7}) \text{ GeV}^{-2}. \tag{2.11}$$

In the case of the lepton sector, that of sleptons are constrained from $\mu \to e\gamma$ (See Fig.2.2) so that

$$\frac{1}{M_1^2}(\frac{\Delta m_{\tilde{l}}^2}{m_{\tilde{l}}^2}) \quad < \quad O(10^{-7}) \text{ GeV}^{-2}. \tag{2.12}$$

If we assume $M_1 \simeq M_2 \simeq 100$ GeV, these constraints can be read as

$$(\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}) \quad < \quad O(\frac{1}{30}), \tag{2.13}$$

$$(\frac{\Delta m_{\tilde{l}}^2}{m_{\tilde{l}}^2}) \quad < \quad O(10^{-3}). \tag{2.14}$$

Because we introduced about one hundred new parameters, another fine-tuning problem arises (SUSY flavor problem). There are three possibilities to escape such phenomenological constraints. In the first case, sfermion masses in the same gauge multiplet are nearly degenerate. [20, 25, 26] In the second case, the fermion mass matrices and the sfermion mass matrices are nearly aligned [27]. In the third case, soft SUSY-breaking masses, which are relevant to the flavor problem, are much larger than the EW scale in a particular manner with which the hierarchy problem does not relapse [28]. In any case, to justify such a special pattern of the soft SUSY-breaking parameters, we can not avoid to discuss the origin of SUSY breaking. In the next section, we introduce a typical model of SUSY breaking which realize such constraints.

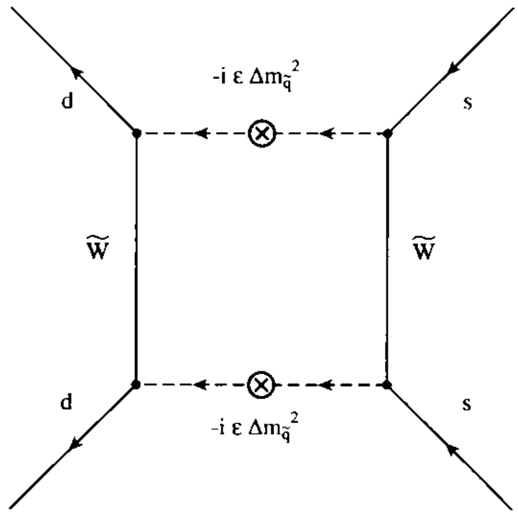


FIG. 2.1: Feynman diagram which induces K^0-$\overline{K}^0$ mixing. The circle-crosses represent insertion of flavor changing off-diagonal elements of the squark mass matrix.

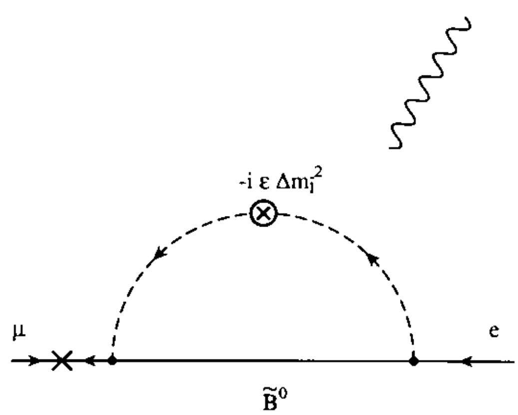


FIG. 2.2: Feynman diagram which induces $\mu \to e\gamma$. The cross denotes chirality flip by the muon mass. The circle-cross represents insertion of lepton flavor violating off-diagonal element of the slepton mass matrix.

IV Minimal supergravity model

We know that a fundamental theory must include the SM and the theory of gravity in accordance with quantum mechanics in a way we have not yet understood. We have introduced SUSY as a solution for the hierarchy problem of the SM. On the other hand, SUSY is an extension of the Poincaré group and its further extension to a local symmetry naturally includes gravity. Such an extension of SUSY to a local symmetry is called supergravity (SUGRA). Unfortunately, there is no consistent quantum theory of SUGRA. However, it is not unreasonable to expect that SUGRA describes an effective theory of the fundamental theory at the Planck scale.

If we extend a global N=1 SUSY model such as the MSSM to SUGRA, all of the interactions of matter chiral superfields , Φ_i, are represented by one function called the Kähler potential:

$$G = -3log(-\frac{\mathcal{K}}{3}) - log(|\mathcal{W}|^2) \quad (2.15)$$

where $\mathcal{W}(\Phi_i)$ is the superpotential of the global SUSY model. A function $\mathcal{K}(\Phi_i^\dagger e^{2gV}, \Phi_j)$ is given by

$$\mathcal{K}(\Phi_i^\dagger e^{2gV}, \Phi_j) = \Phi_i^\dagger e^{2gV} \Phi_i - 3 \quad (2.16)$$

in the case of the renormalizable global SUSY model.

In this setup of SUGRA, we can construct a model which spontaneously breaks SUSY and reproduces the soft SUSY-breaking terms in Eq.(2.10)[29]. In addition to the MSSM sector, this model has a hidden sector which does not couple to the SM gauge group, $SU(3) \times SU(2)_L \times U(1)_Y$, as follows:

$$\mathcal{W} = \mathcal{W}_{MSSM} + \mathcal{W}_{hidden}. \quad (2.17)$$

We assume that SUSY is spontaneously broken in the hidden sector. In the global SUSY case, these two sectors are completely isolated and the SUSY-breaking effects cannot be observed in the MSSM sector. If we couple these two sectors in SUGRA, Those SUSY-breaking effects in the hidden sector are mediated to the MSSM sector thorough higher dimensional operators suppressed by the Planck mass, M_P.

At the Planck scale, it is not reasonable to constrain ourselves to a renormalizable theory, because N=1 SUGRA, itself, can not be renormalized. Thus, there are ambiguities to obtain the Kähler potential which is consistent with the renormalizable superpotential, Eq.(2.17), at the low-energy scale. If we take the leading terms of the Kähler potential expanded by the bilinears of chiral superfields, $\Phi_i^\dagger \Phi_i$ (minimal SUGRA) , the effects of SUSY breaking are parameterized by only four parameters: m_0, M_0, A_0 and B_0 [†]. In such an approximation, we obtain a universal boundary condition for the soft SUSY-breaking parameters at the Planck scale, as follows:

$$
\begin{aligned}
& m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = m_0 \mathbf{1}, \\
& (A_u)_{ij} = m_0 A_0 (y_u)_{ij}, \ (A_d)_{ij} = m_0 A_0 (y_d)_{ij}, \ (A_l)_{ij} = m_0 A_0 (y_l)_{ij}, \\
& B = B_0, \\
& M_1 = M_2 = M_3 = M_0. \qquad (2.18)
\end{aligned}
$$

Then the Lagrangian for soft SUSY-breaking terms becomes as follows:

$$
\begin{aligned}
\mathcal{L}_{Soft} &= -m_0 (\widetilde{q}_i^{\alpha *} \widetilde{q}_{i\alpha} + \widetilde{u}_i^* \widetilde{u}_i + \widetilde{d}_i^* \widetilde{d}_i + \widetilde{l}_i^{\alpha *} \widetilde{l}_{i\alpha} + \widetilde{e}_i^* \widetilde{e}_i) \\
& \quad -[A_0 \{ (y_l)_{ij} \epsilon^{\alpha\beta} h_{1\alpha} \widetilde{e}_i^* \widetilde{l}_{i\beta} + (y_d)_{ij} \epsilon^{\alpha\beta} h_{1\alpha} \widetilde{d}_i^* \widetilde{q}_{i\beta} + (y_u)_{ij} \epsilon^{\alpha\beta} h_{2\alpha} \widetilde{u}_i^* \widetilde{q}_{i\beta} \} \\
& \quad + B_0 \mu \epsilon^{\alpha\beta} h_{1\alpha} h_{2\beta} + \frac{1}{2} M_0 (\overline{\widetilde{B}_R} \widetilde{B}_L + \overline{\widetilde{W}_R} \widetilde{W}_L + \overline{\widetilde{G}_R} \widetilde{G}_L) + h.c.] \qquad (2.19)
\end{aligned}
$$

With these boundary conditions, all of the mass matrices of the scalar partners can be diagonalized simultaneously with the Yukawa couplings, and there is no SUSY flavor problem at the Planck scale.

The above approximation of the Kähler potential can not be always justified. A non-minimal form of the Kähler potential could break this universal boundary conditions. However, this ansatz is very useful as a first approximation to discuss low-energy phenomenology, because it simplifies the model enormously. We do not discuss the case of a non-minimal Kähler potential in this thesis.

V Radiative corrections and mass spectrum of the minimal SUGRA

Even if we take the universal boundary condition at the Planck scale, a radiative correction between the Planck scale and the EW scale modifies the mass spectrum

[†]If we assume the soft SUSY braking terms are determined exactly from the superpotential (2.8) at the Planck scale, B_0 is related to A_0 so that $B_0 = A_0 - 1$. We, however, take B_0 as a free parameter because the origin of the dimensional parameter, μ, in the superpotential is not understood.

of the SUSY partners. A complete set of 1-loop renormalization group equations (RGEs) for the MSSM are summarized in Appendix A. The radiative corrections are classified into two parts. The first part comes from gauge interactions, and the second part comes from Yukawa interactions. The first part is universal for the flavor structure, and determines the rough spectrum of the SUSY partners. The second part is negligibly small relative to the first part, except for the top Yukawa coupling constant. The second part, however, distinguishes the flavor structure. We discuss the fist part in this section and the effects of the top Yukawa coupling are discussed in the next section.

The RGEs for the gauge coupling constants and the gaugino masses are given in Appendix A as follows:

$$(4\pi)^2 M\frac{d}{dM}g_i = b_i g_i^3, \tag{2.20}$$

$$(4\pi)^2 M\frac{d}{dM}M_i = 2b_i g_i^2 M_i, \tag{2.21}$$

where g_3, g_2 and g_1 are the gauge coupling constants for $SU(3)$, $SU(2)_L$ and $U(1)_Y$, respectively. The normalization of the $U(1)$ charge is modified from table 2.1 so that the gauge coupling constant is defined with an extra factor, $\sqrt{\frac{5}{3}}$, because of a reason which will be explained in section I of chapter 3. The coefficients for beta functions are $b_3 = -3$, $b_2 = 1$ and $b_1 = \frac{33}{5}$. If we neglect the Yukawa coupling constants, the RGEs for the soft SUSY-breaking masses of the sfermions are given as follows:

$$(4\pi)^2 M\frac{d}{dM}m_X^2 = -8(c_X^3 g_3^2 M_3^2 + c_X^2 g_2^2 M_2^2 + c_X^1 g_1^2 M_1^2), \tag{2.22}$$

where X distinguishes kinds of sfermions ($X = \{Q, U, D, L, E\}$) and we omit the flavor indices, because the gauge interactions do not distinguish them. In the above formula, c_X^3,c_X^2 and c_X^1 are the quadratic Casimir of the gauge groups and $U(1)$ charge factors. They are given in Table 2.2. This formula is easily solved analytically. If we define $\alpha_i \equiv \frac{g_i^2}{4\pi}$, the solution is described as follows:

$$\frac{1}{\alpha_{iP}} = \frac{1}{\alpha_i} - \frac{b_i}{2\pi}\ln(\frac{M_P}{M_W}), \tag{2.23}$$

$$M_i = (\frac{\alpha_i}{\alpha_{iP}})M_0, \tag{2.24}$$

$$m_X^2 = m_0^2 - 2\{\frac{c_X^3}{b_3}(\frac{\alpha_3^2}{\alpha_{3P}^2} - 1) + \frac{c_X^2}{b_2}(\frac{\alpha_2^2}{\alpha_{2P}^2} - 1) + \frac{c_X^1}{b_1}(\frac{\alpha_1^2}{\alpha_{1P}^2} - 1)\}M_0^2, \tag{2.25}$$

X	Q	U	D	L	E	H_1	H_2
c_X^3	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	0	0	0	0
c_X^2	$\frac{3}{4}$	0	0	$\frac{3}{4}$	0	$\frac{3}{4}$	$\frac{3}{4}$
c_X^1	$\frac{1}{60}$	$\frac{4}{15}$	$\frac{1}{15}$	$\frac{3}{20}$	$\frac{3}{5}$	$\frac{3}{20}$	$\frac{3}{20}$

Table 2.2: The quadratic Casimir of $SU(3)$ and $SU(2)$ and $U(1)$ charge factors which appear in the RGEs of soft SUSY breaking masses.

where M_P and M_W denote the Planck scale and the EW scale, respectively. Subscript P means the value at the Planck scale. If we input experimental values $\alpha_3 \simeq 0.12$, $\alpha_2 \simeq 0.034$ and $\alpha_1 \simeq 0.017$, the soft SUSY-breaking masses at the EW scale become as follows:

$$\begin{aligned}
M_3 &\simeq 3.2M_0, \qquad M_2 \simeq 0.8M_0,\\
M_1 &\simeq 0.33M_0,\\
m_Q^2 &\simeq m_0^2 + 8.5M_0^2, \qquad m_L^2 \simeq m_{H_1}^2 \simeq m_0^2 + 0.6M_0^2,\\
m_U^2 &\simeq m_0^2 + 8M_0^2, \qquad m_E^2 \simeq m_0^2 + 0.16M_0^2,\\
m_D^2 &\simeq m_0^2 + 8M_0^2,
\end{aligned} \tag{2.26}$$

Note that, in this approximation, the soft SUSY-breaking masses of the Higgs scalars, $m_{H_1} \simeq m_{H_2}$, are always positive, and that $SU(2)_L \times U(1)_Y$ can not break to $U(1)_{em}$. These formulas, however, are not correct for the third-generation squarks and H_2, because the top Yukawa coupling is comparable to the gauge couplings and can not be neglected in the RGE.

VI Radiative electroweak symmetry breaking

The top Yukawa contribution to the RGEs are written as follows:

$$(4\pi)^2 M\frac{d}{dM}m_{Q_3}^2 = (\text{gauge}) + 2y_t^\dagger y_t(m_{Q_3}^2 + m_{U_3}^2 + m_{H_2}^2 + A_t^\dagger A_t), \tag{2.27}$$

$$(4\pi)^2 M\frac{d}{dM}m_{U_3}^2 = (\text{gauge}) + 4y_t^\dagger y_t(m_{Q_3}^2 + m_{U_3}^2 + m_{H_2}^2 + A_t^\dagger A_t), \tag{2.28}$$

$$(4\pi)^2 M\frac{d}{dM}m^2_{H_2} = (\text{gauge}) + 6y_t^\dagger y_t(m^2_{Q_3} + m^2_{U_3} + m^2_{H_2} + A_t^\dagger A_t). \quad (2.29)$$

The effect of these contributions is to reduce the soft SUSY-breaking masses of the third-generation squarks and H_2 in Eq.(2.26). Note that the above formulas have a 1-2-3 structure, so that the contribution for H_2 is just three-times larger than that for Q_3. Also, the contribution for U_3 is just two-times larger than that for Q_3 This structure can be understood as a group factor. H_2 is an $SU(3)$ singlet, and the indices of $SU(3)$ of the top Yukawa coupling constant must be closed in loop diagrams which represent radiative corrections for $m^2_{H_2}$ from the top Yukawa coupling. The trace of the **3** representation of $SU(3)$ produces a factor of 3. U_3 is an $SU(2)$ singlet, and the indices of $SU(2)$ of the top Yukawa coupling constant must be closed in the loop diagrams which contribute to $m^2_{U_3}$. Thus, the trace of the **2** representation of $SU(2)$ produces a factor 2. Because of this group factor, $m^2_{H_2}$ is reduced most, and can become negative without breaking the $SU(3)$ symmetry (radiative EW symmetry breaking) [30].

If we keep only the top Yukawa coupling constant in the RGEs in addition to the contribution of the gauge interactions, they can still be solved analytically, except for one integral [31]. Fig.2.3 shows an example for a particular parameter set.

The Higgs potential for the MSSM is calculated from Eq.(2.3), Eq.(2.4) and Eq.(2.8) as follows:

$$\begin{aligned}\mathcal{V}_{Higgs} &= m_1^2|h_1^0|^2 + m_2^2|h_2^0|^2 + (m_3^2 h_1^0 h_2^0 + h.c.) \\ &\quad + \frac{1}{8}(\frac{3}{5}g_1^2 + g_2^2)(|h_1^0|^2 - |h_2^0|^2)^2, \end{aligned} \quad (2.30)$$

where m_1^2, m_2^2 and m_3^2 are determined by the soft SUSY-breaking parameters and the μ parameter as follows:

$$\begin{aligned} m_1^2 = m^2_{H_1} + |\mu|^2, &\qquad m_2^2 = m^2_{H_2} + |\mu|^2, \\ m_3^2 &= B\mu. \end{aligned} \quad (2.31)$$

Note that quadratic terms are completely determined from the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants. In addition to m_0, M_0 and A_0, if we have boundary conditions for μ and B, we can obtain the Higgs potential. Because the quadratic

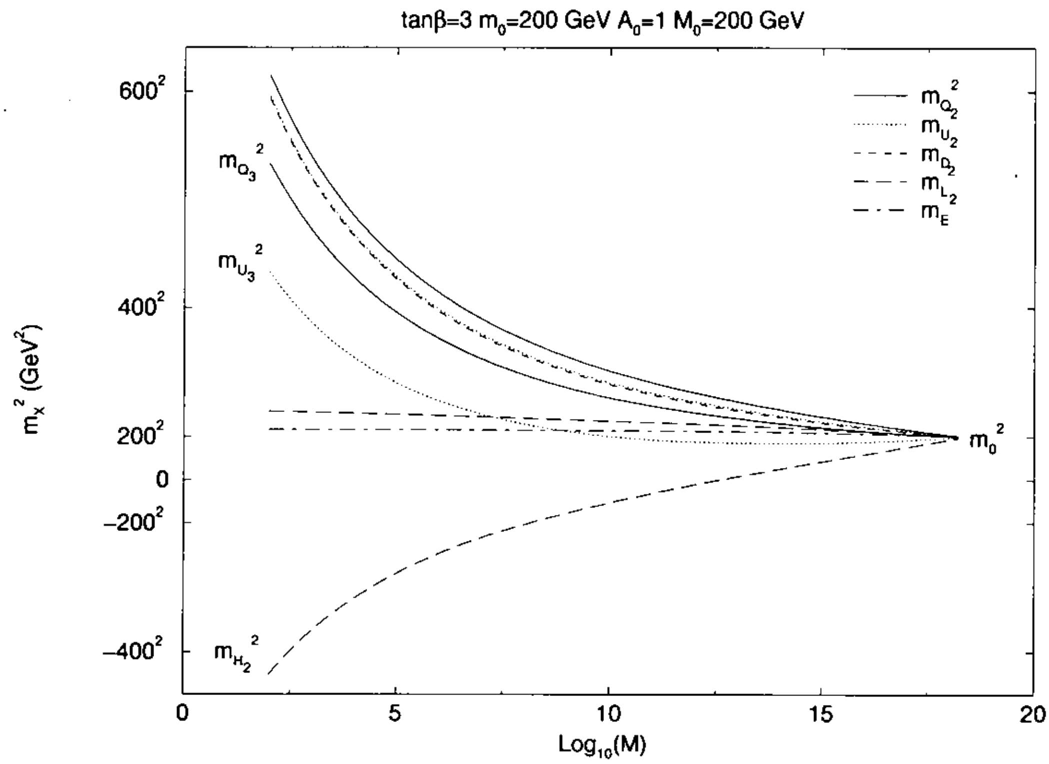


FIG. 2.3: Running of the soft breaking masses in the minimal SUGRA. We take the top quark mass as 175 GeV and the ratio of two Higgs vacuum expectation values $\tan\beta$ equal to 3. As the initial condition of the minimal SUGRA, we take $m_0 = 200\text{GeV}$, $A_0 = 1$ and $M_0 = 200\text{GeV}$.

term has a flat direction, $|h_1^0| = |h_2^0|$, the mass parameters must satisfy the following stability condition:

$$m_1^2 + m_2^2 - 2|m_3|^2 \;\; > \;\; 0. \tag{2.32}$$

In order to break the $SU(2)_L \times U(1)_Y$ symmetry, these parameters also must fulfill the condition

$$m_1^2 m_2^2 - |m_3|^2 < 0. \tag{2.33}$$

In the numerical calculations of this thesis, however, we rather solve μ and B at the EW scale to obtain correct vacuum expectation values (VEVs), which we want, because we do not know the origin of dimensional parameter, μ, in the renormalizable superpotential, which takes a value on the order of the EW scale (μ problem). If we define $\tan\beta$ as the ratio of two VEVs of Higgs fields so that $\tan\beta = \frac{\langle h_2^0 \rangle}{\langle h_1^0 \rangle}$, μ and B can be solved from the minimum of Eq.(2.30) as follows:

$$|\mu|^2 = -(2m_{H_1}^2 + \frac{m_Z^2}{2}) + \frac{2}{1-\tan^{-2}\beta}(m_{H_1}^2 - m_{H_2}^2), \tag{2.34}$$

$$|B\mu| = \frac{\sin 2\beta}{2}\{m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2\}, \tag{2.35}$$

where $m_Z = \frac{1}{2}(\frac{3}{5}g_1^2 + g_2^2)(< h_1^0 >^2 + < h_2^0 >^2)$ is the Z boson mass. The stability condition Eq.(2.32) and the symmetry-breaking condition Eq.(2.33) are translated into constraints that these solutions must be positive.

After the Higgs bosons acquire VEVs at the EW scale, the gauginos and higgsinos are mixed with each other. As a consequence, they form Dirac fields, which are called chargino χ_A^- $(A = 1, 2)$, and Majorana fields, which are called neutralino χ_B^0 $(B = 1-4)$. On the other hand, the sfermions obtain additional masses from the Yukawa and gauge interactions with the Higgs bosons. In addition, they are mixed by the left-right mixing masses, which come from the Yukawa couplings in the superpotential and trilinear scalar couplings in the soft SUSY-breaking terms. Their mass matrices and their diagonalization are summarized in Appendix B.

VII SUSY CP-violating phases in the minimal supergravity model

In addition to the Kobayashi-Maskawa (KM) phase in the SM, the soft SUSY-breaking parameters (M_0, A_0, B_0) in Eq.(2.19) and the μ parameter in Eq.(2.8) can have complex phases. They, however, are not all physically independent. As physical phases of the minimal SUGRA model, we can choose the phase of A_0 and the phase of μ. The phase of the universal gaugino mass, ϕ_{M_0}, can be absorbed by gaugino fields $\tilde{B}$,$\widetilde{W}$ and $\tilde{G}$. To retain Eq.(2.4), we can also rotate the phase of the Weyl fermion fields, Ψ_i, by $\frac{\phi_{M_0}}{2}$. To keep Eq.(2.2), this means that we rotate the total phase of the superpotential by $-\phi_{M_0}$. From Eq.(2.2) and (2.8) we can absorb this total phase by rotating the Higgs fields h_1,h_2 and their fermionic partners simultaneously by $-\phi_{M_0}$, and also rotating the phase of μ by ϕ_{M_0}. The net result is a μ parameter with an additional phase, ϕ_{M_0}. The phase of B_0 can be absorbed by rotating the phase of the Higgs field ,h_2, and its fermionic partner simultaneously. It then appears as a phase of μ and y_u. The total phase of y_u can be absorbed by rotating the phase of right-handed up-type quarks, u_i^c, and their scalar partners simultaneously. The net result is also a μ parameter with an additional phase. These new phases cause electric dipole moments (EDMs) of various particles through 1-loop diagrams, including SUSY partners [40, 41]. We summarize the useful formulas for the EDMs in Appendix F.

Chapter 3

Supersymmetric grand unified theory (SUSY GUT)

I Gauge-coupling unification in the MSSM

We introduced SUSY from a purely theoretical reasoning to remove any fine tuning from the SM. We then constructed the MSSM as the minimal SUSY extension of the SM. Nevertheless, there is a strong experimental indication that the MSSM is a part of nature.

The SM gauge group, $SU(3) \times SU(2)_L \times U(1)_Y$, includes an Abelian subgroup, $U(1)_Y$. In general, normalization of the $U(1)$ generator is arbitrary. However, the $U(1)$ charges of different $SU(3) \times SU(2)_L$ multiplets listed in the table 2.1 are quantized by $\frac{1}{6}$. This fact seems to be unnatural. There is an another "miracle" that the chiral anomaly of the SM is canceled. In other words, the sum of the $U(1)$ charge listed in table 2.1 and the sum of its cube become zero if we take into account the $SU(3)$ and $SU(2)_L$ indices, as follows:

$$(\frac{1}{6}) \times 6 + (-\frac{2}{3}) \times 3 + (\frac{1}{3}) \times 3 + (\frac{1}{2}) \times 2 + (-1) = 0,$$

$$(\frac{1}{6})^3 \times 6 + (-\frac{2}{3})^3 \times 3 + (\frac{1}{3})^3 \times 3 + (\frac{1}{2})^3 \times 2 + (-1)^3 = 0. \tag{3.1}$$

This fact seems to be very unnatural. This condition is indispensable for the SM to be a consistent quantum gauge theory. These facts can be explained naturally if we identify the SM multiplets as being parts of the representations of some kind of simple Lie group, G, which includes the SM group as its subgroup. In such a context, the SM can be realized as part of a unified theory which has gauge symmetry G. Such a unified theory is called a grand unified theory (GUT).

The minimal choice of the GUT group, G, is $SU(5)$ [32, 33, 34]. To cast the SM gauge group in $SU(5)$, we must multiply the factor $\sqrt{\frac{3}{5}}$ to the $U(1)$ charge listed in table 2.1 because the normalization of the $SU(5)$ generators is determined by their commutation relations. Fig.3.1 shows the running of the gauge coupling constants for the MSSM and the SM. They are calculated from Eq.(2.21) while taking into account the $U(1)$ normalization factor. In the MSSM case, these three coupling constants meet at one point of the energy scale, $M_G \simeq 2 \times 10^{16}$. This is a very non-trivial result, because these coupling constants are completely independent in the MSSM. The GUT hypothesis described above can explain this fact naturally. The SM gauge group was originally a part of the gauge group G characterized by one gauge coupling constant above the energy scale M_G. The three coupling constants of the MSSM are split by the radiative corrections after this original symmetry G is broken to $SU(3) \times SU(2)_L \times U(1)_Y$ at the GUT scale M_G. There are various choice of the GUT group G which include the SM group. In this thesis, we discuss the minimal case of $SU(5)$ and the next-minimal case of $SO(10)$ which includes $SU(5)$ as its subgroup [35].

II Minimal SU(5) and SO(10) GUT

The SM group $SU(3) \times SU(2)_L \times U(1)_Y$ include 4 generators which can be diagonalized simultaneously. Then, the rank of the GUT group must be equal to or lager than 4. $SU(5)$ is a simple Lie group of rank 4. The $SU(3)$ index $\hat{a} = 1-3$ and the $SU(2)_L$ index $\alpha = 1, 2$ can be cast in the index of a fundamental representation of $SU(5)$ so that $a = \{\hat{a}, \alpha\}$. Then, the matter superfields in the MSSM can be unified in the **10** representation and the $\bar{\mathbf{5}}$ representation of $SU(5)$ as follows:

$$(T_i)^{ab} = \left(\begin{array}{c|c} \epsilon^{\hat{a}\hat{b}\hat{c}} U^c_{\hat{c}} & -Q^{\hat{a}\beta} \\ \hline Q^{T\alpha\hat{b}} & -\epsilon^{\alpha\beta} E^c \end{array} \right), \quad (\overline{F}_i)_a = \left(\begin{array}{c} D^c_{\hat{a}} \\ \hline -\epsilon_{\alpha\beta} L^\beta \end{array} \right). \tag{3.2}$$

Two Higgs doublets in the MSSM can be cast in the **5** and $\bar{\mathbf{5}}$ representations of $SU(5)$ if we introduce colored Higgs superfields, ${H_C}^{\hat{a}}$ and $\overline{H}_{C\hat{a}}$, in addition to the

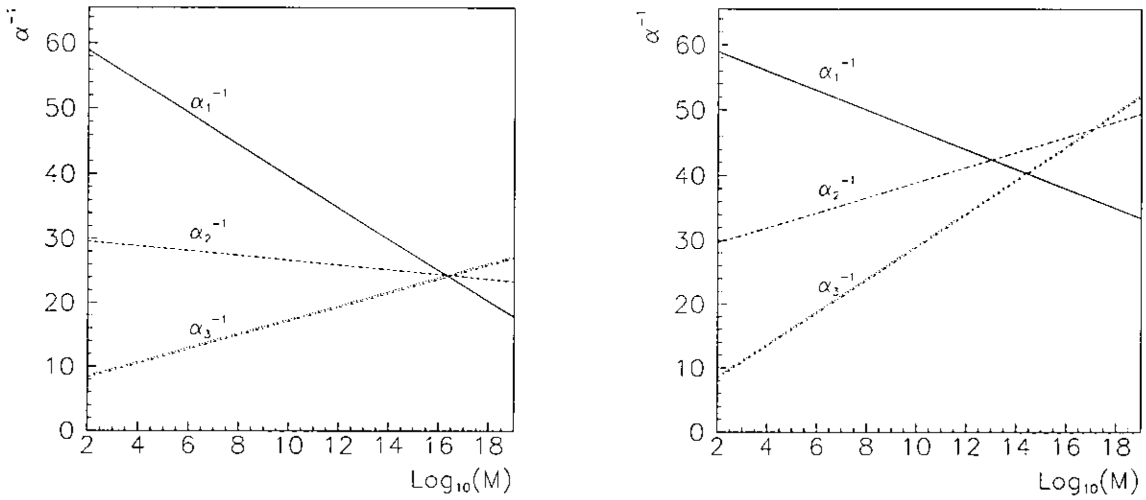


FIG. 3.1: Running of gauge coupling constants in the MSSM (left) and the SM (right). M indicates the renormalization scale in GeV. α_1, α_2 and α_3 denote the square of the gauge coupling constants normalized by 4π for $U(1)$, $SU(2)$ and $SU(3)$ respectively. They are calculated from Eq.(2.21) in the previous chapter. In the SM, the coefficients of the beta functions are calculated so that $b_3 = -7$, $b_2 = -\frac{19}{6}$ and $b_1 = \frac{41}{10}$. A general formula is given in Appendix A.

MSSM superfields as follows:

$$(H)^a = \left(\begin{array}{c} H_C{}^{\hat{a}} \\ \hline H_2^\alpha \end{array} \right), \quad (\overline{H})_a = \left(\begin{array}{c} \overline{H}_{C\hat{a}} \\ \hline -\epsilon_{\alpha\beta} H_1^\beta \end{array} \right). \tag{3.3}$$

Then, the $SU(5)$ invariant renormalizable superpotential which reproduces the Yukawa superpotential of the MSSM, Eq.(2.8), under the above decomposition can be written as

$$\mathcal{W}_{SU(5)} = \frac{1}{8} \epsilon_{abcde} (y_u)_{ij} T_i^{ab} T_j^{cd} H^e + (y_d)_{ij} \overline{F}_{ia} T_j^{ab} \overline{H}_b, \tag{3.4}$$

where i, j are generation indices and a, b, c, d, e are $SU(5)$ indices. The $SU(5)$ invariant antisymmetric tensor, ϵ_{abcde}, is defined so that $\epsilon_{12345} = 1$. y_u is symmetric in the SU(5) model. From the above formula, It can be seen that the Yukawa coupling constants of leptons are related to that of down-type quarks as

$$y_e = y_d^T. \tag{3.5}$$

In addition to the Yukawa superpotential, Eq.(3.4), we need a Higgs sector which is responsible for the spontaneous breaking of $SU(5)$. There is a minimal model which includes an adjoint Higgs superfield, Σ^a_b, in addition to the above superfields [33]; however, we do not discuss the details of SU(5) breaking in this thesis, because our discussion about LFV is almost independent of it.

$SO(10)$ is a simple Lie group of rank 5 which includes SU(5) as a subgroup. The **16** representation of $SO(10)$ can be decomposed into $SU(5)$ representations so that $\mathbf{16} = \bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$. If we introduce three generations of the $SU(5)$ singlet superfield, N_i, all of the matter superfields in the SU(5) (MSSM) can be unified into the **16** representation of $SO(10)$ so that $\Psi_i\{\overline{F}_i, T_i, N_i\}$. The **5** and $\bar{\mathbf{5}}$ representation Higgs superfields, H and $\overline{H}$, in the $SU(5)$ GUT can be cast in a **10** representation of $SO(10)$. The $SO(10)$ invariant renormalizable superpotential which reproduces the Yukawa superpotential of the MSSM Eq.(2.8) can be written as

$$\mathcal{W}_{SO(10)} = \frac{1}{2}(y_u)_{ij}\Psi_i\Phi_u\Psi_j + \frac{1}{2}(y_d)_{ij}\Psi_i\Phi_d\Psi_j, \tag{3.6}$$

where note that we have introduced two **10** representation Higgs superfields, $\Phi_u\{H, \overline{H}'\}$ and $\Phi_d\{H', \overline{H}\}$ for the up- and down-type Yukawa coupling constants. H' and $\overline{H}'$ are other **5** and $\bar{\mathbf{5}}$ multiplets of $SU(5)$. Without these two Higgs superfields, the CKM matrix can not be reproduced in this minimal model. In the minimal $SO(10)$ case, the two Yukawa coupling constants are symmetric about the flavor indices. The GUT relation between the lepton Yukawa coupling constants and those of down-type quarks, Eq.(3.5), also holds in the $SO(10)$ model.

III Bottom tau unification and non-minimal model

The GUT relation for the Yukawa coupling constants Eq.(3.5) at the GUT scale predicts the ratio of the masses of the down-type quarks and charged leptons [36]. If we take into account only the gauge coupling constants in the RGEs for the down-type and lepton Yukawa coupling constants, the ratio can be calculated as follows:

$$\begin{aligned}\frac{y_{d_i}}{y_{e_i}} &\simeq (\frac{\alpha_3}{\alpha_G})^{\frac{8}{9}}(\frac{\alpha_1}{\alpha_G})^{\frac{10}{99}} \\ &\simeq 2.4, \end{aligned} \tag{3.7}$$

where $\alpha_G \simeq \frac{1}{24}$ denotes a unified value of α_i $(i = 1 - 3)$ at the GUT scale. For the third generation, this relation seems to be good, because $m_b \simeq 4$GeV and $m_\tau \simeq 1.777$GeV. Detailed calculations including the Yukawa coupling constants shows that the GUT relation can be satisfied for small and large $\tan\beta$ [37]. On the other hand, for the first and second generation, this relation can not explain the experimental data:

$$\frac{m_s}{m_\mu} \simeq \frac{1 \times 10^2 \text{ MeV}}{106 \text{ MeV}}, \tag{3.8}$$

$$\frac{m_d}{m_e} \simeq \frac{7 \text{ MeV}}{0.511 \text{ MeV}}. \tag{3.9}$$

This mismatch, however, is not a fatal defect of the SUSY GUT. We assumed a minimal form of the superpotential which only includes renormalizable couplings in Eqs.(3.4) and (3.6). Because the GUT scale is only two orders of magnitude below the Planck scale, where the SUSY GUT must be cut off, there is no reason to exclude non-renormalizable higher dimensional operators which are suppressed by the Planck mass, M_P, in the superpotential. They can also contribute to the Yukawa coupling constants of the MSSM after the GUT symmetry is broken. The small Yukawa coupling constants of the first and second generations may be affected much from these contributions. These contributions depend on the details of the models and the mechanism which breaks the GUT symmetry. We do not discuss such details of non-minimal models in this thesis, but rather focus on the universal features of SU(5) and SO(10) SUSY GUT.

IV Radiative corrections and mass spectrum of the SUSY GUT

Because we set the cut-off of the MSSM at the GUT scale, the approximate formulas of the RGE corrections for the first- and the second-generation sfermions in the previous chapter Eq.(2.26) are replaced as follows:

$$\begin{aligned}
& M_3 \simeq 2.9 M_{5G}, \qquad M_2 \simeq 0.82 M_{5G}, \\
& M_1 \simeq 0.41 M_{5G}, \\
& m_Q^2 \simeq m_{QG}^2 + 7.1 M_{5G}^2, \qquad m_L^2 \simeq m_{LG}^2 + 0.5 M_{5G}^2, \\
& m_U^2 \simeq m_{UG}^2 + 6.6 M_{5G}^2, \qquad m_E^2 \simeq m_{EG}^2 + 0.15 M_{5G}^2, \\
& m_D^2 \simeq m_{DG}^2 + 6.6 M_{5G}^2 \qquad m_{H_1}^2 \simeq m_{H_1G}^2 + 0.5 M_{5G}^2,
\end{aligned} \tag{3.10}$$

where subscript G means the value at the GUT scale. M_{5G} represents a unified value of the gaugino masses at the GUT scale. The ratio of three gaugino masses satisfy well-known relation $M_3 : M_2 : M_1 \simeq 7 : 2 : 1$. These boundary conditions for the soft breaking terms of the MSSM at the GUT scale are determined by the physics above the GUT scale. We simply assume that the SUSY GUT with the soft SUSY-breaking terms describes the nature up to the Planck scale, and that SUSY is broken at the Planck scale by the minimal SUGRA described in section IV of the previous chapter. This assumption is essential for the discussion of this thesis, whereas another possibilities also can be considered [25].

The soft breaking terms for the SU(5) SUSY GUT are parameterized as follows:

$$\begin{aligned}
\mathcal{L}_{soft} &= -(m_T^2)_{ij} \tilde{T}_{ia}^{\dagger} \tilde{T}_j^a - (m_{\overline{F}}^2)_{ij} \tilde{\overline{F}}_i^{a\dagger} \tilde{\overline{F}}_{ja} - m_H^2 H_a^{\dagger} H^a - m_{\overline{H}}^2 \overline{H}^{a\dagger} \overline{H}_a \\
& -\{\frac{1}{8} (A_u)_{ij} \epsilon_{abcde} \tilde{T}_i^{ab} \tilde{T}_j^{cd} H^e + (A_d)_{ij} \tilde{\overline{F}}_{ia} \tilde{T}_j^{ab} \overline{H}_b \\
& + \frac{1}{2} M_5 \overline{\lambda_{5R}} \lambda_{5L} + h.c.\}.
\end{aligned} \tag{3.11}$$

where we used same symbols for superfields and their component fields for abbreviation. $\tilde{T}$, $\tilde{\overline{F}}$ are scalar components of the superfields T, $\overline{F}$ and λ_5 is the $SU(5)$ gaugino. These parameters satisfy the universal boundary condition at the Planck scale similar to the case of the MSSM, as follows:

$$\begin{aligned}
& M_5 = M_0, \\
& m_T^2 = m_{\overline{F}}^2 = m_0^2 \mathbf{1}, \; m_H^2 = m_{\overline{H}}^2 = m_0^2, \\
& (A_u)_{ij} = m_0 A_0 (y_u)_{ij}, \; (A_d)_{ij} = m_0 A_0 (y_d)_{ij}.
\end{aligned} \tag{3.12}$$

These conditions receive radiative corrections between the Planck scale and the GUT scale. RGEs for the minimal SU(5) SUSY GUT are summarized in Appendix A. These radiative corrections are also classified to the contribution from the gauge interaction and the contribution from the Yukawa interactions. The latter contribution has an important meaning for flavor physics. It is a main subject of this thesis, and we discuss the details of this correction in the next chapter. In this section, however, we discuss the former contribution to see the rough spectrum. Because we assumed a minimal model which includes an adjoint Higgs field, the coefficient of the gauge beta function becomes $b_5 = -3$. Thus, the radiative correction for the gaugino mass becomes as follows:

$$M_{5G} \simeq 1.1 M_0. \tag{3.13}$$

If we neglect the Yukawa coupling constants in the RGEs, using a similar formula to Eq.(2.23), the soft SUSY-breaking mass parameters for the first and second generations at the GUT scale are approximated as follows:

$$m_{TG}^2 \simeq m_0^2 + 0.47 M_0^2, \tag{3.14}$$

$$m_{\overline{F}G}^2 \simeq m_{\overline{H}G} \simeq m_0^2 + 0.32 M_0^2. \tag{3.15}$$

The quadratic Casimir for **10** ($\bar{\mathbf{5}}$) representation of $SU(5)$ is $c_T^5 = \frac{18}{5}$ ($c_{\overline{F}}^5 = \frac{12}{5}$). $SU(3) \times SU(2)_L \times U(1)_Y$ decomposition gives boundary conditions for the soft SUSY-breaking parameters of the MSSM at the GUT scale, as follows:

$$A_e = A_d^T, \tag{3.16}$$

$$m_E^{2T} = m_U^{2T} = m_Q^2 = m_{TG}^2, \ m_L^2 = m_D^2 = m_{\overline{F}G}^2,$$

$$m_{H_1}^2 = m_{\overline{H}G}^2, \ m_{H_2}^2 = m_{HG}^2, \ M_1 = M_2 = M_3 = M_{5G}. \tag{3.17}$$

Combined with Eq.(3.10), we obtain an approximated formulas at the EW scale, as follows:

$$M_3 \simeq 3.2 M_0, \qquad M_2 \simeq 0.88 M_0,$$

$$M_1 \simeq 0.45 M_0,$$

$$m_Q^2 \simeq m_0^2 + 8.9M_0^2, \qquad m_L^2 \simeq m_{H_1}^2 \simeq m_0^2 + 0.94M_0^2,$$
$$m_U^2 \simeq m_0^2 + 8.4M_0^2, \qquad m_E^2 \simeq m_0^2 + 0.65M_0^2,$$
$$m_D^2 \simeq m_0^2 + 8.2M_0^2. \tag{3.18}$$

Because of the running between the Planck scale and the GUT scale, the contributions from the gaugino mass to the slepton masses becomes large compared to the case of the MSSM, Eq.(2.26).

In the case of the SO(10) SUSY GUT, the soft SUSY-breaking terms can be written as

$$\begin{aligned}\mathcal{L}_{soft} &= -(m_\Psi^2)_{ij}\widetilde{\Psi}_i^\dagger\widetilde{\Psi}_j - m_{\Phi_u}^2\Phi_u^\dagger\Phi_u - m_{\Phi_d}^2\Phi_d^\dagger\Phi_d \\ &\quad -\{\frac{1}{2}(A_u)_{ij}\widetilde{\Psi}_i\Phi_u\widetilde{\Psi}_j + \frac{1}{2}(A_d)_{ij}\widetilde{\Psi}_i\Phi_d\widetilde{\Psi}_j + \frac{1}{2}M_{10}\overline{\lambda_{10R}}\lambda_{10L} \\ &\quad +h.c.\},\end{aligned} \tag{3.19}$$

where we used same symbols for superfields and their component fields. $\widetilde{\Psi}$ is the scalar component of the superfield Ψ and λ_{10} is the $SO(10)$ gaugino. These parameters satisfy the universal boundary condition at the Planck scale, as follows:

$$M_{10} = M_0,$$
$$m_\Psi^2 = m_0^2\mathbf{1}, \qquad m_{\Phi_u}^2 = m_{\Phi_d}^2 = m_0^2,$$
$$(A_u)_{ij} = m_0A_0(y_u)_{ij}, \qquad (A_d)_{ij} = m_0A_0(y_d)_{ij}. \tag{3.20}$$

The RGEs for the minimal $SO(10)$ SUSY GUT are also summarized in Appendix A. The radiative corrections for the first and second generations are approximated as follows:

$$m_{\Psi G}^2 \simeq m_0^2 + 0.74M_0^2, \tag{3.21}$$

$$m_{\Phi_d G}^2 \simeq m_0^2 + 0.59M_0^2. \tag{3.22}$$

The quadratic Casimir for **16** (**10**) representation of $SO(10)$ is $c_\Psi^{10} = \frac{45}{8}$ ($c_{\Phi_{u,d}}^{10} = \frac{9}{2}$). Decomposition to the SM group shows that the boundary conditions for the soft

SUSY-breaking parameters of the MSSM at the GUT scale is given by:

$$
\begin{aligned}
&A_e = A_d, \\
&m_E^2 = m_L^2 = m_D^2 = m_U^2 = m_Q^2 = m_{\Psi G}^2, \\
&m_{H_1}^2 = m_{\Phi_d G}^2,\ m_{H_2}^2 = m_{\Phi_u G}^2,\ M_1 = M_2 = M_3 = M_{10G}.
\end{aligned}
\tag{3.23}
$$

Combined with Eq.(3.10), the approximated spectrum for the first and second generation at the EW scale becomes as follows:

$$
\begin{aligned}
M_3 &\simeq 3.2M_0, & M_2 &\simeq 0.88M_0, \\
M_1 &\simeq 0.45M_0, & & \\
m_Q^2 &\simeq m_0^2 + 9.2M_0^2, & m_L^2 &\simeq m_0^2 + 1.4M_0^2, \\
m_U^2 &\simeq m_0^2 + 8.7M_0^2, & m_E^2 &\simeq m_0^2 + 0.92M_0^2, \\
m_D^2 &\simeq m_0^2 + 8.6M_0^2, & m_{H_1}^2 &\simeq m_0^2 + 1.2M_0^2.
\end{aligned}
\tag{3.24}
$$

In this section, we took into account only the renormalization effects from the gauge interactions. Contributions from the Yukawa interactions modify the above spectrum. In particular, the effect of the top Yukawa coupling can not be neglected. The third-generation squarks and H_2 become considerably lighter than the above spectrum. The radiative EW symmetry-breaking scenario described in section VI of the previous chapter also works in the SUSY GUT. In addition to the radiative EW symmetry breaking, another important consequence of the large top Yukawa coupling constant is new contributions to flavor physics. In the next chapter, we consider the LFV processes induced by these renormalization effects of the Yukawa couplings in SUSY GUT.

Chapter 4

SUSY GUT and lepton flavor violation (LFV)

I LFV in the SUSY GUT

Now that we have reviewed the SUSY GUT and the minimal SUGRA, let us go to the main subject of this thesis: LFV in the SUSY GUT. As discussed in section IV of chapter 2, there is no source of LFV in the MSSM if we assume the flavor blind condition for the soft SUSY-breaking terms at some high-energy scale, such as the Planck scale. Radiative corrections between the high-energy scale to the EW scale do not change the situation, because the MSSM interactions conserve lepton flavor. However, if we assume SUSY GUT, the GUT interaction which violates lepton flavor can become a source of LFV through the radiative correction to the soft SUSY breaking terms between the Planck scale and the GUT scale [38]. In this chapter we explain the mechanism which induces LFV in the $SU(5)$ and $SO(10)$ SUSY GUTs and discuss qualitative features of LFV interactions specific to these two theories. There are various LFV processes, such as $\mu \to e\gamma$, $\mu \to 3e$, μ-e conversion, $\tau \to l\gamma$, and $\tau \to 3l$. In particular, we focus on the $\mu \to e\gamma$ and $\mu \to 3e$ processes in this thesis because we can define various P- and T-odd asymmetries relative to the muon polarization. We discuss a possibility to use these observables to distinguish the different SUSY theories in subsequent chapters.

II Effective Lagrangian for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes

Before explaining the mechanism which generates LFV processes in the SUSY GUT, we present the effective Lagrangian for the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes. Using the electromagnetic gauge invariance and the Fierz rearrangement, we can write without any loss of generality *:

$$\begin{aligned}\mathcal{L} &= -\frac{4G_F}{\sqrt{2}}\{m_\mu A_R\overline{\mu_R}\sigma^{\mu\nu}e_L F_{\mu\nu} + m_\mu A_L\overline{\mu_L}\sigma^{\mu\nu}e_R F_{\mu\nu} \\ &\quad +\hat{g}_1(\overline{\mu_R}e_L)(\overline{e_R}e_L) + \hat{g}_2(\overline{\mu_L}e_R)(\overline{e_L}e_R) \\ &\quad +\hat{g}_3(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_R}\gamma_\mu e_R) + \hat{g}_4(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_L}\gamma_\mu e_L) \\ &\quad +\hat{g}_5(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_L}\gamma_\mu e_L) + \hat{g}_6(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_R}\gamma_\mu e_R) + h.c.\}, \end{aligned} \tag{4.1}$$

where G_F is the Fermi coupling constant and m_μ is the muon mass. The chirality projection is defined by the projection operators $P_R = \frac{1+\gamma_5}{2}$ and $P_L = \frac{1-\gamma_5}{2}$. $\sigma_{\mu\nu}$ is defined as $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. $A_L(A_R)$ is a dimensionless photon-penguin coupling constant which contributes to $\mu^+ \to e_L^+\gamma$ ($\mu^+ \to e_R^+\gamma$). These couplings also induce the $\mu^+ \to e^+e^+e^-$ process. $\hat{g}_i$'s $(i = 1-6)$ are dimensionless four-fermion coupling constants which only contribute to $\mu^+ \to e^+e^+e^-$. $\hat{g}_1$ and $\hat{g}_2$ are scalar type coupling constants and $\hat{g}_i$'s $(i = 3-6)$ are vector-type coupling constants. $A_{L,R}$ and $\hat{g}_i$'s $(i = 1-6)$ are generally complex numbers, and are calculated based on a particular model with LFV interactions. We first discuss them based on the SU(5) SUSY GUT and introduce the SO(10) SUSY GUT later.

III SU(5) SUSY GUT

As discussed in section IV of the previous chapter, at the Planck scale the soft SUSY breaking parameters satisfy flavor-blind universal conditions which are implied in the minimal SUGRA model Eq.(3.12). With these conditions the lepton and slepton mass matrices can be diagonalized simultaneously at the Planck scale, and therefore

*We include the contributions from off-shell photon-penguin amplitudes in the four-fermion coupling constants there is no LFV at this scale. The radiative correction from the gauge interaction discussed in the last section of the previous chapter does not change the situation. However, radiative corrections from the Yukawa couplings between the Planck scale to the GUT scale break such a universality about flavor indices under these conditions. Especially, the correction from the large top Yukawa coupling constant can not be neglected. As a result, the magnitude of the 3-3 element of the mass matrix for the **10** scalar fields becomes smaller than the 1-1 and 2-2 elements. On the basis where y_u is diagonalized at the Plank scale, the mass matrix for the **10** scalar fields at the GUT scale is approximately given by:

$$
\begin{aligned}
m_T^2 &\simeq \begin{pmatrix} m^2 & & \\ & m^2 & \\ & & m^2 + \Delta m^2 \end{pmatrix}, \\
\Delta m^2 &\simeq -\frac{3}{8\pi^2}|(y_u)_{33}|^2 m_0^2 (3 + |A_0|^2) \ln(\frac{M_P}{M_G}),
\end{aligned}
\tag{4.2}
$$

where M_P and M_G denote the reduced Planck mass ($\sim 2 \times 10^{18}$GeV)and the GUT scale ($\sim 2 \times 10^{16}$GeV). This correction amounts to about 50% of their original values and the lepton and slepton mass matrices are no longer diagonalized simultaneously. This becomes a source of LFV, which could induce observable effects in $\mu^+ \to e^+\gamma$[6]. On the basis where y_u is diagonalized at the Planck scale, y_u at the GUT scale still remains approximately diagonal. On this basis, y_e is diagonalized in the following way:

$$V_R y_e V_L^\dagger = \mathit{diagonal}, \tag{4.3}$$

where V_L and V_R are unitary matrices and, using Eq.(3.5), V_R is given by

$$(V_R)_{ij} = (V_{CKM}^0)_{ji}, \tag{4.4}$$

where V_{CKM}^0 is CKM matrix at the GUT scale.

It is useful to make unitary transformations on E_i and L_j to go to the basis where y_e is diagonalized at the GUT scale. In the new basis, the off-diagonal element of m_E^2 is given by

$$(m_E^2)_{ij} \simeq -\frac{3}{8\pi^2}(V_{CKM}^0)_{3i}(V_{CKM}^0)_{3j}^*|(y_u)_{33}|^2 m_0^2(3 + |A_0|^2)\ln(\frac{M_P}{M_G}). \tag{4.5}$$

The off-diagonal element of the slepton mass matrix becomes a source of LFV.

In an actual numerical analysis, we solved the MSSM renormalization group equation from the GUT scale to the electroweak scale, and determined the masses and mixings for SUSY particles. We also required the electroweak symmetry breaking to occur properly to give the correct Z-boson mass, as discussed in section VI of chapter 2. From the MSSM Lagrangian at the electroweak scale we can derive the LFV coupling constants, $A_{L,R}$ and $\hat{g}_{1-6}$, through 1-loop diagrams involving the slepton, gaugino and higgsino. The complete formulas are given in Appendix I.

In the SU(5) model, only the right-handed slepton mass matrix can develop off-diagonal terms if the ratio of the vacuum expectation values of two Higgs fields ($\tan\beta = \frac{\langle H_2^0\rangle}{\langle H_1^0\rangle}$) is not very large. In such a case, only A_L, $\hat{g}_3$ and $\hat{g}_5$ have sizable contributions. Restricting to small or moderate $\tan\beta$ cases, all effective coupling constants are proportional to the product of the CKM matrix element, $\lambda_\tau = (V_{CKM}^0)_{32}, (V_{CKM}^0)_{31}^*$ since the LFV transition occurs through $(m_E^2)_{21}$ or $(m_E^2)_{32}^*(m_E^2)_{31}$. This situation does not change even if we take into account the LFV transition due to the left-right mixing of the slepton mass matrix. This means that the CP-violating phase of Yukawa coupling constants cannot make a phase difference among the LFV coupling constants. Thus, as we will discuss in the subsection II.1 of the next chapter it can not generate any observable effects in the LFV processes.

There is another important source of CP-violating phases in soft SUSY breaking terms, as discussed in section VII of chapter 2. Within the SUGRA model, we can introduce four phases, phases of M_0, A_0, B and μ, but not all of them are physically independent. By field redefinition, we can take the phases of A_0 and μ as being independent phases. Since these phases also induce the electron, neutron and Hg EDMs [40, 41], we take into account these EDM constraints to obtain the allowed region of the SUSY phases.

Up to now we have considered that the Yukawa coupling constants are given by Eq. (3.4), so that the lepton and down-type quark Yukawa coupling constants are related at the GUT scale by Eq. (3.5). On the other hand, as discussed in section III of the previous chapter, it is known that this relation does not reproduce realistic mass relations for charged leptons and down-type quarks in the first and second generations. It is therefore important to study how the prediction for LFV processes depends on the details of the origin of the Yukawa coupling constant in the MSSM Lagrangian. One way to generate a realistic mass matrix is to introduce higher dimensional operators in the SU(5) superpotential. Once this is done, the simple relationship between the charged lepton and down-type quark Yukawa coupling constants does not hold. Although the effect of higher dimensional operators is suppressed by $O\left(\frac{M_G}{M_P}\right)$, the masses and mixings for the first and second generations can receive large corrections to the GUT relation. If $\tan\beta$ is not very large, LFV is still induced only for the right-handed slepton sector, and Eq. (4.5) holds with a replacement of V^0_{CKM} by V_R^T, which is not necessary related to the CKM matrix elements. In the following, therefore, we treat λ_τ as a free parameter. Since the $\mu^+ \to e^+\gamma$ and the $\mu^+ \to e^+e^+e^-$ branching ratios are proportional to $|\lambda_\tau|^2$, we present these branching ratios divided by $|\lambda_\tau|^2$. If $\tan\beta$ is as large as 30, the bottom Yukawa coupling constant can induce the LFV in the left-handed slepton sector. In such a case, if we include the effect of higher dimensional operators at the GUT scale, there are photon-penguin diagrams which are proportional to m_τ; these contributions tend to dominate over other contributions, as shown in [42]. Because the LFV branching ratios depend on many unknown parameters in such a case, we do not consider this possibility here.

IV SO(10) SUSY GUT

In contrast with the SU(5) SUSY GUT, in the SO(10) SUSY GUT, all matter fields are unified in a single representation, Ψ, of SO(10), and the masses of all squarks and sleptons of the third generation receive a large correction due to the renormalization effect by the top Yukawa coupling constant. In the y_u-diagonalized basis, the difference between the mass of the third-generation sfermion and that of the first and second generation is given by

$$\Delta m^2_{\Psi} \simeq -\frac{5}{8\pi^2}|(y_u)_{33}|^2 m_0^2(3+|A_0|^2)\ln(\frac{M_P}{M_G}), \quad (4.6)$$

where the symmetric matrix, y_e, can be expressed as:

$$y_e = U^T P \hat{y}_e U,$$

$$P = \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}. \tag{4.7}$$

Here $\hat{y}_e$ is a real diagonal matrix, and therefore the unitary matrix, U, is related to the CKM matrix at the GUT scale as

$$U = V^{0\dagger}_{CKM}. \tag{4.8}$$

If we go to the y_e-diagonalized basis at the GUT scale, the off-diagonal elements of slepton mass matrices become as follows:

$$(m_E^2)_{ij} \simeq -\frac{5}{8\pi^2} e^{-i(\phi_i-\phi_j)} (V^0_{CKM})_{3i} (V^0_{CKM})^*_{3j} |(y_u)_{33}|^2 m_0^2 (3+|A_0|^2) \ln(\frac{M_P}{M_G}), \tag{4.9}$$

$$(m_L^2)_{ij} \simeq -\frac{5}{8\pi^2} (V^0_{CKM})^*_{3i} (V^0_{CKM})_{3j} |(y_u)_{33}|^2 m_0^2 (3+|A_0|^2) \ln(\frac{M_P}{M_G}). \tag{4.10}$$

Since the left-handed slepton also has the LFV effect in the case of the SO(10) SUSY GUT, there are dominant photon-penguin diagrams which are proportional to m_τ in the slepton left-right mixing, as discussed in [7] . These contributions come from neutralino-charged-slepton loop diagrams (see Fig.4.1). There is an another diagram which only contributes to A_R (see Fig.4.2). In spite of no m_τ enhancement, this chargino-sneutrino loop diagram is comparable to the neutralino diagrams which are proportional to m_τ if the chargino mass is not very much larger than the slepton masses. However, it becomes dominant as the slepton masses become larger. A detailed discussion using approximate formulas is given in Appendix D.

In addition to the KM phase, there are two physical phases in Eq. (4.7) up to an overall phase. A combination of these phases and the KM phase is responsible for the electron EDM through the diagrams in Fig.4.3 [7, 43]. Because this diagram is obtained by simply replacing the LFV coupling constants in Fig.4.1, there is a simple relation between the electron EDM and the $\mu^+ \to e^+\gamma$ branching ratio, if the photon-penguin diagram proportional to m_τ dominates in the $\mu^+ \to e^+\gamma$ amplitude [7]. Defining a phase as

$$Im[e^{i(\phi_3-\phi_1)}\{(V^0_{CKM})_{31}(V^0_{CKM})^*_{33}\}^2] = |(V^0_{CKM})_{31}(V^0_{CKM})^*_{33}|^2 \sin\phi, \tag{4.11}$$

the relation is given by

$$|d_e| = 1.3\sqrt{\frac{B(\mu \to e\gamma)}{10^{-12}}}|\sin\phi| \quad (10^{-27} e \cdot cm). \tag{4.12}$$

However, the diagram proportional to m_τ does not necessarily dominate over other diagrams when the chargino-neutralino loop contribution becomes large. In such a case, the above relation does not hold.

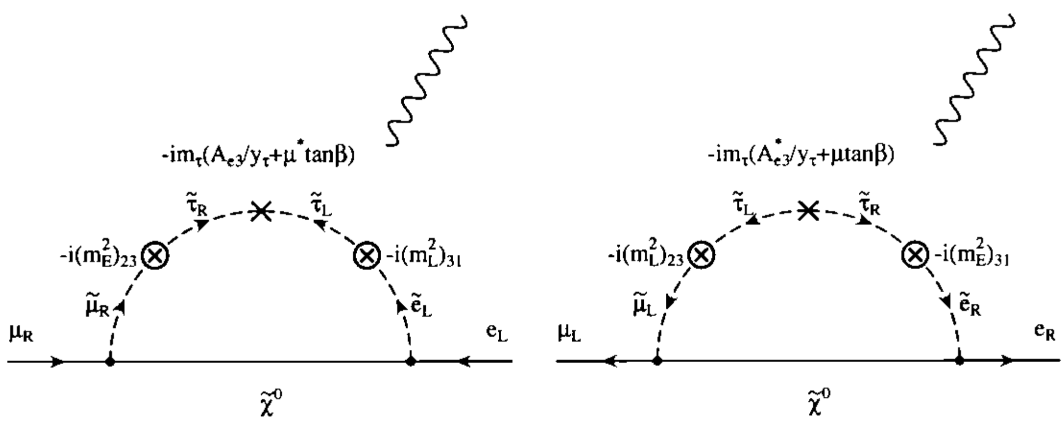


FIG. 4.1: Two dominant diagrams which are proportional to m_τ in neutralino-charged-slepton loop diagrams. They contribute to A_R and A_L, respectively. The cross denotes a chirality flip and the circle-crosses denote LFV coupling constants, which are given by Eq. 4.9, if we neglect the renormalization effect between the GUT scale and the EW scale.

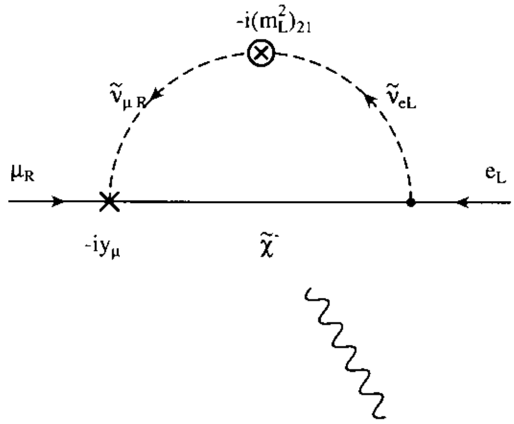


FIG. 4.2: Dominant diagram in Chargino-sneutrino loop diagrams which only contributes to A_R. The cross denotes chirality flip and the circle-cross denotes a LFV coupling constant.

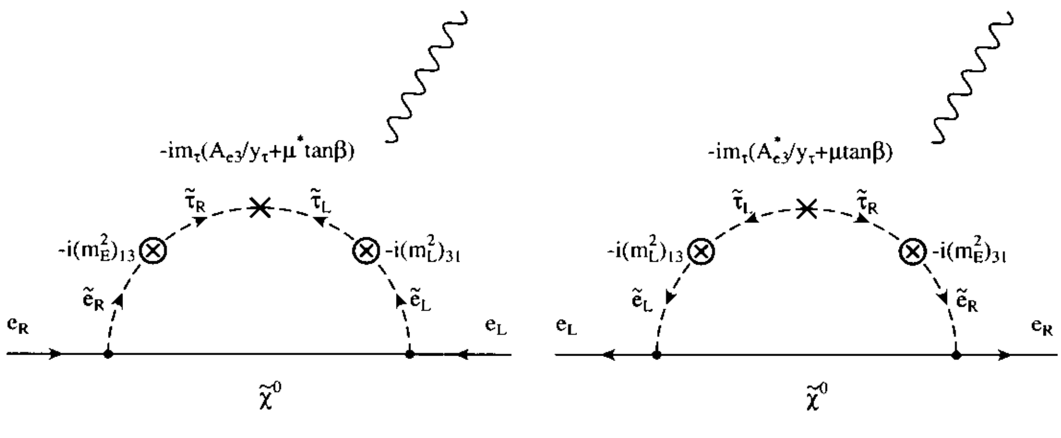


FIG. 4.3: Dominant diagrams which contribute to the electron EDM. The cross denotes a chirality flip and the circle-crosses denote LFV coupling constants.

Chapter 5

LFV processes with polarized muons

The previous chapter discussed the LFV processes in particular models : SU(5) and SO(10) SUSY GUT. In this chapter, however, we return to the general effective Lagrangian, Eq.(4.1), and develop a model-independent framework for analyzing $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes with polarized muons. We fix the kinematics of the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes in the center-of-mass system of the muon, and define various P- and T-odd asymmetries relative to the muon polarization. In the next chapter we discuss the SU(5) and SO(10) SUSY GUT numerically using these observables.

I $\mu^+ \to e^+\gamma$ process

First, we discuss the $\mu^+ \to e^+\gamma$ process with polarized muons. Using the effective coupling constants defined in Eq.(4.1), the differential branching ratio for $\mu^+ \to e^+\gamma$ is given by

$$\begin{aligned}\frac{dB(\mu^+ \to e^+\gamma)}{d\cos\theta} &= 192\pi^2\{|A_L|^2(1+P\cos\theta)+|A_R|^2(1-P\cos\theta)\} &(5.1)\\ &= \frac{B(\mu^+ \to e^+\gamma)}{2}\{1+A(\mu^+ \to e^+\gamma)P\cos\theta\}, &(5.2)\end{aligned}$$

where the total branching ratio for $\mu^+ \to e^+\gamma$ $(B(\mu^+ \to e^+\gamma))$ and the P-odd asymmetry $(A(\mu^+ \to e^+\gamma))$ are defined as

$$B(\mu^+ \to e^+\gamma) = 384\pi^2(|A_L|^2+|A_R|^2), \qquad (5.3)$$

$$A(\mu^+ \to e^+\gamma) = \frac{|A_L|^2 - |A_R|^2}{|A_L|^2 + |A_R|^2}. \tag{5.4}$$

Here, P is the muon polarization and θ is the angle between the positron momentum and the polarization direction.

II $\mu^+ \longrightarrow e^+e^+e^-$ process

II.1 Kinematics of $\mu^+ \to e^+e^+e^-$

Next, we discuss the $\mu^+ \to e^+e^+e^-$ process with polarized muons. The kinematics of this process is determined by two energy variables of decay positrons and two angle variables which indicate the direction of the muon polarization with respect to the decay plane. In Fig. 5.1 we take the z-axis as the direction of the decay electron momentum ($\vec{p_3}$) and the z-x plane as the decay plane. Polar angles (θ, φ) $(0 \leq \theta \leq \pi, 0 \leq \varphi < 2\pi)$ indicate the direction of the muon polarization, $\vec{P}$. We take the convention that the decay positron having a larger energy is named positron 1 and the other is positron 2 and $(p_1)_x \geq 0$. We define the energy variables as $x_1 = \frac{2E_1}{m_\mu}$ and $x_2 = \frac{2E_2}{m_\mu}$, where E_1 and E_2 are the energy of positron 1 and 2, respectively. In this convention (x_1,x_2) represents one point of the Dalitz plot (Fig. 5.2). In our calculation we neglect the electron mass compared to the muon mass, except for the total branching ratio. To evaluate the total branching ratio we have to take into account the electron mass properly in order to avoid a logarithmic singularity .

Using the coupling constants in the Lagrangian in Eq.(4.1) the differential branching ratio for $\mu^+ \to e^+e^+e^-$ is written as follows:

$$\begin{aligned}
\frac{dB}{dx_1 dx_2 d\cos\theta d\varphi} &= \frac{3}{2\pi}[C_1\alpha_1(x_1,x_2)(1+P\cos\theta) + C_2\alpha_1(x_1,x_2)(1-P\cos\theta) \\
&+ C_3\{\alpha_2(x_1,x_2) + P\beta_1(x_1,x_2)\cos\theta + P\gamma_1(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_4\{\alpha_2(x_1,x_2) - P\beta_1(x_1,x_2)\cos\theta - P\gamma_1(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_5\{\alpha_3(x_1,x_2) + P\beta_2(x_1,x_2)\cos\theta + P\gamma_2(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_6\{\alpha_3(x_1,x_2) - P\beta_2(x_1,x_2)\cos\theta - P\gamma_2(x_1,x_2)\sin\theta\cos\varphi\}
\end{aligned}$$

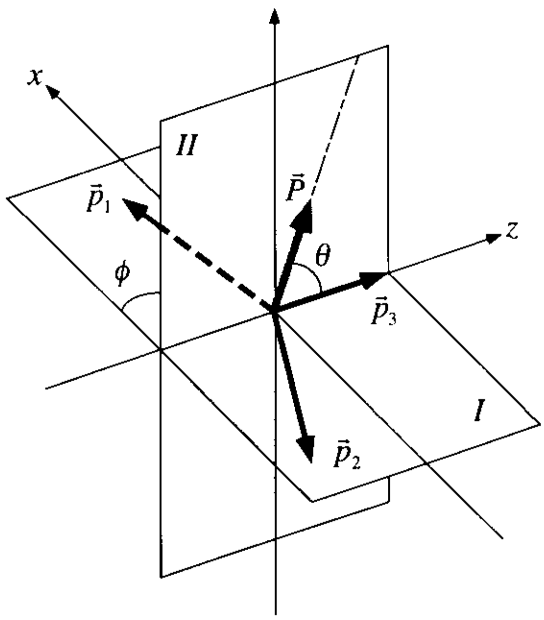


FIG. 5.1: Kinematics of the $\mu^+ \to e^+e^+e^-$ decay in the center-of-mass system of muon. Plane I represents the decay plane on which the momentum vectors $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ lie, where $\vec{p}_1$ and $\vec{p}_2$ are momenta of two e^+'s and $\vec{p}_3$ is momentum of e^- respectively. Plane II is the plane which the muon polarization vector $\vec{P}$ and $\vec{p}_3$ make.

$$
\begin{aligned}
&+ C_7\{\alpha_4(x_1,x_2)(1 - P\cos\theta) + P\gamma_3(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_8\{\alpha_4(x_1,x_2)(1 + P\cos\theta) - P\gamma_3(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_9\{\alpha_5(x_1,x_2)(1 + P\cos\theta) - P\gamma_4(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_{10}\{\alpha_5(x_1,x_2)(1 - P\cos\theta) + P\gamma_4(x_1,x_2)\sin\theta\cos\varphi\} \\
&+ C_{11}P\gamma_3(x_1,x_2)\sin\theta\sin\varphi - C_{12}P\gamma_4(x_1,x_2)\sin\theta\sin\varphi], \quad (5.5)
\end{aligned}
$$

where C_i are expressed by the coupling constants $\hat{g}_i$ $(i = 1-6)$ and $A_{L,R}$ as:

$$
\begin{aligned}
&C_1 = \frac{|\hat{g}_1|^2}{16} + |\hat{g}_3|^2,\ C_2 = \frac{|\hat{g}_2|^2}{16} + |\hat{g}_4|^2, \\
&C_3 = |\hat{g}_5|^2,\ C_4 = |\hat{g}_6|^2, C_5 = |eA_R|^2,\ C_6 = |eA_L|^2, \\
&C_7 = Re(eA_R\hat{g}_4^*),\ C_8 = Re(eA_L\hat{g}_3^*),\ C_9 = Re(eA_R\hat{g}_6^*),\ C_{10} = Re(eA_L\hat{g}_5^*),
\end{aligned}
$$

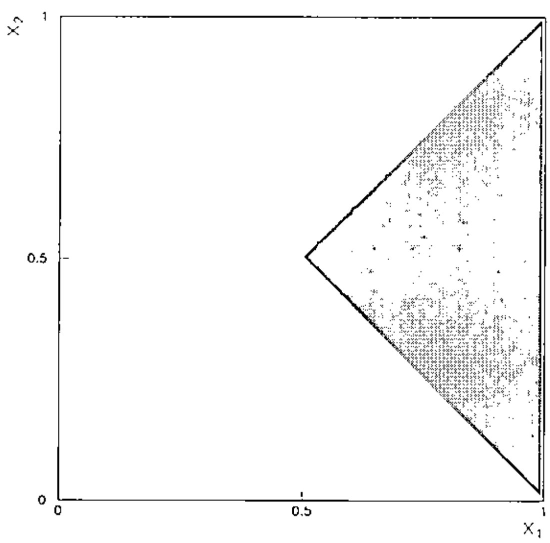


FIG. 5.2: Kinematical region of the $\mu^+ \to e^+e^+e^-$ decay in the center-of-mass system of the muon. x_1 (x_2) represents a larger (smaller) energy of decay positrons normalized by $\frac{m_\mu}{2}$. The light shaded region is allowed.

$$C_{11} = Im(eA_R\hat{g}_4^* + eA_L\hat{g}_3^*),\ C_{12} = Im(eA_R\hat{g}_6^* + eA_L\hat{g}_5^*), \tag{5.6}$$

where $e(>0)$ is the positron charge and P is the magnitude of the polarization vector. Functions α_i, β_i and γ_i are defined as follows:

$$\alpha_1(x_1,x_2) = 8(2-x_1-x_2)(x_1+x_2-1), \tag{5.7}$$

$$\alpha_2(x_1,x_2) = 2\{x_1(1-x_1)+x_2(1-x_2)\}, \tag{5.8}$$

$$\alpha_3(x_1,x_2) = 8\{\frac{2x_2^2-2x_2+1}{1-x_1}+\frac{2x_1^2-2x_1+1}{1-x_2}\}, \tag{5.9}$$

$$\alpha_4(x_1,x_2) = 32(x_1+x_2-1), \tag{5.10}$$

$$\alpha_5(x_1,x_2) = 8(2-x_1-x_2), \tag{5.11}$$

$$\beta_1(x_1,x_2) = 2\frac{(x_1+x_2)(x_1^2+x_2^2)-3(x_1+x_2)^2+6(x_1+x_2)-4}{(2-x_1-x_2)}, \tag{5.12}$$

$$
\begin{aligned}
\beta_2(x_1,x_2) &= \frac{8}{(1-x_1)(1-x_2)(2-x_1-x_2)} \times \\
&\quad \{2(x_1+x_2)(x_1^3+x_2^3) - 4(x_1+x_2)(2x_1^2+x_1x_2+2x_2^2) \\
&\quad +(19x_1^2+30x_1x_2+19x_2^2) - 12(2x_1+2x_2-1)\}, \qquad (5.13)
\end{aligned}
$$

$$
\gamma_1(x_1,x_2) = 4\frac{\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}(x_2-x_1)}{(2-x_1-x_2)}, \qquad (5.14)
$$

$$
\gamma_2(x_1,x_2) = 32\sqrt{\frac{(x_1+x_2-1)}{(1-x_1)(1-x_2)}}\frac{(x_1+x_2-1)(x_2-x_1)}{(2-x_1-x_2)}, \qquad (5.15)
$$

$$
\gamma_3(x_1,x_2) = 16\sqrt{\frac{(x_1+x_2-1)}{(1-x_1)(1-x_2)}}(x_1+x_2-1)(x_2-x_1), \qquad (5.16)
$$

$$
\gamma_4(x_1,x_2) = 8\sqrt{\frac{(x_1+x_2-1)}{(1-x_1)(1-x_2)}}(2-x_1-x_2)(x_2-x_1). \qquad (5.17)
$$

In Eq. (5.5) there are three classes of terms: the first contribution arises from the four-fermion coupling constants (C_{1-4}) and the second from the photon-penguin coupling constants ($C_{5,6}$); the third comes from interferences between the four-fermion couplings and the photon-penguin couplings (C_{7-12}). In our approximation, neglecting the electron mass, there is no interference between the photon-penguin couplings and among the four-fermion couplings by themselves because the chirality of the electrons can not be matched between these couplings. For the same reason, the scalar-type coupling constants, $\hat{g}_1$ and $\hat{g}_2$, can not interfere with the photon-penguin coupling constants, A_R and A_L. The angular dependence with respect to the polarization direction is classified into four types, namely, terms proportional to (i) 1, (ii) $\cos\theta$, (iii) $\sin\theta\cos\varphi$, and (iv) $\sin\theta\sin\varphi$. Under the parity operation (P), θ, φ transform as follows:

$$
\begin{aligned}
\theta &\to \pi-\theta, \\
\varphi &\to \begin{cases} \pi-\varphi & (0\le\varphi<\pi) \\ 3\pi-\varphi & (\pi\le\varphi<2\pi) \end{cases}, \qquad (5.18)
\end{aligned}
$$

so that terms proportional to (ii) and (iii) are P-odd. On the other hand the timereversal operation (T) induces the following transformation:

$$\theta \to \theta, \; \varphi \to 2\pi - \varphi. \tag{5.19}$$

Thus, only terms proportional to C_{11} and C_{12} are T-odd quantities. Notice that these terms are given by imaginary parts of the interference terms between the photon-penguin and vector-type four-fermion coupling constants. This means that the effects of CP violation can be seen only through a phase difference between these two coupling constants. As we discussed in the previous chapter, in the case of $SU(5)$ SUSY GUT, the KM phase of the Yukawa coupling constants is factorized as a overall phase of LFV coupling constants. We can not observe such a CP-violating parameter through the $\mu^+ \to e^+e^+e^-$ process.

II.2 Branching ratios and P and T-odd asymmetries

It is convenient to define integrated asymmetries in order to separate four angular dependences, although in principle we can determine C_i separately by fitting the experimental data in full phase space. In the Dalitz plot, α_3 and β_2 have a singularity as $\frac{1}{1-x_{1,2}}$ in the region near to the kinematical boundary ($x_{1,2} \sim 1$). γ_2, γ_3 and γ_4 have a weaker singularity as $\frac{1}{\sqrt{1-x_{1,2}}}$. α_3, β_2, and γ_2 arise as the square of the photon-penguin amplitudes, whereas γ_3 and γ_4 come from interferences between the photon-penguin and four-fermion terms. On the contrary, contributions from the square of the four-fermion coupling constants have no singularity on the edge, and have a rather flat shape. These singular behaviors are cut off if we take into account the electron mass. To show this behavior explicitly, we first integrate over a smaller positron energy, x_2, while fixing the larger positron energy, x_1, and define the following differential branching ratio and three types of asymmetries (a_{P_1},a_{P_2} and a_T) as a function of the larger positron energy x_1 ($\frac{1}{2} \le x_1 \le 1$):

$$\begin{aligned}
\frac{dB(x_1)}{dx_1} &\equiv \int_{1-x_1}^{x_1} dx_2 \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\varphi \frac{dB}{dx_1 dx_2 d\cos\theta d\varphi} \\
&= 3\{(C_1 + C_2)F_1(x_1) + (C_3 + C_4)F_2(x_1) \\
&\quad +(C_5 + C_6)F_3(x_1) + (C_7 + C_8)F_4(x_1)
\end{aligned}$$

$$+(C_9+C_{10})F_5(x_1)\}, \tag{5.20}$$

$$\begin{aligned}
a_{P_1}(x_1) &\equiv \frac{1}{P\frac{dB(x_1)}{dx_1}}\Big(\int_{1-x_1}^{x_1}dx_2\int_0^1 d\cos\theta\int_0^{2\pi}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi} \\
&\quad -\int_{1-x_1}^{x_1}dx_2\int_{-1}^{0}d\cos\theta\int_0^{2\pi}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi}\Big) \\
&= \frac{3}{2}\frac{1}{\frac{dB(x_1)}{dx_1}}\{(C_1-C_2)F_1(x_1)+(C_3-C_4)G_1(x_1) \\
&\quad +(C_5-C_6)G_2(x_1)-(C_7-C_8)F_4(x_1) \\
&\quad +(C_9-C_{10})F_5(x_1)\},
\end{aligned} \tag{5.21}$$

$$\begin{aligned}
a_{P_2}(x_1) &\equiv \frac{-1}{P\frac{dB(x_1)}{dx_1}}\Big(\int_{1-x_1}^{x_1}dx_2\int_{-1}^1 d\cos\theta\int_0^{\frac{\pi}{2}}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi} \\
&\quad -\int_{1-x_1}^{x_1}dx_2\int_{-1}^{1}d\cos\theta\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi} \\
&\quad +\int_{1-x_1}^{x_1}dx_2\int_{-1}^{1}d\cos\theta\int_{\frac{3}{2}\pi}^{2\pi}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi}\Big) \\
&= \frac{3}{2}\frac{1}{\frac{dB(x_1)}{dx_1}}\{(C_3-C_4)H_1(x_1)+(C_5-C_6)H_2(x_1) \\
&\quad +(C_7-C_8)H_3(x_1)-(C_9-C_{10})H_4(x_1)\},
\end{aligned} \tag{5.22}$$

$$\begin{aligned}
a_T(x_1) &\equiv \frac{-1}{P\frac{dB(x_1)}{dx_1}}\Big(\int_{1-x_1}^{x_1}dx_2\int_{-1}^1 d\cos\theta\int_0^{\pi}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi} \\
&\quad -\int_{1-x_1}^{x_1}dx_2\int_{-1}^{1}d\cos\theta\int_{\pi}^{2\pi}d\varphi\frac{dB}{dx_1dx_2d\cos\theta d\varphi}\Big) \\
&= \frac{3}{2}\frac{1}{\frac{dB(x_1)}{dx_1}}\{C_{11}H_3(x_1)-C_{12}H_4(x_1)\}.
\end{aligned} \tag{5.23}$$

In these formula, F_i, G_i and H_i are functions of variable x_1; their analytic forms are found in Appendix E. $\frac{dB(x_1)}{dx_1}$, $a_{P_1}(x_1)$, $a_{P_2}(x_1)$ and $a_T(x_1)$ are defined to extract terms (i)-(iv) with different angular dependences, and $a_T(x_1)$ is the T-odd quantity. In the above expression, $F_3(x_1)$ in $\frac{dB(x_1)}{dx_1}$ and $G_2(x_1)$ in a_{P_1} have a $\frac{1}{1-x_1}$ singularity.

Introducing the cutoff δ for variable x_1 and integrating over $\frac{1}{2} \leq x_1 \leq 1-\delta$, we define the integrated branching ratio, B, and three asymmetries (A_{P_1},A_{P_2} and A_T) as follows':

$$
\begin{aligned}
B[\delta] &= \int_{\frac{1}{2}}^{1-\delta} dx_1 \frac{dB(x_1)}{dx_1} \\
&= 3\{(C_1+C_2)I_1[\delta]+(C_3+C_4)I_2[\delta]+(C_5+C_6)I_3[\delta] \\
&\quad +(C_7+C_8)I_4[\delta]+(C_9+C_{10})I_5[\delta]\}, \qquad (5.24)\\
A_{P_1}[\delta] &= \frac{1}{B[\delta]}\int_{\frac{1}{2}}^{1-\delta} dx_1 a_1(x_1)\frac{dB}{dx_1}(x_1) \\
&= \frac{3}{2B[\delta]}\{(C_1-C_2)I_1[\delta]+(C_3-C_4)J_1[\delta]+(C_5-C_6)J_2[\delta] \\
&\quad -(C_7-C_8)I_4[\delta]+(C_9-C_{10})I_5[\delta]\}, \qquad (5.25)\\
A_{P_2}[\delta] &= \frac{1}{B[\delta]}\int_{\frac{1}{2}}^{1-\delta} dx_1 a_2(x_1)\frac{dB}{dx_1}(x_1) \\
&= \frac{3}{2B[\delta]}\{(C_3-C_4)K_1[\delta]+(C_5-C_6)K_2[\delta]+(C_7-C_8)K_3[\delta] \\
&\quad -(C_9-C_{10})K_4[\delta]\}, \qquad (5.26)\\
A_T[\delta] &= \frac{1}{B[\delta]}\int_{\frac{1}{2}}^{1-\delta} dx_1 a_3(x_1)\frac{dB}{dx_1}(x_1) \\
&= \frac{3}{2B[\delta]}\{C_{11}K_3[\delta]-C_{12}K_4[\delta]\}. \qquad (5.27)
\end{aligned}
$$

I_i, J_i and K_i are functions of the cutoff δ; their analytic forms are also found in Appendix E. Note that $I_3[\delta]$ and $J_2[\delta]$ have a logarithmic singularity at $\delta = 0$. Because of this logarithmic dependence, the terms $|A_L|^2$ and $|A_R|^2$ dominate over all other terms in the branching ratio if the coupling constants eA_L, eA_R and $\hat{g}_i$ have similar magnitudes. On the other hand, because the numerator of A_T does not have a singular behavior, A_T, itself, is suppressed when we take a very small δ. In the latter numerical analysis of SUSY GUT cases we introduce the cutoff δ to optimize the T-odd asymmetry.

We have to take into account the electron mass properly to obtain a precise value of the total branching ratio. If the photon-penguin contribution dominates the branching ratio, we can derive a model-independent relation between the two branching ratios [39], as follows:

$$\begin{aligned}
\frac{B(\mu^+ \to e^+e^+e^-)}{B(\mu^+ \to e^+\gamma)} &\simeq \frac{\alpha}{3\pi}(\ln(\frac{m_\mu^2}{m_e^2}) - \frac{11}{4}), \\
&\simeq 0.0061,
\end{aligned} \tag{5.28}$$

where α is the fine structure constant. Neglecting the terms suppressed by $\frac{m_e}{m_\mu}$, the total branching ratio is , therefore, given by

$$\begin{aligned}
B(\mu^+ \to e^+e^+e^-) &= 2(C_1 + C_2) + (C_3 + C_4) + 32\{\log(\frac{m_\mu^2}{m_e^2}) - \frac{11}{4}\}(C_5 + C_6) \\
&\quad +16(C_7 + C_8) + 8(C_9 + C_{10}).
\end{aligned} \tag{5.29}$$

Chapter 6

Results of numerical calculations

Now that we have defined various P- and T-odd observables in the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes, let us present results of our numerical analysis based on these observables for the SU(5) and SO(10) SUSY GUT. We also discuss our calculation of the electron, neutron and Hg EDMs as constraints on the CP-violating phases of the soft SUSY-breaking terms. Following the procedure discussed in chapter 4, we solve the renormalization group equations with the universal condition for the SUSY breaking terms at the Plank scale. Though approximate formula for the slepton mass and mass difference are given in chapters 3 and 4 to explain qualitative features, we solved the renormalization group equations from the Planck scale to the electroweak scale numerically while taking into account the full flavor-mixing matrix for fermions and sfermions. To determine allowed range of SUSY parameter space we used the results of various SUSY particle searches at LEP and Tevatron and the branching ratio, $B(b \to s\gamma)$. The details concerning these constraints are described in Ref. [44]*. We took the top quark mass as $m_t = 175$ GeV. Because we calculate the LFV branching ratios divided by $|\lambda_\tau|^2$, the result is almost independent of the CKM matrix elements. For definiteness, we used the input parameters of the CKM matrix elements as $|(V_{CKM})_{cb}| = 0.041$, $|(V_{CKM})_{td}| = 0.006$ and $|(V_{CKM})_{us}| = 0.22$. Requiring the radiative electroweak symmetry breaking described in the section VI of the chapter 2, the free parameters of the SUGRA model can be taken as $\tan\beta$, M_0, m_0, $|A_0|$ and the phase of A_0 (θ_{A_0}) and that of μ (θ_μ).

*The branching ratio $B(b \to s\gamma)$ is updated as $2.0 \times 10^{-4} < B(B \to X_s\gamma) < 4.5 \times 10^{-4}$ [45]

I SU(5) GUT

Let us first discuss the case without the CP violating phases in the SU(5) GUT. In Fig. 6.1 we present the following quantities:

$$\frac{B(\mu^+ \to e^+\gamma)}{|\lambda_\tau|^2}, \frac{B(\mu^+ \to e^+e^+e^-)}{|\lambda_\tau|^2}, \frac{B(\mu^+ \to e^+e^+e^-)}{B(\mu^+ \to e^+\gamma)},$$

$$A(\mu^+ \to e^+\gamma),\ A_{P_1}\ , A_{P_2}, \tag{6.1}$$

in the plane of $m_{\tilde{e}_R}$ and $|A_0|$ for $\tan\beta = 3$, $M_2 = 150\text{GeV}$, $\theta_{A_0} = \theta_\mu = 0$. Here, λ_τ is defined by the mixing matrix which diagonalizes the right-handed slepton mass matrix at the electroweak scale in the basis where the charged lepton mass matrix is diagonalized. For the asymmetries we take the cutoff parameter $\delta = 0.02$. If $|\lambda_\tau| = 10^{-2}$, $B(\mu^+ \to e^+\gamma)$ can be 10^{-11} and $B(\mu \to e^+e^+e^-)$ can be at the 10^{-13} level, but if λ_τ is given by the corresponding CKM matrix element, $|\lambda_\tau|$ becomes $(3-5)\times 10^{-4}$, so that the branching ratios are smaller by three orders of magnitude. In Fig. 6.1(c) the ratio of two branching fractions is shown. If the photon-penguin contribution dominates over the four-fermion ones, this ratio is given by Eq. (5.28). We can see that for the large-parameter region the ratio is enhanced. In particular, near $m_{\tilde{e}_R}$ = 400-600 GeV almost exact cancellation occurs for the photon-penguin amplitudes [8]. In Fig. 6.1(d) $A(\mu^+ \to e^+\gamma)$ is shown. It is close to 100% except for the small region where an almost exact cancellation occurs. The P-odd asymmetries, A_{P_1} and A_{P_2}, are shown in Fig. 6.1(e) and Fig. 6.1(f). A_{P_1} changes from -30% to 40% and A_{P_2} changes from -10% to 15%. For $\delta = 0.02$ the asymmetries A_{P_1} and A_{P_2} are expressed as follows:

$$\begin{aligned} A_{P_1} &\simeq \frac{3}{2B}\{0.6(C_1 - C_2) - 0.12(C_3 - C_4) + 5.6(C_5 - C_6) \\ &\quad -4.7(C_7 - C_8) + 2.5(C_9 - C_{10})\}, \end{aligned} \tag{6.2}$$

$$\begin{aligned} A_{P_2} &\simeq \frac{3}{2B}\{0.1(C_3 - C_4) + 10(C_5 - C_6) \\ &\quad +2(C_7 - C_8) - 1.6(C_9 - C_{10})\}. \end{aligned} \tag{6.3}$$

In the SU(5) case, because only $\hat{g}_3$, $\hat{g}_5$ and A_L have sizable contributions, we obtain the following expressions:

$$A_{P_1} \simeq \frac{3}{2B}\{0.6|\hat{g}_3|^2 - 0.12|\hat{g}_5|^2 - 5.6|eA_L|^2 + 4.7Re(eA_L\hat{g}_3^*) - 2.5Re(eA_L\hat{g}_5^*)\}, \tag{6.4}$$

$$A_{P_2} \simeq \frac{3}{2B}\{0.1|\hat{g}_5|^2 - 10|eA_L|^2 - 2Re(eA_L\hat{g}_3^*) + 1.6Re(eA_L\hat{g}_5^*)\}. \tag{6.5}$$

In the above formula we can see that the coefficients for $|A_L|^2$, $Re(A_L\hat{g}_3^*)$ and $Re(A_L\hat{g}_5^*)$ are large. Therefore, these asymmetries represent the dependence of the square of photon-penguin terms and interference terms. It is interesting to see that we can over-determine the three coupling constants ($\hat{g}_3$, $\hat{g}_5$ and A_L) from observables $B(\mu^+ \to e^+\gamma)$, $B(\mu^+ \to e^+e^+e^-)$, A_{P_1} and A_{P_2} if we assume the SU(5) SUSY GUT without the SUSY CP violating phases. For example, we can determine $\hat{g}_3$, $\hat{g}_5$ and A_L from the three observables $B(\mu^+ \to e^+\gamma)$, $B(\mu \to e^+e^+e^-)$ and A_{P_1}; then, A_{P_2} can be predicted. In addition we should have $A(\mu^+ \to e^+\gamma) = 100\%$ and $A_T = 0$.

Next, we include the SUSY CP-violating phases and discuss the EDM constraints and T-odd asymmetry. We calculate the electron and neutron EDMs according to Ref. [46]. Useful formulas for their calculations are summarized in Appendix F. For the Hg EDM, we use the result of Ref. [41]. d_{Hg} is given by:

$$d_{Hg} = -(C_d^C - C_u^C - 0.012C_s^C) \times 3.2 \cdot 10^{-2} e, \tag{6.6}$$

where C_u^C, C_d^C and C_s^C are chromomagnetic moments discussed in Appendix F.

In order to see the θ_{A_0} and θ_μ dependences on the EDMs and A_T, we first show these quantities for a specific set of SUSY parameters. In Fig. 6.2, the electron, neutron and Hg EDMs and A_T are shown for $\tan\beta = 3$, $M_2 = 300\text{GeV}$, $m_{\tilde{e}_R} = 650\text{GeV}$, $|A_0| = 1$ in the parameter region $-\pi < \theta_{A_0} \leq \pi$ and $-0.05\pi \leq \theta_\mu \leq 0.05\pi$. The experimental bounds on the EDMs are given by $|d_e| < 4 \times 10^{-27}(e \cdot cm)$ [47], $|d_n| < 0.63 \times 10^{-25}(e \cdot cm)$ [48] and $|d_{Hg}| < 9 \times 10^{-28}(e \cdot cm)$ [49]. As is well known from Ref. [50], because the EDMs are very sensitive to θ_μ, θ_μ is strongly constrained. On the other hand θ_{A_0} can be large. In this particular parameter set, $\theta_{A_0} = \frac{\pi}{2}$ is not excluded by three EDM constraints. The maximum value of the T-odd asymmetry, A_T in the allowed region in this figure is 15%. Note that A_T is proportional to

$\sin\theta_{A_0}$ in a good approximation because the magnitude of θ_μ is strongly constraint by the EDMs.

In Fig. 6.3 we show the quantities in Eq.(6.1) and A_T for $\tan\beta = 3$, $M_2 = 300\text{GeV}$, $\theta_{A_0} = \frac{\pi}{2}$, $\theta_\mu = 0$. We also show the constraints from the electron, neutron and Hg EDMs. Within the EDM constraints A_T can be 10%. As shown in Fig. 6.2, when we vary θ_μ around $\theta_\mu = 0$, the EDM values change considerably but A_T is almost constant. Therefore, the allowed region by the EDM constraints moves in Fig. 6.3 if we take θ_μ as being a slightly different value from 0. On the other hand, the contours for the branching ratios and the asymmetries in this figure are almost exactly the same. In this figure we also show the parameter region which is not allowed by the EDM constraints, even if we change θ_μ around $\theta_\mu = 0$ for $\theta_{A_0} = \frac{\pi}{2}$. Within the allowed region, the maximum value of A_T is 15%.

Similar plots are shown for $\tan\beta = 10$ in Fig. 6.4. In this case, also, the maximum value of A_T is about 15%. Note that, in the case with the CP violating phases, we can still determine the complex coupling constants ($\hat{g}_3$, $\hat{g}_5$ and A_L) up to a total phase from the two branching ratios, $B(\mu^+ \to e^+\gamma)$, $B(\mu^+ \to e^+e^+e^-)$, and three asymmetries: A_{P_1}, A_{P_2} and A_T.

II SO(10) GUT

In the SO(10) case, from Eq. (4.7) there are two physical phases which contribute to the EDMs and the $\mu^+ \to e^+\gamma$ amplitudes. In the $\mu^+ \to e^+\gamma$ amplitudes the term proportional to m_τ has a dependence of $e^{i(\phi_3-\phi_2)}(V^0_{CKM})_{32}\{(V^0_{CKM})^*_{33}\}^2(V^0_{CKM})_{31}$ and other contributions are proportional to $(V^0_{CKM})^*_{32}(V^0_{CKM})_{31}$. Therefore, the branching ratio $\mu^+ \to e^+\gamma$ depends on the relative phase of the two terms. In the following we consider the case where there is no relative phase, so that the amplitude is proportional to λ_τ. Also, we do not consider EDM constraints from Eq. (4.12) explicitly, since this can be suppressed when ϕ is small.

In Fig. 6.5 the branching ratios and the asymmetries are shown for the SO(10) model. We first show the case without the SUSY CP-violating phases. Input SUSY parameters are taken as $\tan\beta = 3$, $M_2 = 150\text{GeV}$, $\theta_{A_0} = 0$ and $\theta_\mu = 0$. We see that $B(\mu^+ \to e^+\gamma)/|\lambda_\tau|^2$ can be 10^{-3}. This value is enhanced by 2-4 orders of magnitude compared to the SU(5) case. The ratio of two branching fractions is almost constant, because the photon-penguin diagrams give dominant contributions to $\mu^+ \to e^+e^+e^-$. The $\mu^+ \to e^+\gamma$ asymmetry $A(\mu^+ \to e^+\gamma)$ varies from -20% to -90%. This is in contrast to the previous belief that A_L and A_R have a similar magnitude in this model. As discussed in chapter 4, although the diagram proportional to m_τ gives the same contribution to A_L and A_R, there is a chargino loop diagram which only contributes to A_R. In spite of no m_τ enhancement, the contribution from the latter diagram can be comparable to that from the former one, especially when the slepton mass is larger than the chargino mass. The dominant contributions to A_L and A_R are discussed based on approximate formulas in a special parameter region in Appendix D. In Fig. 6.5(e), (f) the P-odd asymmetries for $\mu^+ \to e^+e^+e^-$ are shown and these asymmetries are small compared to the SU(5) case. A_{P1} is less than 10% and A_{P2} is less than 14%. In this case the C_5 and C_6 terms dominate in Eqs. (6.2) and (6.3), so that these asymmetries are proportional to $A(\mu^+ \to e^+\gamma)$ and expressed, as follows:

$$A_{P_1} \simeq -\frac{1}{10}A(\mu^+ \to e^+\gamma), \tag{6.7}$$

$$A_{P_2} \simeq -\frac{1}{6}A(\mu^+ \to e^+\gamma). \tag{6.8}$$

It is interesting to see that we can predict two observables in the $\mu^+ \to e^+e^+e^-$ process from the $\mu^+ \to e^+\gamma$ asymmetry. We have also investigated the case with $\tan\beta = 10$. We found that the parity asymmetries for $\mu^+ \to e^+\gamma$ and A_{P1}, A_{P2} have similar magnitudes as in Fig. 6.5; namely, $A(\mu^+ \to e^+\gamma)$ varies $-20\% - -100\%$, A_{P1} varies $2\% - 10\%$ and A_{P2} varies $4\% - 16\%$ in the same parameter space.

In Fig. 6.6 we consider the case with the SUSY CP-violating phase, and take the input parameters as $\tan\beta = 3$, $M_2 = 300\text{GeV}$, $\theta_{A_0} = \frac{\pi}{2}$ and $\theta_\mu = 0$. The branching ratio and other asymmetries have similar magnitudes compared to the case in Fig. 6.5. We can see that the T-odd asymmetry, A_T, is less than 0.01%, because only the photon-penguin amplitude becomes large.

Some remarks are in order:

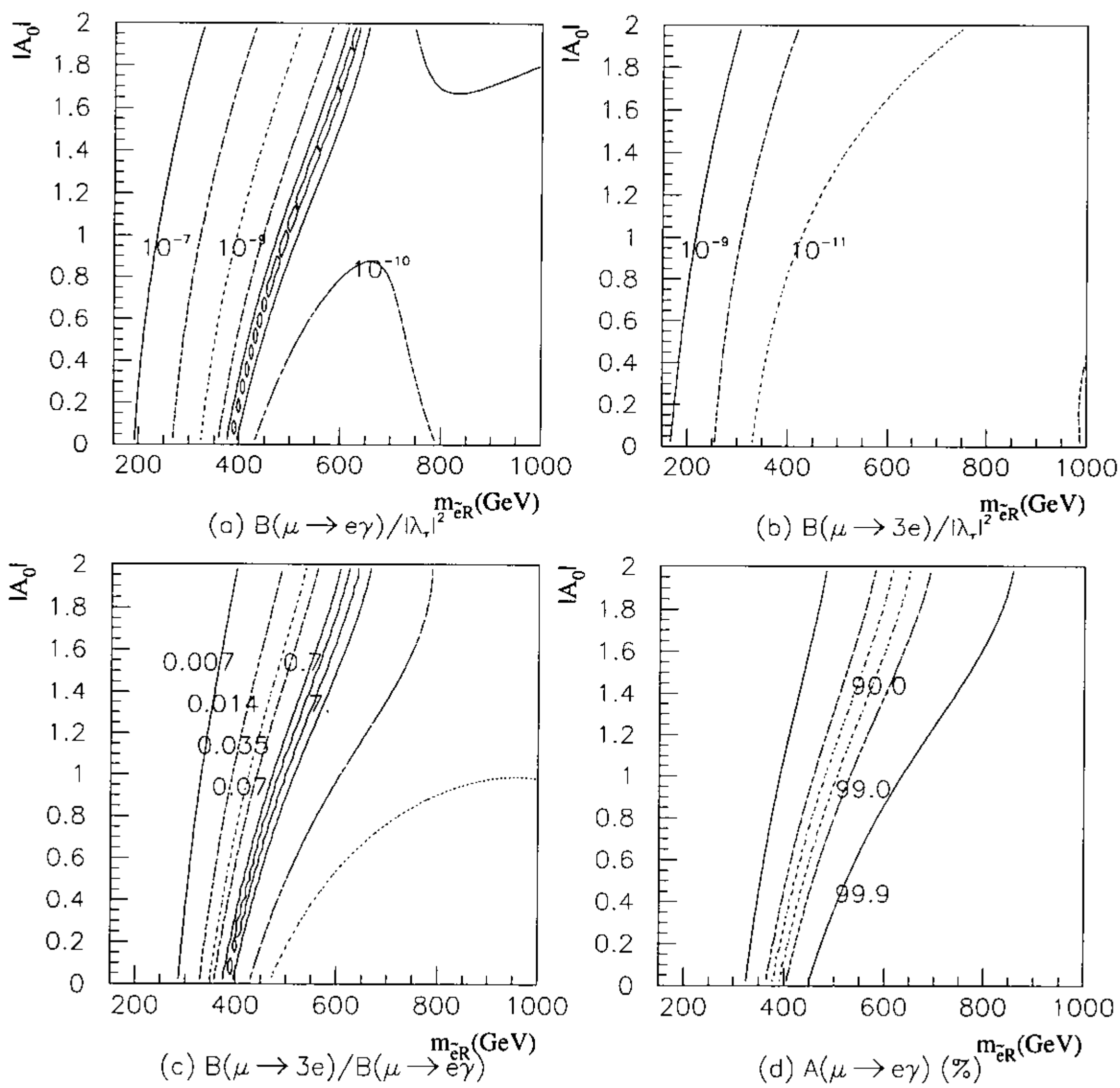
1. When we take into account the phase in Eq. (4.11), the EDM is generated as discussed in Eq. (4.12). We note that the T-odd asymmetry cannot be large even in such a case, because the photon-penguin diagram dominates over the four-fermion contributions.

2. If the $\mu^+ \to e^+\gamma$ asymmetry is sizable, the simple relationship between the EDM and the $\mu^+ \to e^+\gamma$ branching ratio, as in Eq. (4.12), does not hold. This is because the EDM amplitude is no longer proportional to the $\mu^+ \to e^+\gamma$ amplitude due to the chargino loop contribution.

3. Even if we include the relative phases between the term proportional to m_τ and other contribution in the $\mu^+ \to e^+\gamma$ amplitude, we expect a large $A(\mu^+ \to e^+\gamma)$ as long as the two contributions have a similar magnitude. By a numerical calculation we have checked that the asymmetry varies from -100% to 100% if we include the relative phases. Qualitatively, this feature can be understood by the approximate formulas in Appendix C. From Eq.(D.1) we can see that the neutralino and chargino contributions to A_R can interfere either constructively or destructively, depending on the relative phase, so that $A(\mu^+ \to e^+\gamma)$ can change its sign.

III Differential branching ratio and asymmetries

Up to now, we have only discussed the integrated branching ratio and asymmetries of $\mu^+ \to e^+e^+e^-$. In the actual experiment, the differential quantities are useful to distinguish different models. For example, in Figs. 6.7 and 6.8, we show the differential branching ratio and asymmetries for a particular parameter set in the SU(5) and SO(10) models. $\frac{dB}{dx_1}$, a_{P_1}, a_{P_2} and a_T are plotted for the parameter set of $\tan\beta = 3$, $m_{\tilde{e}_R} = 700$ GeV, $M_2 = 300$ GeV, $|A_0| = 0.5$, $\theta_{A_0} = \frac{\pi}{2}$ and $\theta_\mu = 0$. We can see a clear difference between the SU(5) and SO(10) models. The differential branching has a steep peak near to $x_1 = 1$ for the SO(10) case, whereas the distribution is broader for the SU(5) case. This is because the photon-penguin contribution has a $\frac{1}{1-x_1}$ behavior near to $x_1 = 1$ and the four-fermion operators give a broad spectrum. We can also see that the T-odd asymmetry has the peak at x_1 close to 1. This fact arises from the $\frac{1}{\sqrt{1-x_1}}$ behavior in the γ_3 and γ_4 near $x_1 = 1$. Because of this feature of the distribution, we have chosen $\delta = 0.02$ to optimize the T-odd asymmetry.



$|A_0|$
$m_{\tilde{e}R}$(GeV)
10^{-7}
10^{-9}
10^{-10}
(a) $B(\mu \to e\gamma)/|\lambda_\tau|^2$
10^{-9}
10^{-11}
(b) $B(\mu \to 3e)/|\lambda_\tau|^2$
0.007
0.014
0.035
0.07
0.7
7.
(c) $B(\mu \to 3e)/B(\mu \to e\gamma)$
90.0
99.0
99.9
(d) $A(\mu \to e\gamma)$ (%)

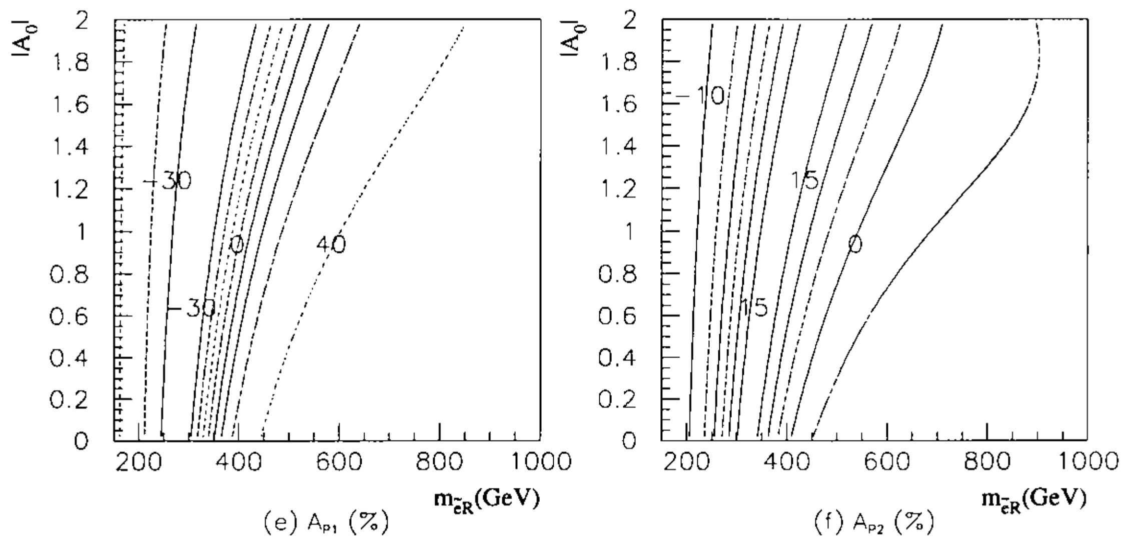


FIG. 6.1: Observables in the SU(5) model without the SUSY CP-violating phases in the $m_{\tilde{e}_R}$-$|A_0|$ plane. We fix the SUSY parameters as $\tan\beta = 3$, $M_2 = 150$ GeV and $\mu > 0$ and the top quark mass as 175 GeV. (a) Branching ratio for $\mu^+ \to e^+\gamma$ normalized by $|\lambda_\tau|^2 \equiv |(V_R)_{23}(V_R)^*_{13}|^2$. (b) Branching ratio for $\mu^+ \to e^+e^+e^-$ normalized by $|\lambda_\tau|^2$. (c) Ratio of two branching fractions $\frac{B(\mu\to 3e)}{B(\mu\to e\gamma)}$. (d) P-odd asymmetry for $\mu^+ \to e^+\gamma$. (e) P-odd asymmetries, A_{P_1}, for $\mu^+ \to e^+e^+e^-$. (f) P-odd asymmetries, A_{P_2}, for $\mu^+ \to e^+e^+e^-$. The cut-off parameter δ is taken to be 0.02.

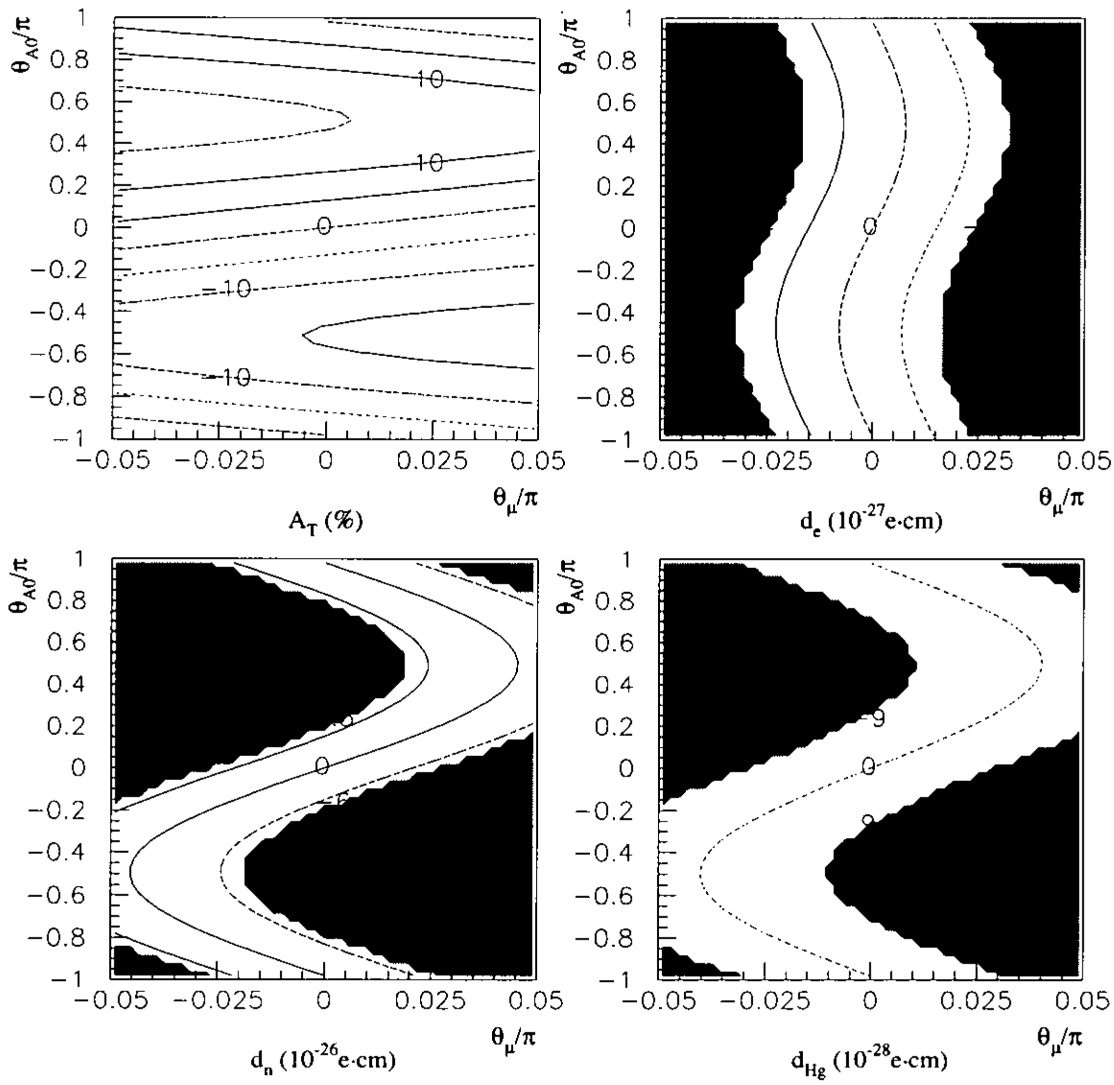
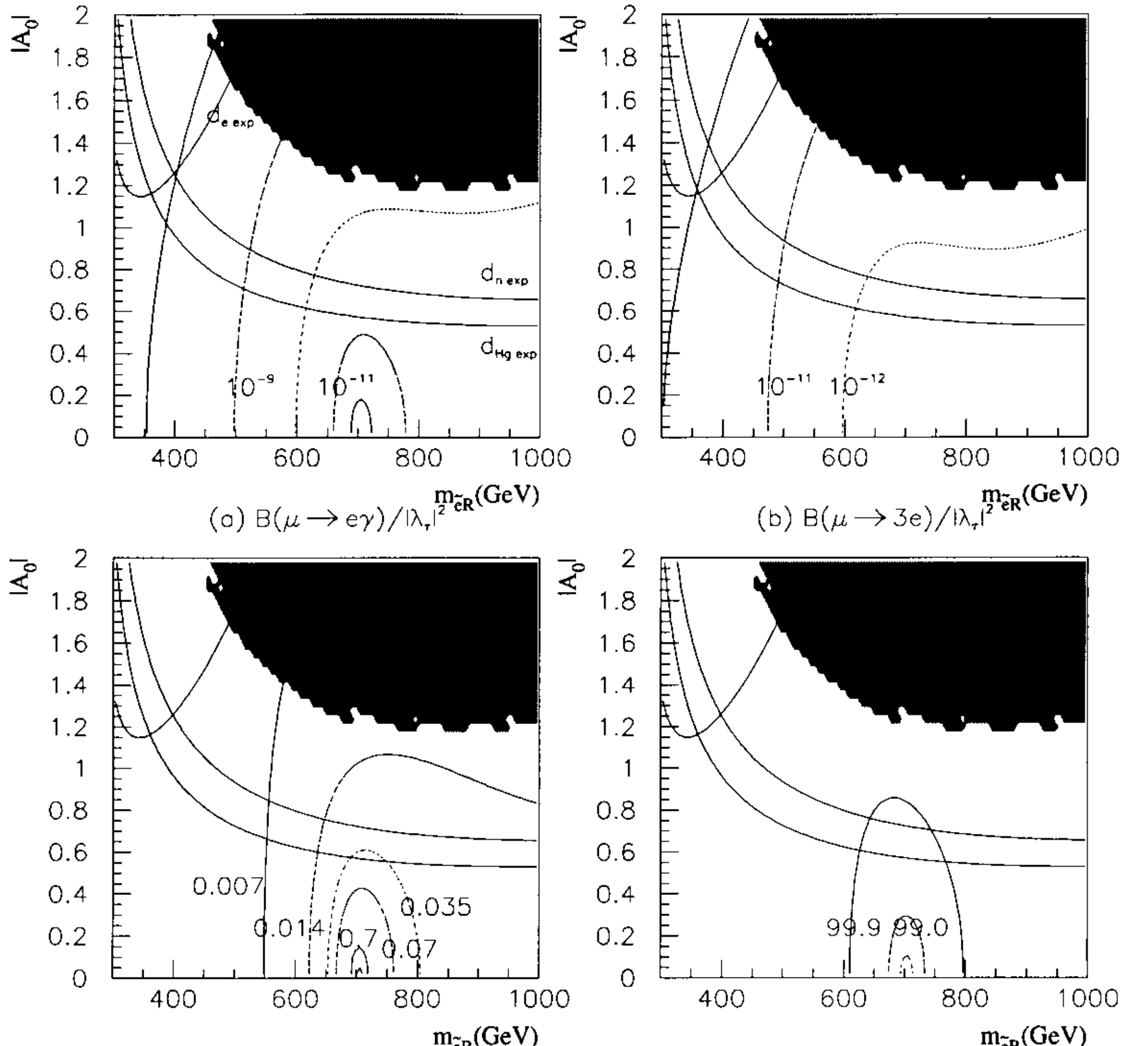


FIG. 6.2: θ_{A_0} and θ_μ dependences on the EDMs and A_T. We take a specific set of SUSY parameters ($\tan\beta = 3$, $M_2 = 300$ GeV, $m_{\tilde{e}_R} = 650$GeV and $|A_0| = 1$) in the parameter region $-\pi < \theta_{A_0} \leq \pi$ and $-0.05\pi \leq \theta_\mu \leq 0.05\pi$. The dark shaded regions are excluded by the EDM experiments



$|A_0|$
$d_{e\,exp}$
$d_{n\,exp}$
$d_{Hg\,exp}$
10^{-9}
10^{-11}
$m_{\tilde{e}R}$(GeV)
(a) $B(\mu \to e\gamma)/|\lambda_\tau|^2$
10^{-11}
10^{-12}
(b) $B(\mu \to 3e)/|\lambda_\tau|^2$
0.007
0.014
0.7
0.07
0.035
(c) $B(\mu \to 3e)/B(\mu \to e\gamma)$
99.9
99.0
(d) $A(\mu \to e\gamma)$ (%)

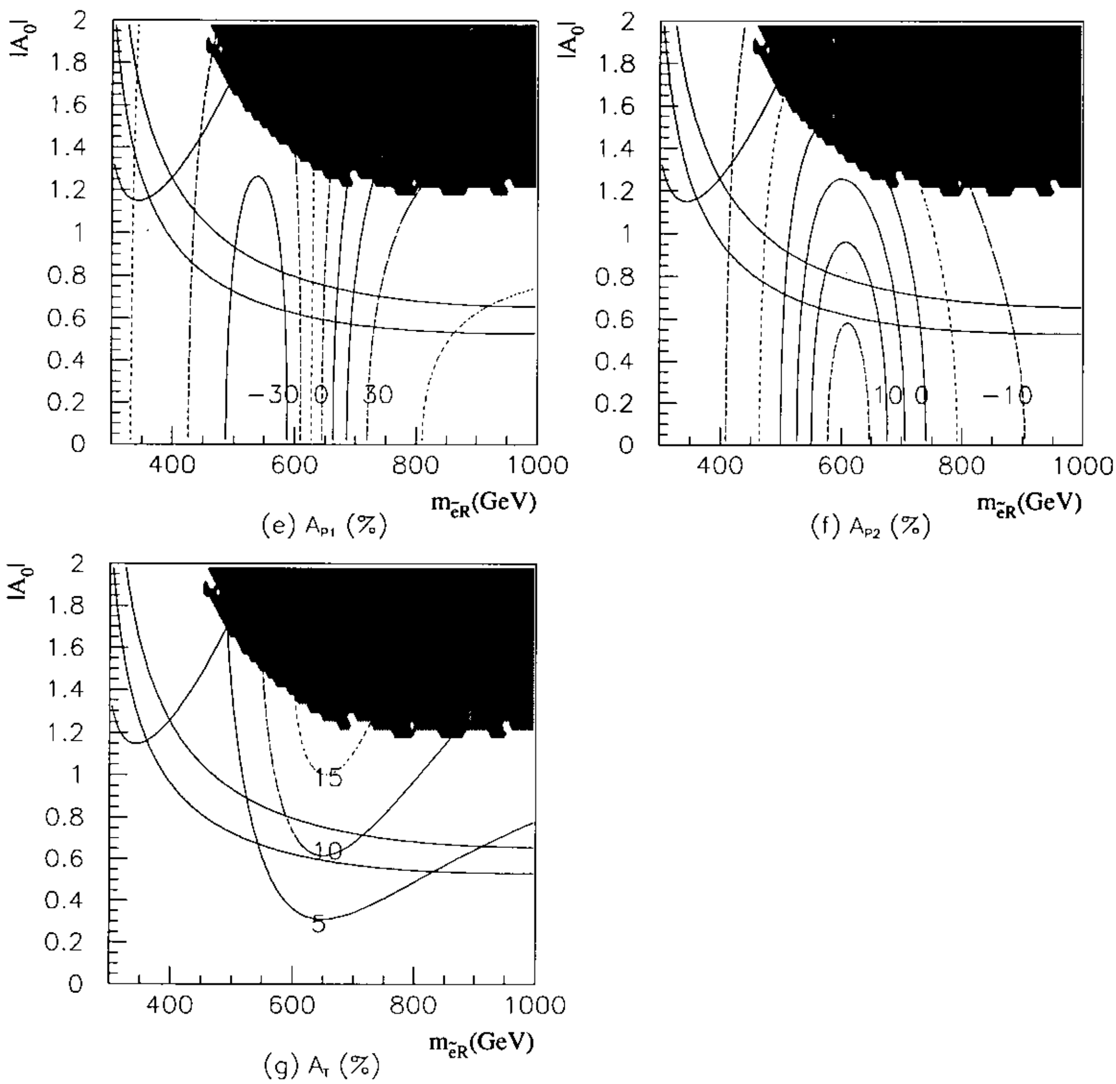
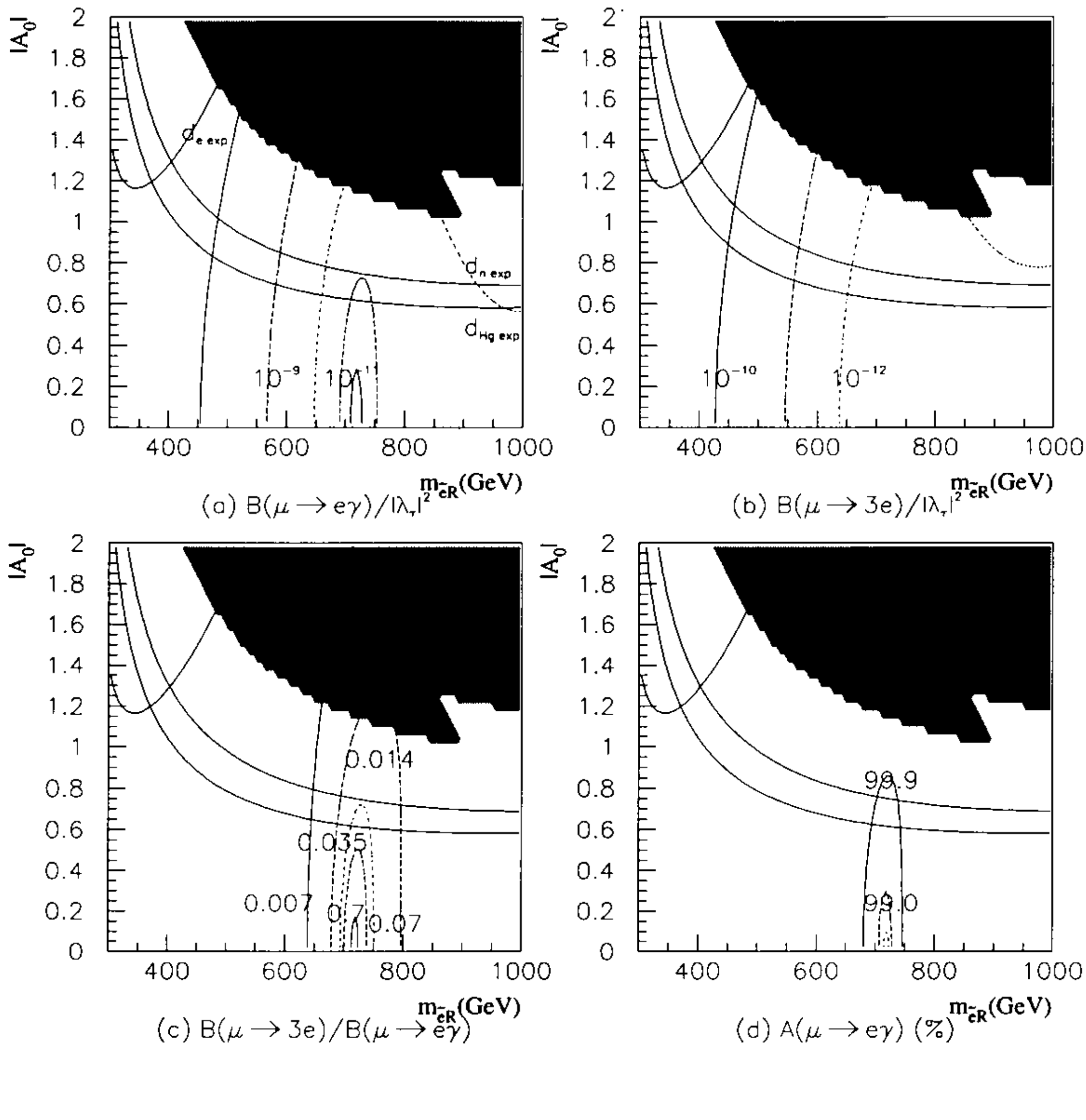


FIG. 6.3: Observables in the SU(5) model with the SUSY CP-violating phases in the $m_{\tilde{e}_R}$-$|A_0|$ plane. We fix the SUSY parameters as $\tan\beta = 3$, $M_2 = 300$ GeV, $\theta_{A_0} = \frac{\pi}{2}$ and $\theta_\mu = 0$ and the top quark mass as 175 GeV. (a)-(f) are same as Fig. 6.1. (g) T-odd asymmetry for $\mu^+ \to e^+e^+e^-$. The cut-off parameter, δ, is taken to be 0.02. The experimental bounds from the electron, neutron and Hg EDMs are also shown in each figure. The left upper line corresponds to the electron EDM, the right upper line to the neutron EDM and the right lower line to the Hg EDM. The lower side of each bound is allowed by these experiments. A dark shaded region is excluded by the EDM bounds, even if we allow θ_μ to take a slightly different value from 0.



|A0|
m~eR(GeV)
d e exp
d n exp
d Hg exp
10^-9
10^-11
(a) B(μ → eγ)/|λτ|²
10^-10
10^-12
(b) B(μ → 3e)/|λτ|²
0.014
0.035
0.007
0.7
0.07
(c) B(μ → 3e)/B(μ → eγ)
99.9
99.0
(d) A(μ → eγ) (%)

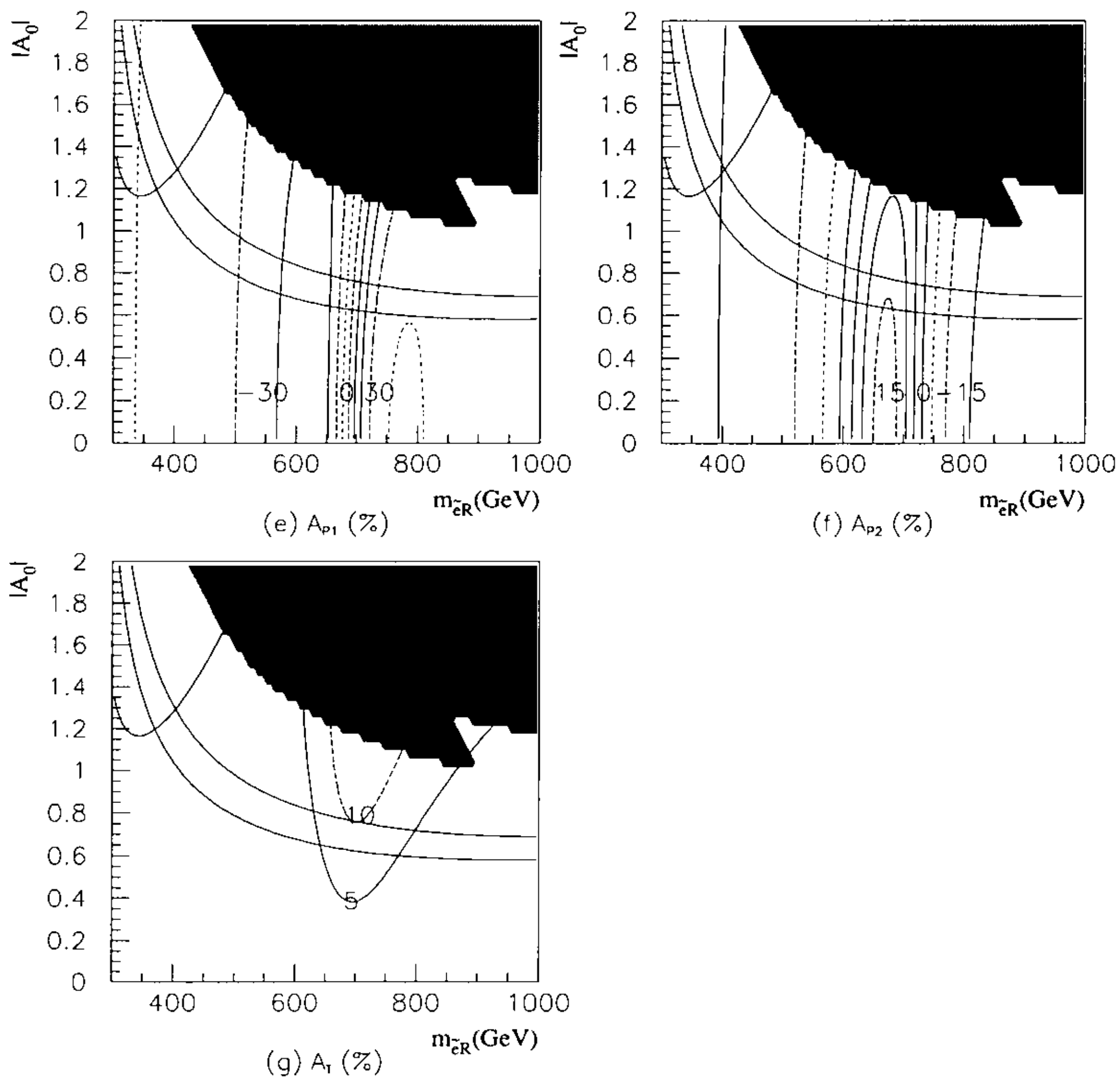
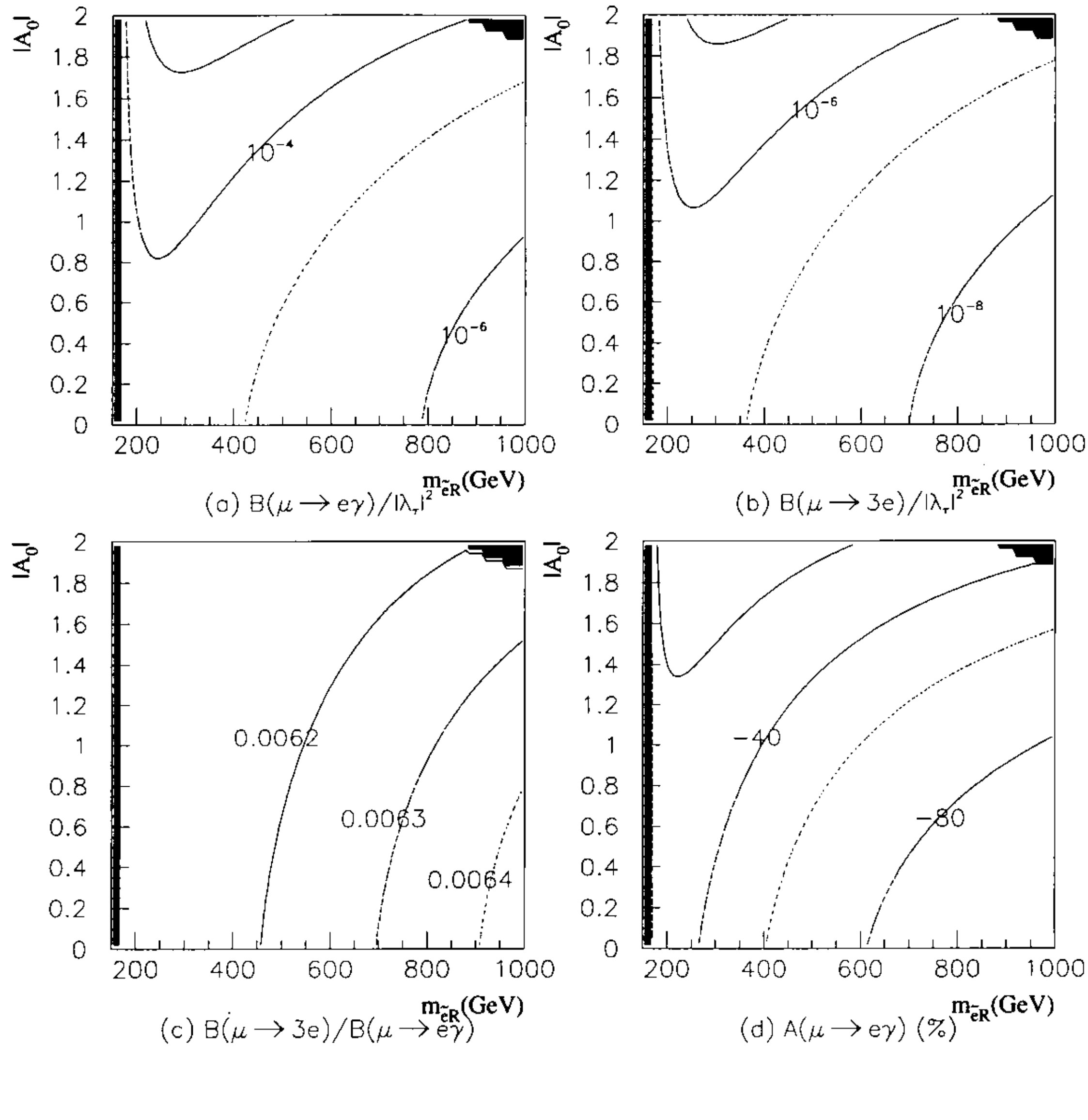


FIG. 6.4: Observables in the SU(5) model for $\tan\beta = 10$ in the $m_{\tilde{e}_R}$-$|A_0|$ plane. The other parameters are the same as in Fig. 6.3.



$|A_0|$
$m_{\tilde{e}R}$(GeV)
10^{-4}
10^{-6}
(a) $B(\mu \to e\gamma)/|\lambda_\tau|^2$
10^{-6}
10^{-8}
(b) $B(\mu \to 3e)/|\lambda_\tau|^2$
0.0062
0.0063
0.0064
(c) $B(\mu \to 3e)/B(\mu \to e\gamma)$
−40
−80
(d) $A(\mu \to e\gamma)$ (%)

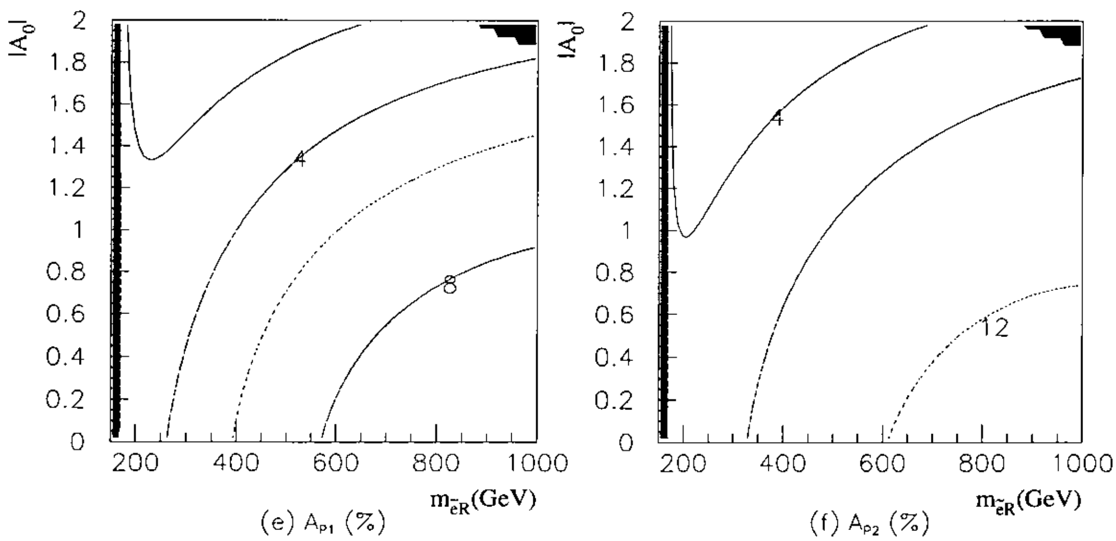
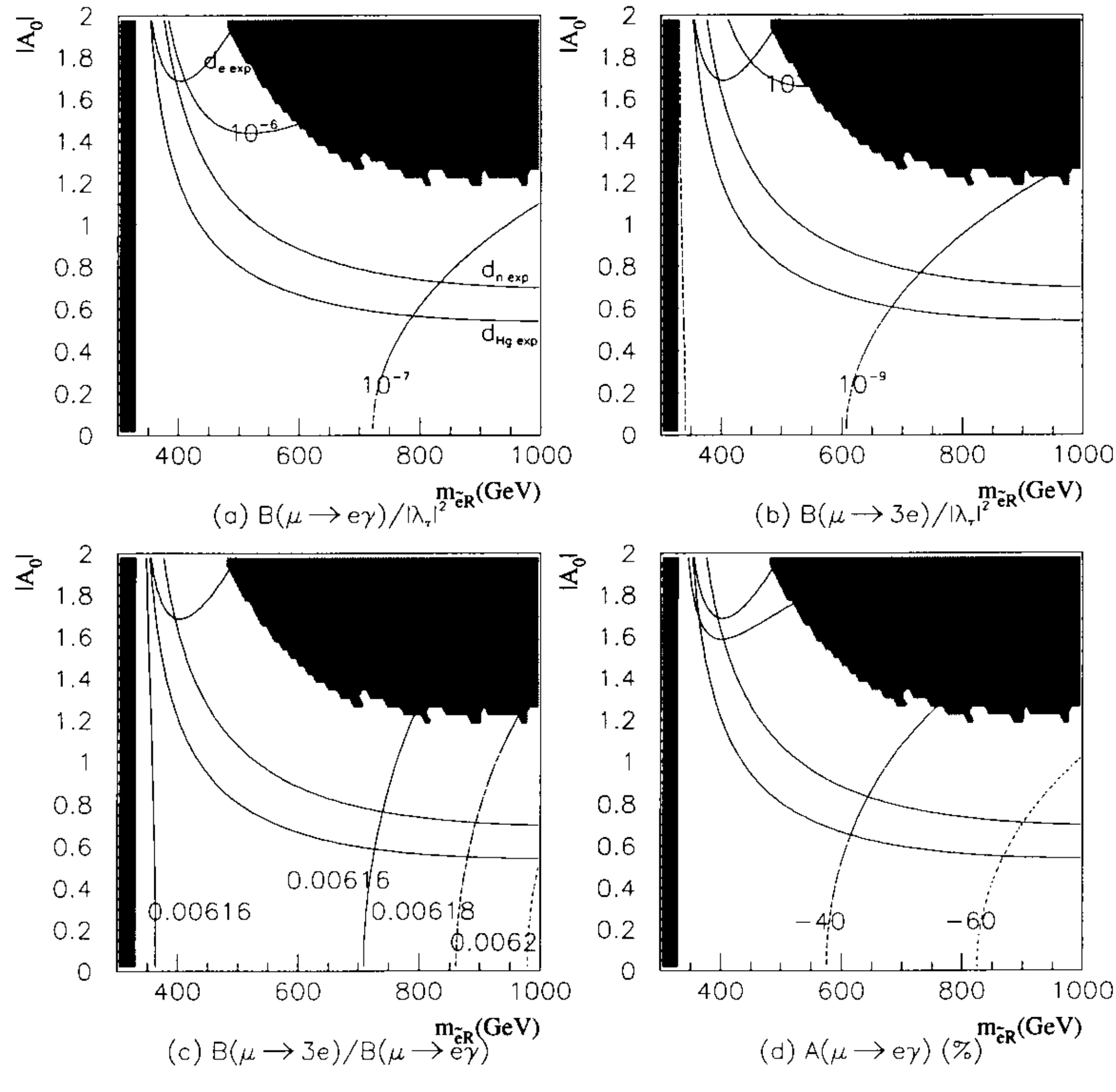


FIG. 6.5: Observables in the SO(10) model without the SUSY CP-violating phase in the $m_{\tilde{e}_R}$-$|A_0|$ plane. The input parameters are the same as in Fig. 6.1. The upper right black region is excluded by phenomenological constraints and the left black region is not allowed in the minimal SUGRA model.


$|A_0|$
$d_{e\ exp}$
10^{-6}
$d_{n\ exp}$
$d_{Hg\ exp}$
10^{-7}
$m_{\tilde{e}R}$(GeV)
(a) $B(\mu \to e\gamma)/|\lambda_\tau|^2$
10^{-9}
(b) $B(\mu \to 3e)/|\lambda_\tau|^2$
0.00616
0.00616
0.00618
0.0062
(c) $B(\mu \to 3e)/B(\mu \to e\gamma)$
−40
−60
(d) $A(\mu \to e\gamma)$ (%)

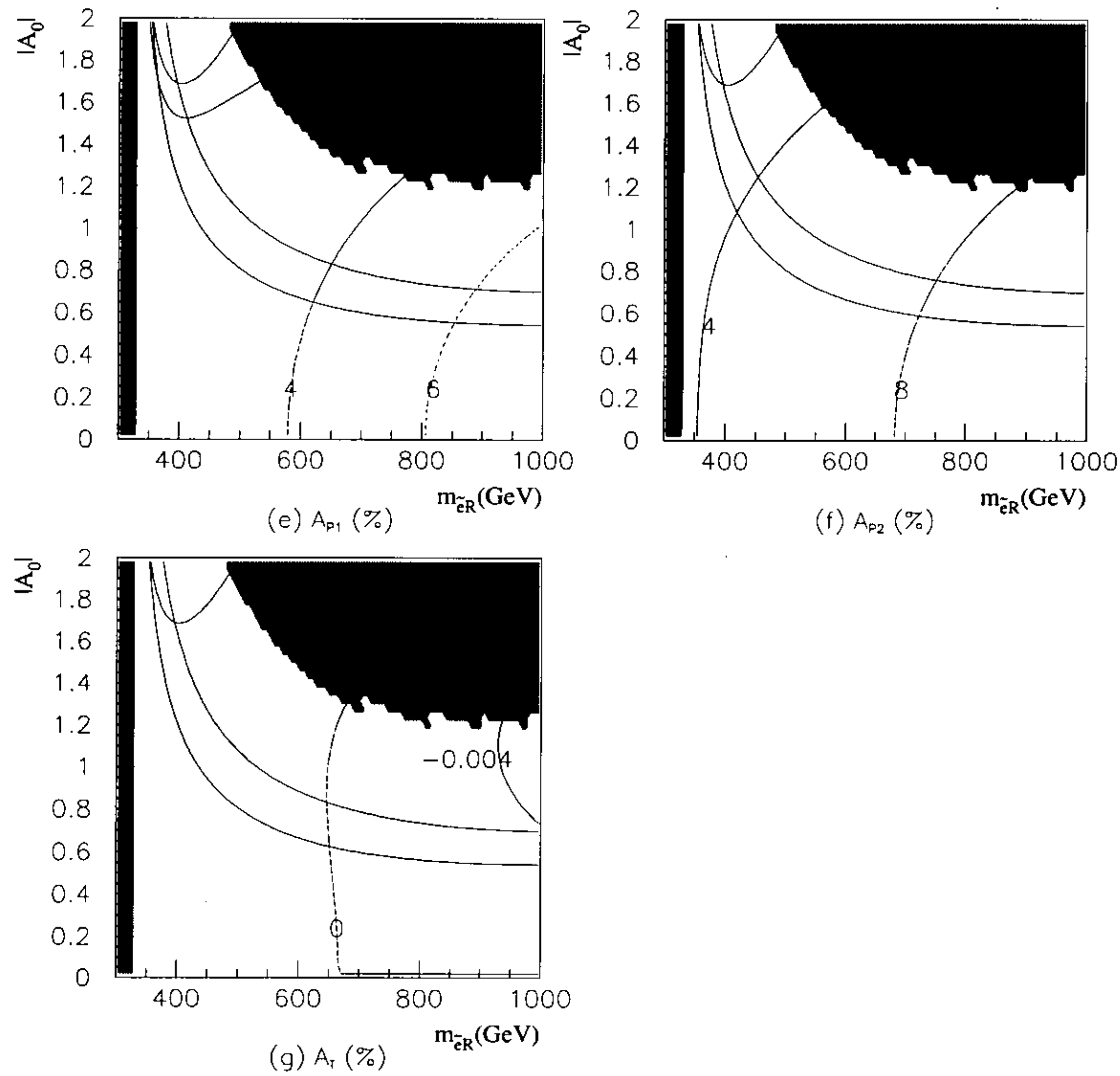


FIG. 6.6: Observables in the SO(10) model with the SUSY CP-violating phase in the $m_{\tilde{e}_R}$-$|A_0|$ plane. The input parameters are the same as in Fig. 6.3.

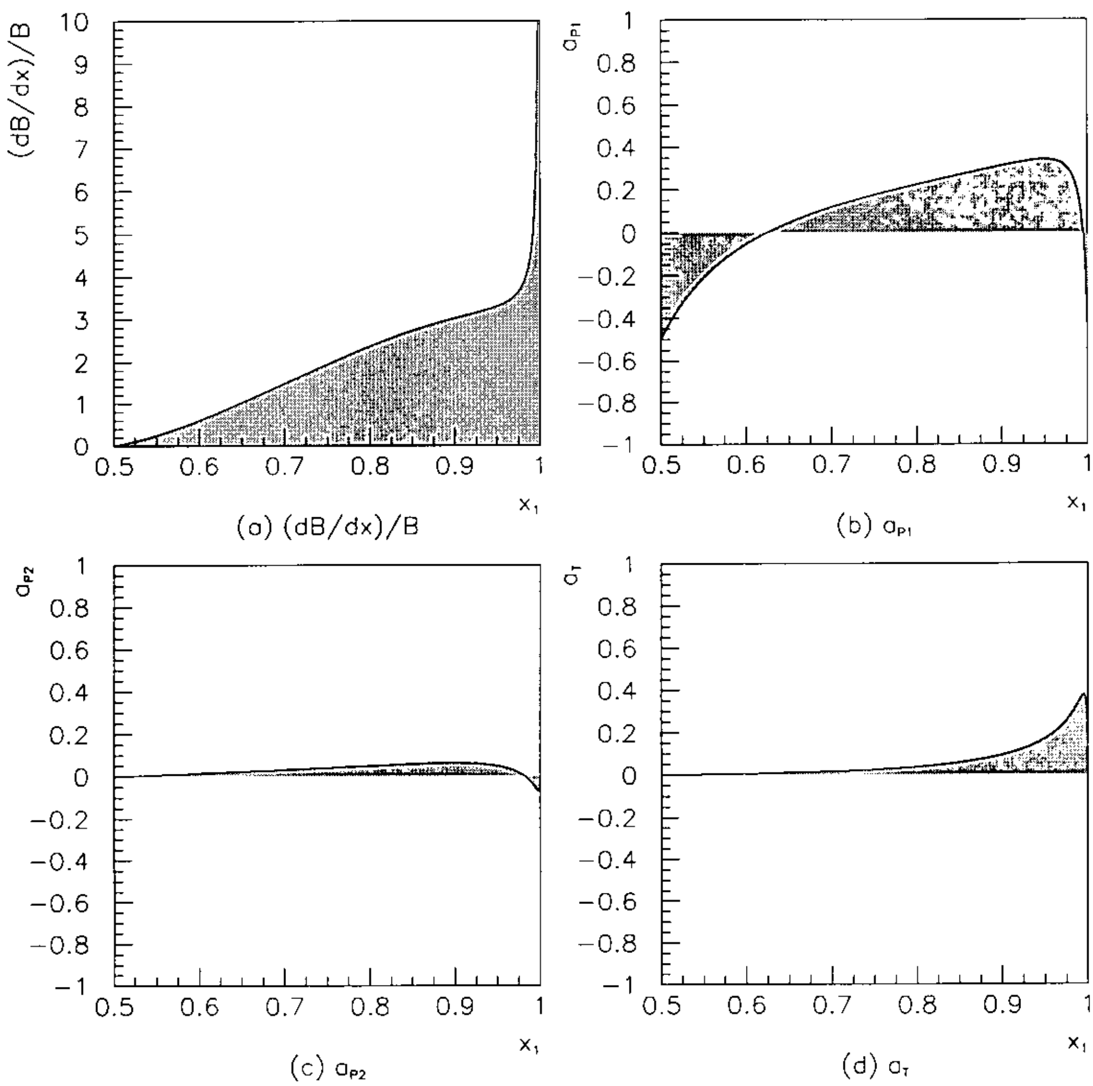


FIG. 6.7: Differential branching ratio and asymmetries for the $\mu^+ \rightarrow e^+e^+e^-$ process in the SU(5) model as a function of x_1, which is a larger energy of decay positrons $(\frac{2E_1}{m_\mu})$. We fix the SUSY parameters as $\tan\beta = 3$, $M_2 = 300$ GeV, $m_{\tilde{e}_R} = 700$GeV, $|A_0| = 0.5$, $\theta_{A_0} = \frac{\pi}{2}$ and $\theta_\mu = 0$. (a) Differential branching ratio for the $\mu^+ \rightarrow e^+e^+e^-$ normalized by the total branching ratio. (b) Differential P-odd asymmetry, a_{P_1}. (c) Differential P-odd asymmetry, a_{P_2}. (d) Differential T-odd asymmetry, a_T.

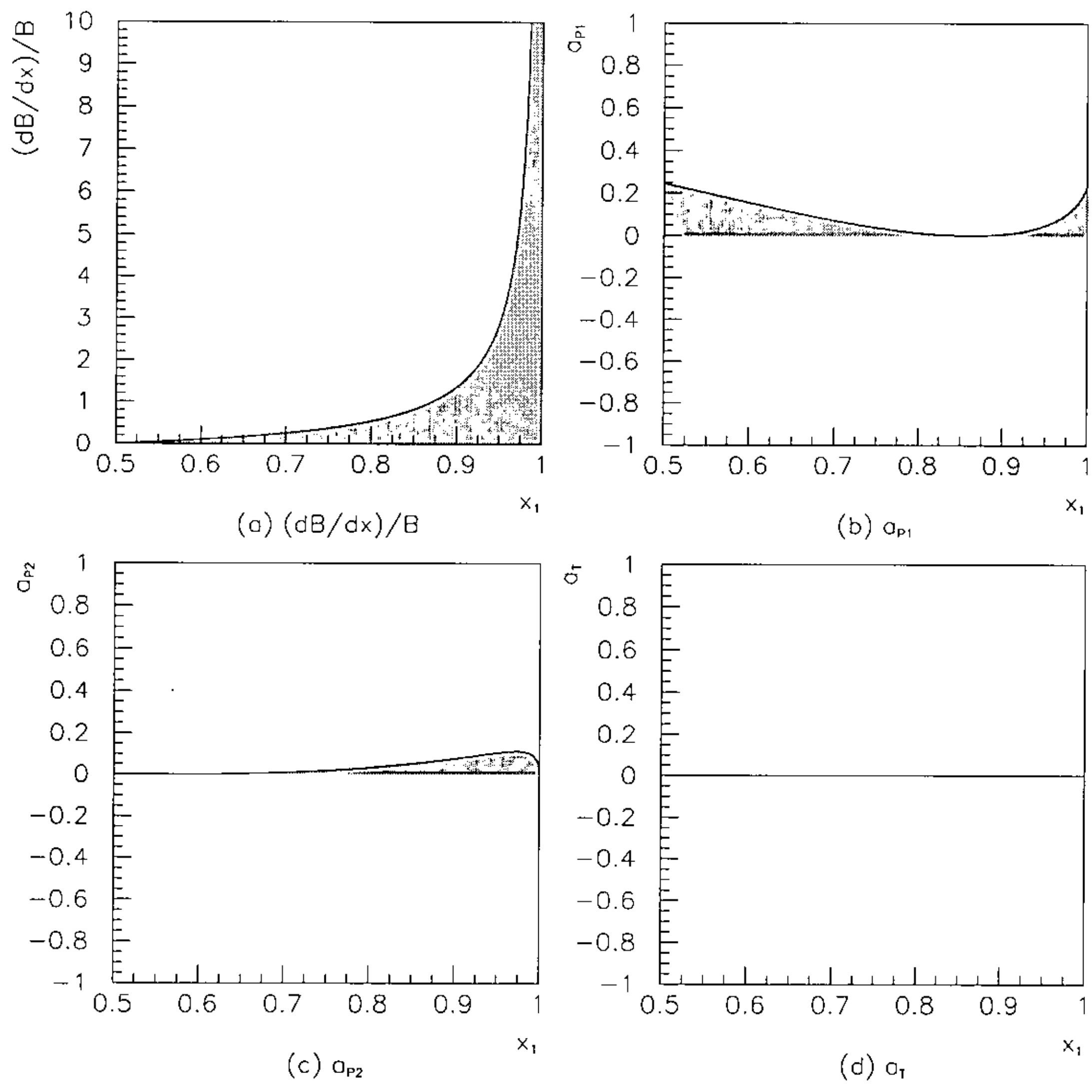


FIG. 6.8: Differential branching ratio and asymmetries for the $\mu^+ \to e^+e^+e^-$ process in the SO(10) model as a function of x_1. The input parameters are the same as in Fig. 6.7.

Chapter 7

Conclusions

We developed a model-independent formalism for the processes $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ with a polarized muon, and defined convenient observables, such as the P-odd and T-odd asymmetries. Using an explicit calculation based on SU(5) and SO(10) SUSY GUT, we show that various combinations of the LFV coupling constants can be determined from measurements of the branching ratio and asymmetries. In the SO(10) case, the P-odd asymmetry in $\mu^+ \to e^+\gamma$ varies from +100% to −100%, whereas it is +100% for the SU(5) case. The P-odd asymmetries in $\mu^+ \to e^+e^+e^-$ are simply proportional to the $\mu^+ \to e^+\gamma$ asymmetry in the SO(10) case, and can be predicted from it. On the other hand, with the branching ratios and the P-odd asymmetries in the $\mu^+ \to e^+e^+e^-$ process, we can over-determine the coupling constants in the effective Lagrangian in the SU(5) SUSY GUT if there is no SUSY CP-violating phases. We also calculated the T-odd asymmetry in the $\mu^+ \to e^+e^+e^-$ process with the SUSY CP-violating phases, and compared it with the neutron, electron and Hg EDMs. In the SU(5) case we can still determine these coupling constants using additional information concerning the T-odd asymmetry. The T-odd asymmetry can reach 15% within the constraints of the EDMs. In the SO(10) case, the T-odd asymmetry is small as a result of the dominance of photon-penguin diagram. These results are summarized in Table 7.1. We stress that although the magnitude of the branching ratio has a large uncertainty due to an unknown parameter, λ_τ, asymmetries and the ratio of two branching ratios are independent of this ambiguity. Thus, these quantities are useful to distinguish different models.

The experimental prospects for measuring these quantities depend on the branching ratio. For the SO(10) model, we expect that the $\mu^+ \to e^+\gamma$ branching ratio can be 10^{-12} when λ_τ is given by the corresponding CKM matrix elements. In such a case, the $\mu^+ \to e^+\gamma$ asymmetry can be measurable in an experiment with a sensitivity on the 10^{-14} level. For the SU(5) model, to obtain a $\mu^+ \to e^+\gamma$ branching ratio on order of 10^{-12} and a $\mu^+ \to e^+e^+e^-$ branching ratio of 10^{-14}, we have to assume that λ_τ is larger than several times 10^{-3}. If the branching ratio turns out to be so large, $\mu^+ \to e^+e^+e^-$ experiments with a sensitivity on the 10^{-16} level could reveal various asymmetries. Because various asymmetries are defined with respect to the muon polarization, experimental searches for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ with polarized muons are very important to uncover the nature of the LFV interactions.

	SU(5) SUSY GUT	SO(10) SUSY GUT
$A(\mu^+ \to e^+\gamma)$	+100%	+100% – −100%
$\frac{B(\mu^+ \to e^+e^+e^-)}{B(\mu^+ \to e^+\gamma)}$	0.007 – $O(1)$	constant ($\sim$ 0.0062)
A_{P_1}	−30% – +40%	$A_{P1} \simeq -\frac{1}{10}A(\mu^+ \to e^+\gamma)$
A_{P_2}	−20% – +20%	$A_{P2} \simeq -\frac{1}{6}A(\mu^+ \to e^+\gamma)$
$\lvert A_T \rvert$	$\lesssim$ 15%	$\lesssim$ 0.01%

Table 7.1: Summary of the results

Chapter 8

Acknowledgments

First of all, I would like to greatly thank Prof. Yasuhiro Okada, who has been my supervisor since I started a phenomenological study of high-energy physics. He is a person who gave me a chance to jump into studying theoretical physics. He trained me with his enthusiastic energy for promoting high-energy phenomenology. I could not accomplish this work without thoughtful and helpful discussions with him and his continuous encouragement.

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I wish to thank members of the theoretical and computational physics of KEK. With their hospitality and cooperation, I could accomplish this thesis.

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I wish to express my gratitude to Prof. Ken-ichi Imai, Prof. Hideto Enyo, Prof. Koichiro Asahi, Prof. Yasuhiro Masuda, Prof. Hirohiko M. Shimizu, members of group for particle and nuclear physics (PN group) in Kyoto University and members of PEN group in KEK. Their frontier spirit influenced me greatly. From experimental studies of physics with them, which ranged from experiments with accelerators to a cosmic-ray search at a mountain, I have learned much pragmatic wisdom in studying science and an extent and depth of nature which could not be obtained without struggling with the real world by my hands. I hope these invaluable experiences are reflected in my study of physics. I also greatly appreciate their encouragement, without which I could not make a decision to jumped into a new field.

Finally, I would like to express my great thanks to my parents, Shin-ichi Okumura and Hisako Okumura. They first led me to the wonder of nature in my childhood. They have offered me more than enough chances to pursue my dream. They have encouraged and supported me during my long period of professional training with an understanding of my devotion to science. Now, sincerely, I would like to dedicate this thesis to them.

Appendix A

Renormalization group equations

In this Appendix we summarize 1-loop renormalization group equations (RGEs) which are used in our numerical calculations.

I Gauge coupling constant

We first present a general formula of RGE for the gauge coupling constant. In 1-loop level, running of the gauge coupling constant is completely determined by the quadratic Casimir of the gauge group and the matter contents, which couple to the gauge field. If there are N_f Weyl fermion fields of representation R_f and N_s complex scalar fields of representation R_s, the coefficient of the beta function is given by:

$$(4\pi)^2 M\frac{\partial}{\partial M}g \quad = \quad b_G g^3, \tag{A.1}$$

$$b_G \quad = \quad -\frac{11}{3}C_2(G)+\sum_f \frac{2}{3}N_f C_2(R_f)+\sum_s \frac{1}{3}N_s C_2(R_s), \tag{A.2}$$

where $C_2(R)$ denotes the quadratic Casimir of the representation R of the gauge group. The adjoint representation is referred as G and sum is performed over different representations. In the case of Abelian gauge theory, the above formula can be used by replacing $C_2(G)=0$ and $C_2(R_f)=Y_f^2$ ($C_2(R_s)=Y_s^2$) where Y_f (Y_s) represents the $U(1)$ charge of the fermion (scalar) field.

In the case of N=1 SUSY, a gauge field has a fermionc partner of adjoint representation and a matter fermion has a scalar partner of the same representation.

Then, we can obtain the formula for N=1 SUSY theories as follows:

$$b_G = -3C_2(G) + \sum_{\Phi} N_\Phi C_2(R_\Phi), \tag{A.3}$$

where N_Φ represents the number of chiral superfields of representation R_Φ which couple to the gauge field.

II MSSM

The running of gauge coupling constants of the MSSM is derived from the general formula in the previous section as follows:

$$(4\pi)^2 M\frac{d}{dM}g_i = b_i g_i^3 \quad (i = 1-3), \tag{A.4}$$

$$b_3 = -3,\ b_2 = 1,\ b_1 = \frac{33}{5}. \tag{A.5}$$

where the coefficients of the beta functions are obtained from the quadratic Casimir and $U(1)$ charge factors listed in Table 2.2 and that of adjoint representations.

The anomalous dimensions from wavefunction renormalization associated with the Yukawa interactions are summarized as follows:

$$\begin{aligned}
&\gamma_Q = y_u^\dagger y_u + y_d^\dagger y_d, && \gamma_L = y_e^\dagger y_e, \\
&\gamma_U = 2y_u y_u^\dagger, && \gamma_E = 2y_e y_e^\dagger, \\
&\gamma_D = 2y_d y_d^\dagger, && \\
&\gamma_{H_1} = (3tr(y_d^\dagger y_d) + tr(y_e^\dagger y_e))\mathbf{1}, && \gamma_{H_2} = 3tr(y_u^\dagger y_u)\mathbf{1}.
\end{aligned} \tag{A.6}$$

Using these anomalous dimensions, the running of the Yukawa coupling constants in the superpotential is described as

$$(4\pi)^2 M\frac{d}{dM}y_e = -2\left(\frac{3}{2}g_2^2 + \frac{9}{10}g_1^2\right) y_e + \gamma_{H_1} y_e + \gamma_E y_e + y_e \gamma_L, \tag{A.7}$$

$$(4\pi)^2 M\frac{d}{dM}y_d = -2\left(\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{7}{30}g_1^2\right) y_d + \gamma_{H_1} y_d + \gamma_D y_d + y_d \gamma_Q, \tag{A.8}$$

$$(4\pi)^2 M\frac{d}{dM}y_u = -2\left(\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{13}{30}g_1^2\right) y_u + \gamma_{H_2} y_u + \gamma_U y_u + y_u \gamma_Q. \tag{A.9}$$

Next we present the RGEs for the soft SUSY-breaking parameters. The running of gaugino masses is determined by the coefficients of the beta functions of the gauge coupling constants as follows:

$$(4\pi)^2 M\frac{d}{dM}M_i \;=\; 2b_i g_i^2 M_i \qquad (i=1-3). \tag{A.10}$$

Note that they do not depend on the other soft SUSY-breaking parameters.

The RGEs for trilinear scalar coupling constants are described as follows:

$$\begin{aligned}
(4\pi)^2 M\frac{d}{dM}A_e &= -2\left(\frac{3}{2}g_2^2+\frac{9}{10}g_1^2\right)A_e \\
&\quad -4\left(\frac{3}{2}g_2^2M_2+\frac{9}{10}g_1^2M_1\right)y_e \\
&\quad +A_e\gamma_L+\gamma_E A_e+A_e\gamma_{H_1} \\
&\quad +2\left\{3tr(y_e^\dagger A_e)y_e+tr(y_e^\dagger A_e)y_e+2A_e y_e^\dagger y_e+y_e y_e^\dagger A_e\right\} \qquad \text{(A.11)}\\
(4\pi)^2 M\frac{d}{dM}A_d &= -2\left(\frac{8}{3}g_3^2+\frac{3}{2}g_2^2+\frac{7}{30}g_1^2\right)A_d \\
&\quad -4\left(\frac{8}{3}g_3^2M_3+\frac{3}{2}g_2^2M_2+\frac{7}{30}g_1^2M_1\right)y_d \\
&\quad +A_d\gamma_Q+\gamma_D A_d+\gamma_{H_1}A_d \\
&\quad +2\left\{3tr(y_d^\dagger A_d)y_d+2A_d y_d^\dagger y_d+y_d y_u^\dagger A_u+y_d y_d^\dagger A_d\right\}, \qquad \text{(A.12)}\\
(4\pi)^2 M\frac{d}{dM}A_u &= -2\left(\frac{8}{3}g_3^2+\frac{3}{2}g_2^2+\frac{13}{30}g_1^2\right)A_u \\
&\quad -4\left(\frac{8}{3}g_3^2M_3+\frac{3}{2}g_2^2M_2+\frac{13}{30}g_1^2M_1\right)y_u \\
&\quad +A_u\gamma_Q+\gamma_U A_u+\gamma_{H_2}A_u \\
&\quad +2\left\{3tr(y_u^\dagger A_u)y_u+2A_u y_u^\dagger y_u+y_u y_u^\dagger A_u+y_u y_d^\dagger A_d\right\}. \qquad \text{(A.13)}
\end{aligned}$$

They do not depend on the soft SUSY-breaking scalar masses.

Finally we present the RGEs for scalar masses. In the case of MSSM, the gauge group includes a $U(1)$ subgroup and there is a contribution from the $U(1)$

D-term, which is parameterized by

$$S = tr(m_Q^2) - 2tr(m_U^2) + tr(m_D^2) - tr(m_L^2) + tr(m_E^2) - m_{H_1}^2 + m_{H_2}^2. \quad \text{(A.14)}$$

Then the running of scalar masses become as follows:

$$\begin{aligned}
(4\pi)^2 M \frac{d}{dM} m_E^2 &= -\frac{24}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S \\
&\quad + \gamma_E m_E^2 + m_E^2 \gamma_E \\
&\quad + 2\{2 y_e (m_L^2 + m_{H_1}^2) y_e^\dagger + 2 A_e A_e^\dagger\}, && \text{(A.15)} \\
(4\pi)^2 M \frac{d}{dM} m_L^2 &= -4\left(\frac{3}{2} g_2^2 M_2^2 + \frac{3}{10} g_1^2 M_1^2\right) + \frac{3}{10} g_1^2 S \\
&\quad + \gamma_L m_L^2 + m_L^2 \gamma_L \\
&\quad + 2\{2 y_e^\dagger (m_E^2 + m_{H_1}^2) y_e + 2 A_e^\dagger A_e\}, && \text{(A.16)} \\
(4\pi)^2 M \frac{d}{dM} m_D^2 &= -4\left(\frac{8}{3} g_3^2 M_3^2 + \frac{2}{15} g_1^2 M_1^2\right) - \frac{1}{5} g_1^2 S \\
&\quad + \gamma_D m_D^2 + m_D^2 \gamma_D \\
&\quad + 2\{2 y_d (m_Q^2 + m_{H_1}^2) y_d^\dagger + 2 A_d A_d^\dagger\}, && \text{(A.17)} \\
(4\pi)^2 M \frac{d}{dM} m_U^2 &= -4\left(\frac{8}{3} g_3^2 M_3^2 + \frac{8}{15} g_1^2 M_1^2\right) + \frac{2}{5} g_1^2 S \\
&\quad + \gamma_U m_U^2 + m_U^2 \gamma_U \\
&\quad + 2\{2 y_u (m_Q^2 + m_{H_2}^2) y_u^\dagger + 2 A_u A_u^\dagger\}, && \text{(A.18)} \\
(4\pi)^2 M \frac{d}{dM} m_Q^2 &= -4\left(\frac{8}{3} g_3^2 M_3^2 + \frac{3}{2} g_2^2 M_2^2 + \frac{1}{30} g_1^2 M_1^2\right) - \frac{1}{10} g_1^2 S \\
&\quad + \gamma_Q m_Q^2 + m_Q^2 \gamma_Q \\
&\quad + 2\{y_u^\dagger (m_U^2 + m_{H_2}^2) y_u + y_d^\dagger (m_D^2 + m_{H_2}^2) y_d \\
&\quad + A_u^\dagger A_u + A_d^\dagger A_d\}, && \text{(A.19)} \\
(4\pi)^2 M \frac{d}{dM} m_{H_1}^2 &= -4\left(\frac{3}{2} g_2^2 M_2^2 + \frac{3}{10} g_1^2 M_1^2\right) + \frac{3}{10} g_1^2 S
\end{aligned}$$

$$+2\gamma_{H_1} m_{H_1}^2 + 2\{3tr(y_d m_Q^2 y_d^\dagger) + 3tr(y_d^\dagger m_D^2 y_d)$$

$$+tr(y_e m_L^2 y_e^\dagger) + tr(y_e^\dagger m_E^2 y_e)$$

$$+3tr(A_d A_d^\dagger) + tr(A_e A_e^\dagger)\}, \qquad \text{(A.20)}$$

$$(4\pi)^2 M \frac{d}{dM} m_{H_2}^2 = -4\left(\frac{3}{2} g_2^2 M_2^2 + \frac{3}{10} g_1^2 M_1^2\right) - \frac{3}{10} g_1^2 S$$

$$+2\gamma_{H_2} m_{H_2}^2 + 2\{3tr(y_u m_Q^2 y_u^\dagger) + 3tr(y_u^\dagger m_U^2 y_u)$$

$$+3tr(A_u A_u^\dagger)\}. \qquad \text{(A.21)}$$

III minimal SU(5) SUSY GUT

If we assume the minimal model with an adjoint Higgs, the running of the gauge coupling constant becomes as follows

$$(4\pi)^2 M \frac{d}{dM} g_5 = 2b_5 g_5^3, \qquad \text{(A.22)}$$

$$b_5 = -3. \qquad \text{(A.23)}$$

The Yukawa contributions in the anomalous dimensions which come from the wavefunction renormalization and the RGEs for the Yukawa coupling constants are calculated as follows:

$$\begin{aligned} &\gamma_{\bar{F}} = y_d y_d^\dagger, && \gamma_T = (3y_u^\dagger y_u + 2y_d^\dagger y_d), \\ &\gamma_{\bar{H}} = 4tr(y_d^\dagger y_d)\mathbf{1}, && \gamma_H = 3tr(y_u^\dagger y_u)\mathbf{1}, \end{aligned} \qquad \text{(A.24)}$$

$$(4\pi)^2 M \frac{d}{dM} y_d = -\frac{96}{5} g_5^2 y_d + \gamma_{\bar{H}} y_u + \gamma_{\bar{F}} y_d + y_d \gamma_T, \qquad \text{(A.25)}$$

$$(4\pi)^2 M \frac{d}{dM} y_u = -\frac{84}{5} g_5^2 y_u + \gamma_H y_u + \gamma_T^T y_u + y_u \gamma_T. \qquad \text{(A.26)}$$

The RGEs of the soft SUSY-breaking parameters for the minimal SU(5) SUSY GUT are summarized as follows:

$$(4\pi)^2 M\frac{d}{dM}M_5 \;\; = \;\; b_5 g_5^2 M_5, \tag{A.27}$$

$$\begin{aligned}
(4\pi)^2 M\frac{d}{dM}A_d \;\; &= \;\; -\frac{84}{5}g_5^2 A_d - \frac{168}{5}g_5^2 M_5 y_d \\
&+\gamma_{\bar{F}} A_d + A_d\gamma_T + \gamma_{\bar{H}} A_d \\
&+2\{4tr(y_d^\dagger A_d)y_d + y_d(3y_u^\dagger A_u + 2y_d^\dagger A_d) \\
&+(4A_d y_d^\dagger)y_d\}, \qquad (A.28) \\
(4\pi)^2 M\frac{d}{dM}A_u \;\; &= \;\; -\frac{96}{5}g_5^2 A_u - \frac{192}{5}g_5^2 M_5 y_u \\
&+\gamma_T^T A_u + A_u\gamma_T + \gamma_H A_u \\
&+2\{3tr(y_u^\dagger A_u)y_u + y_u(3y_u^\dagger A_u + 2y_d^\dagger A_d) \\
&+(3A_u y_u^\dagger + 2A_d^T y_d^*)y_u\}, \qquad (A.29)
\end{aligned}$$

$$\tag{A.30}$$

$$\begin{aligned}
(4\pi)^2 M\frac{d}{dM}m_{\bar{F}}^2 \;\; &= \;\; -\frac{96}{5}g_5^2 M_5^2 \\
&+\gamma_{\bar{F}}^* m_{\bar{F}}^2 + m_{\bar{F}}^2\gamma_{\bar{F}}^T \\
&+2\{4y_d^*(m_T^{2T} + m_{\bar{H}}^2)y_d^T + 4A_d^* A_d^T\}, \qquad (A.31) \\
(4\pi)^2 M\frac{d}{dM}m_T^2 \;\; &= \;\; -\frac{144}{5}g_5^2 M_5^2 \\
&+\gamma_T^\dagger m_T^2 + m_T^2\gamma_T \\
&+2\{3y_u^\dagger(m_T^{2T} + m_H^2)y_u + 2y_d^\dagger(m_{\bar{F}}^{2T} + m_{\bar{H}}^2)y_d \\
&+3A_u^\dagger A_u + 4A_d^\dagger A_d\}, \qquad (A.32)
\end{aligned}$$

$$
\begin{aligned}
(4\pi)^2 M \frac{d}{dM} m_{\bar{H}}^2 &= -\frac{96}{5} g_5^2 M_5^2 \\
&\quad +2\gamma_{\bar{H}}^2 m_{\bar{H}}^2 \\
&\quad +2\{4tr(y_d m_T^2 y_d^\dagger + y_d^T m_{\bar{F}}^2 y_d^*) + 4tr(A_d^\dagger A_d)\}, && \text{(A.33)} \\
(4\pi)^2 M \frac{d}{dM} m_H^2 &= -\frac{96}{5} g_5^2 M_5^2 \\
&\quad +2\gamma_H m_H^2 \\
&\quad +2\{6tr(y_u^\dagger m_T^{2T} y_u) + 3tr(A_u^\dagger A_u)\}. && \text{(A.34)} \\
& && \text{(A.35)}
\end{aligned}
$$

IV minimal SO(10) SUSY GUT

We arrange the RGE for the gauge coupling constant of the $SO(10)$ SUSY GUT as follows [7]:

$$
\begin{aligned}
(4\pi)^2 M \frac{d}{dM} g_{10} &= b_{10} g_{10}^3, && \text{(A.36)} \\
b_{10} &= -3. && \text{(A.37)}
\end{aligned}
$$

The Yukawa contributions in the anomalous dimensions which come from the wavefunction renormalization and the RGEs for the Yukawa coupling constants are calculated as follows:

$$
\begin{aligned}
&\gamma_\Psi = 5(y_u^\dagger y_u + y_d^\dagger y_d), \\
&\gamma_{\Phi_u} = 4tr(y_u^\dagger y_u)\mathbf{1}, \qquad \gamma_{\Phi_d} = 4tr(y_d^\dagger y_d)\mathbf{1},
\end{aligned} \tag{A.38}
$$

$$
\begin{aligned}
(4\pi)^2 M \frac{d}{dM} y_u &= -\frac{63}{2} g_{10}^2 y_u + \gamma_\Psi^T y_u + y_u \gamma_\Psi + \gamma_{\Phi_u} y_u, && \text{(A.39)} \\
(4\pi)^2 M \frac{d}{dM} y_d &= -\frac{63}{2} g_{10}^2 y_d + \gamma_\Psi^T y_d + y_d \gamma_\Psi + \gamma_{\Phi_d} y_d. && \text{(A.40)}
\end{aligned}
$$

The RGEs of the soft SUSY-breaking parameters for the minimal SO(10) SUSY GUT are summarized as follows:

$$
(4\pi)^2 M \frac{d}{dM} M_{10} = b_{10} g_{10}^2 M_{10}, \tag{A.41}
$$

$$
\begin{aligned}
(4\pi)^2 M \frac{d}{dM} A_u &= -\frac{63}{2} g_{10}^2 A_u - 63 g_{10}^2 M_{10} y_u \\
&\quad + \gamma_\Psi^T A_u + A_u \gamma_\Psi + \gamma_{\Phi_u} A_u \\
&\quad + 2\{5 y_u^T y_u^* A_u + 5 A_u y_u^\dagger y_u \\
&\quad + 4tr(y_u^\dagger A_u) y_u + 4tr(y_d^\dagger A_u) y_d\}, \qquad \text{(A.42)} \\
(4\pi)^2 M \frac{d}{dM} A_d &= -\frac{63}{2} g_{10}^2 A_d - 63 g_{10}^2 M_{10} y_d \\
&\quad + \gamma_\Psi^T A_d + A_d \gamma_\Psi + \gamma_{\Phi_d} A_d \\
&\quad + 2\{5 y_d^T y_d^* A_d + 5 A_d y_d^\dagger y_d \\
&\quad + 4tr(y_u^\dagger A_d) y_u + 4tr(y_d^\dagger A_d) y_d\}, \qquad \text{(A.43)}
\end{aligned}
$$

$$
\begin{aligned}
(4\pi)^2 M \frac{d}{dM} m_\Psi^2 &= -45 g_{10}^2 M_{10}^2 \\
&\quad + \gamma_\Psi^\dagger m_\Psi^2 + m_\Psi^2 \gamma_\Psi \\
&\quad + 10\{y_u^\dagger m_\Psi^{2T} y_u + (y_u^\dagger y_u) m_{\Phi_u}^2 \\
&\quad + y_d^\dagger m_\Psi^{2T} y_d + (y_d^\dagger y_d) m_{\Phi_d}^2 \\
&\quad + A_u^\dagger A_u + A_d^\dagger A_d\}, \qquad \text{(A.44)} \\
(4\pi)^2 M \frac{d}{dM} m_{\Phi_u}^2 &= -36 g_{10}^2 M_{10}^2 \\
&\quad + 2\gamma_{\Phi_u} m_{\Phi_u}^2 \\
&\quad + 8\{2tr(y_u^\dagger m_\Psi^{2T} y_u) + tr(A_u^\dagger A_u)\}, \qquad \text{(A.45)} \\
(4\pi)^2 M \frac{d}{dM} m_{\Phi_d}^2 &= -36 g_{10}^2 M_{10}^2 \\
&\quad + 2\gamma_{\Phi_d} m_{\Phi_d}^2
\end{aligned}
$$

$$+8\{2tr(y_d^\dagger m_\Psi^{2T} y_d) + tr(A_d^\dagger A_d)\}. \qquad \text{(A.46)}$$

Appendix B

Mass matrices and mass eigen states in the MSSM

After the EW symmetry is broken, couplings to the Higgs fields become new sources of mass parameters which mix the $SU(2) \times U(1)$ eigen states. In this Appendix we fix our notations of the MSSM relevant to the definition of various mass matrices and diagonalization of them for our numerical calculations.

I Fermion mass matrices

The matter fermions acquire masses from the Yukawa couplings to the Higgs fields as follows:

$$(m_e)_{ij} = -(y_e)_i \delta_{ij} \frac{v}{\sqrt{2}} \cos\beta, \tag{B.1}$$

$$(m_d)_{ij} = -(y_d)_i ({V_{CKM}}^\dagger)_{ij} \frac{v}{\sqrt{2}} \cos\beta, \tag{B.2}$$

$$(m_u)_{ij} = (y_u)_i \delta_{ij} \frac{v}{\sqrt{2}} \sin\beta, \tag{B.3}$$

where v and β are defined so that:

$$v \equiv \sqrt{2(\langle h_2^0 \rangle^2 + \langle h_1^0 \rangle^2)}, \tag{B.4}$$

$$\tan\beta \equiv \frac{\langle h_2^0 \rangle}{\langle h_1^0 \rangle}. \tag{B.5}$$

In the above formula, we take a basis of the superfields in which the lepton and up-type Yukawa coupling constants are diagonal. .

II Neutralino and chargino mass matrices

Winos, bino and higgsinos are mixed each other through the gauge couplings to the Higgs fields and form new mass eigen states named neutralino $\tilde{\chi}^0$ and chargino $\tilde{\chi}^-$. The neutralino mass matrix is written as follows:

$$\mathcal{L}_N = -\frac{1}{2}\begin{pmatrix} \overline{\widetilde{B}}_R & \overline{\widetilde{W}_{3R}} & \overline{\widetilde{h}^0_{1R}}^c & \overline{\widetilde{h}^0_{2R}}^c \end{pmatrix} \mathcal{M}_N \begin{pmatrix} \widetilde{B}_L \\ \widetilde{W}_{3L} \\ \widetilde{h}^0_{1L} \\ \widetilde{h}^0_{2L} \end{pmatrix} + h.c.,$$

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_z \sin\theta_W \cos\beta & m_z \sin\theta_W \sin\beta \\ 0 & M_2 & m_z \cos\theta_W \cos\beta & -m_z \cos\theta_W \sin\beta \\ -m_z \sin\theta_W \cos\beta & m_z \cos\theta_W \cos\beta & 0 & -\mu \\ m_z \sin\theta_W \sin\beta & -m_z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix}, \tag{B.6}$$

where m_z is the Z boson mass and θ_W is the Weinberg angle ($\tan\theta_W \equiv \frac{3g_1}{5g_2}$). This symmetric matrix can be diagonalized with an unitary matrix, as follows:

$$O_N \mathcal{M}_N O_N^T = diag(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4}), \tag{B.7}$$

$$O_{NR} \equiv O_N, \tag{B.8}$$

$$O_{NL} \equiv O_N{}^*. \tag{B.9}$$

The chargino mass matrix is written as follows:

$$\mathcal{L}_C = -\begin{pmatrix} \overline{\widetilde{W}^-}_R & \overline{\widetilde{h}^-_2}^c_R \end{pmatrix} \mathcal{M}_C \begin{pmatrix} \widetilde{W}^-_L \\ \widetilde{h}^-_{1L} \end{pmatrix} + h.c.,$$

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}, \tag{B.10}$$

where m_W is W boson mass. This matrix can be diagonalized with unitary matrices, as follows:

$$O_{CR} \mathcal{M}_C O_{CL}^\dagger = diag(m_{\tilde{\chi}^-_1}, m_{\tilde{\chi}^-_2}). \tag{B.11}$$

III Sfermion mass matrices

Sfermions acquire masses from the Yukawa couplings and gauge couplings and they are mixed each other. The charged slepton and sneutrino mass matrices are written as follows:

$$\mathcal{L}_{\tilde{l}} = -\begin{pmatrix} \tilde{l}^{-\dagger} & \tilde{e}^{\dagger} \end{pmatrix} m_{\tilde{e}}^2 \begin{pmatrix} \tilde{l}^{-} \\ \tilde{e} \end{pmatrix} - \tilde{l}^{0\dagger} m_{\tilde{\nu}}^2 \tilde{l}^0,$$

$$m_{\tilde{e}}^2 = \begin{pmatrix} m_L^2 + m_e^{\dagger} m_e + m_z^2 \cos 2\beta(-\frac{1}{2} + \sin^2\theta_W) & \frac{v}{\sqrt{2}} \cos\beta(A_e + y_e \mu^* \tan\beta)^{\dagger} \\ \frac{v}{\sqrt{2}} \cos\beta(A_e + y_e \mu^* \tan\beta) & m_E^2 + m_e m_e^{\dagger} - m_z^2 \cos 2\beta \sin^2\theta_W \end{pmatrix},$$

$$m_{\tilde{\nu}}^2 = m_L^2 - \frac{1}{2} m_z^2 \cos 2\beta. \tag{B.12}$$

They are diagonalized with unitary matrices as follows:

$$U_e m_{\tilde{e}}^2 U_e^{\dagger} = diag(m_{\tilde{e}_1}^2, m_{\tilde{e}_2}^2, m_{\tilde{e}_3}^2, m_{\tilde{e}_4}^2, m_{\tilde{e}_5}^2, m_{\tilde{e}_6}^2), \tag{B.13}$$

$$U_\nu m_{\tilde{\nu}}^2 U_\nu^{\dagger} = diag(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2). \tag{B.14}$$

The squark mass matrices are written as follows:

$$\mathcal{L}_{\tilde{q}} = -\begin{pmatrix} \tilde{q}^{-\frac{1}{3}\dagger} & \tilde{d}^{\dagger} \end{pmatrix} m_{\tilde{d}}^2 \begin{pmatrix} \tilde{q}^{-\frac{1}{3}} \\ \tilde{d} \end{pmatrix} - \begin{pmatrix} \tilde{q}^{\frac{2}{3}\dagger} & \tilde{u}^{\dagger} \end{pmatrix} m_{\tilde{u}}^2 \begin{pmatrix} \tilde{q}^{\frac{2}{3}} \\ \tilde{u} \end{pmatrix},$$

$$m_{\tilde{d}}^2 = \begin{pmatrix} m_Q^2 + m_d^{\dagger} m_d + m_z^2 \cos 2\beta(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W) & \frac{v}{\sqrt{2}} \cos\beta(A_d + y_d \mu^* \tan\beta)^{\dagger} \\ \frac{v}{\sqrt{2}} \cos\beta(A_d + y_d \mu^* \tan\beta) & m_D^2 + m_d m_d^{\dagger} - \frac{1}{3} m_z^2 \cos 2\beta \sin^2\theta_W \end{pmatrix},$$

$$m_{\tilde{u}}^2 = \begin{pmatrix} m_Q^2 + m_u^{\dagger} m_u + m_z^2 \cos 2\beta(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W) & -\frac{v}{\sqrt{2}} \sin\beta(A_u + y_u \mu^* \cot\beta)^{\dagger} \\ -\frac{v}{\sqrt{2}} \sin\beta(A_u + y_u \mu^* \cot\beta) & m_U^2 + m_u m_u^{\dagger} + \frac{2}{3} m_z^2 \cos 2\beta \sin^2\theta_W \end{pmatrix}. \tag{B.15}$$

They are diagonalized with unitary matrices, as follows:

$$U_d m_{\tilde{d}}^2 U_d^{\dagger} = diag(m_{\tilde{d}_1}^2, m_{\tilde{d}_2}^2, m_{\tilde{d}_3}^2, m_{\tilde{d}_4}^2, m_{\tilde{d}_5}^2, m_{\tilde{d}_6}^2), \tag{B.16}$$

$$U_u m_{\tilde{u}}^2 U_u^{\dagger} = diag(m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_3}^2, m_{\tilde{u}_4}^2, m_{\tilde{u}_5}^2, m_{\tilde{u}_6}^2). \tag{B.17}$$

IV Neutralino and chargino vertices

The neutralino and chargino vertices for leptons and sleptons are written as follows:

$$\begin{aligned}\mathcal{L} &\equiv \sum_{i=1}^{3}\sum_{A=1}^{4}\sum_{X=1}^{6} \overline{e_i}(N^L_{iAX}P_L + N^R_{iAX}P_R)\tilde{\chi}^0_A\tilde{e}_X \\ &+\sum_{i=1}^{3}\sum_{A=1}^{2}\sum_{X=1}^{3} \overline{e_i}(C^L_{iAX}P_L + C^R_{iAX}P_R)\tilde{\chi}^-_A\tilde{\nu}_X + h.c., \end{aligned} \tag{B.18}$$

$$\begin{aligned} N^L_{iAX} &= -g_2\{\sqrt{2}\tan\theta_W(O_{NL})^*_{A1}(U_e)^*_{Xi+3} \\ &+\frac{(m_e)_i}{\sqrt{2}m_W\cos\beta}(O_{NL})^*_{A3}(U_e)^*_{Xi}\}, \end{aligned} \tag{B.19}$$

$$\begin{aligned} N^R_{iAX} &= -g_2[-\frac{\sqrt{2}}{2}\{(O_{NR})^*_{A2}+\tan\theta_W(O_{NR})^*_{A1}\}(U_e)^*_{Xi} \\ &+\frac{(m_e)_i}{\sqrt{2}m_W\cos\beta}(O_{NR})^*_{A3}(U_e)^*_{Xi+3}], \end{aligned} \tag{B.20}$$

$$C^L_{iAX} = g_2\frac{(m_e)_i}{\sqrt{2}m_W\cos\beta}(O_{CL})^*_{A2}(U_\nu)^*_{Xj}, \tag{B.21}$$

$$C^R_{iAX} = -g_2(O_{CR})^*_{A1}(U_\nu)^*_{Xi}. \tag{B.22}$$

The neutralino, chargino and gluino vertices for quarks and squarks are written as follows:

$$\begin{aligned}\mathcal{L} &\equiv \sum_{i=1}^{3}\sum_{A=1}^{4}\sum_{X=1}^{6}\{\overline{d_i}(N^{dL}_{iAX}P_L + N^{dR}_{iAX}P_R)\tilde{\chi}^0_A\tilde{d}_X \\ &\qquad + \overline{u_i}(N^{uL}_{iAX}P_L + N^{uR}_{iAX}P_R)\tilde{\chi}^0_A\tilde{u}_X\} \\ &+\sum_{i=1}^{3}\sum_{A=1}^{2}\sum_{X=1}^{6}\{\overline{d_i}(C^{dL}_{iAX}P_L + C^{dR}_{iAX}P_R)\tilde{\chi}^-_A\tilde{u}_X \\ &\qquad + \overline{u_i}(C^{uL}_{iAX}P_L + C^{uR}_{iAX}P_R)\tilde{\chi}^-_A\tilde{d}_X\} \\ &+\sum_{i=1}^{3}\sum_{X=1}^{6}\{\overline{d_i}(\Gamma^{dL}_{iX}P_L + \Gamma^{dR}_{iX}P_R)\tilde{G}^aT^a\tilde{d}_X \end{aligned}$$

$$+\,\overline{u_i}(\Gamma^{uL}_{iX}P_L + \Gamma^{uR}_{iX}P_R)\tilde{G}^a T^a \tilde{u}_X\} + h.c., \tag{B.23}$$

$$\begin{aligned} N^{dL}_{iAX} &= -g_2\{\sqrt{2}\left(\frac{1}{3}\right)\tan\theta_W (O_{NL})^*_{A1}(U_d)^*_{Xi+3} \\ &+ \sum_{j=1}^{3}\frac{(m_d)_{ij}}{\sqrt{2}m_W\cos\beta}(O_{NL})^*_{A3}(U_d)^*_{Xj}\}, \end{aligned} \tag{B.24}$$

$$\begin{aligned} N^{dR}_{iAX} &= -g_2[\sqrt{2}\{\left(-\frac{1}{2}\right)(O_{NR})^*_{A2} + \left(\frac{1}{6}\right)\tan\theta_W(O_{NR})^*_{A1}\}(U_d)^*_{Xi} \\ &+ \sum_{j=1}^{3}\frac{(m_d^\dagger)_{ij}}{\sqrt{2}m_W\cos\beta}(O_{NR})^*_{A3}(U_d)^*_{Xj+3}], \end{aligned} \tag{B.25}$$

$$\begin{aligned} N^{uL}_{iAX} &= -g_2\{\sqrt{2}\left(-\frac{2}{3}\right)\tan\theta_W(O_{NL})^*_{A1}(U_u)^*_{Xi+3} \\ &+ \frac{(m_u)_i}{\sqrt{2}m_W\sin\beta}(O_{NL})^*_{A3}(U_u)^*_{Xi}\}, \end{aligned} \tag{B.26}$$

$$\begin{aligned} N^{uR}_{iAX} &= -g_2[\sqrt{2}\{\left(\frac{1}{2}\right)(O_{NR})^*_{A2} + \left(\frac{1}{6}\right)\tan\theta_W(O_{NR})^*_{A1}\}(U_u)^*_{Xi} \\ &+ \frac{(m_u)_i}{\sqrt{2}m_W\sin\beta}(O_{NR})^*_{A3}(U_u)^*_{Xi+3}], \end{aligned} \tag{B.27}$$

$$C^{dL}_{iAX} = g_2\sum_{j=1}^{3}\frac{(m_d)_{ij}}{\sqrt{2}m_W\cos\beta}(O_{CL})^*_{A2}(U_u)^*_{Xj}, \tag{B.28}$$

$$C^{dR}_{iAX} = -g_2\{(O_{CR})^*_{A1}(U_u)^*_{Xi} - \frac{(m_u)_i}{\sqrt{2}m_W\sin\beta}(O_{CR})^*_{A2}(U_u)^*_{Xi+3}\}, \tag{B.29}$$

$$C^{uL}_{iAX} = g_2\frac{(m_u)_i}{\sqrt{2}m_W\cos\beta}(O_{CR})_{A2}(U_d)^*_{Xi}, \tag{B.30}$$

$$C^{uR}_{iAX} = -g_2\{(O_{CL})_{A1}(U_d)^*_{Xi} - \sum_{j=1}^{3}\frac{(m_d^\dagger)_{ij}}{\sqrt{2}m_W\cos\beta}(O_{CL})_{A2}(U_d)^*_{Xj+3}\}, \tag{B.31}$$

$$\Gamma^{dL}_{iX} = -\sqrt{2}g_3(U_d^*)_{Xi+3}, \tag{B.32}$$

$$\Gamma^{dR}_{iX} = \sqrt{2} g_3 (U^*_d)_{Xi}, \tag{B.33}$$

$$\Gamma^{uL}_{iX} = -\sqrt{2} g_3 (U^*_u)_{Xi+3}, \tag{B.34}$$

$$\Gamma^{uR}_{iX} = \sqrt{2} g_3 (U^*_u)_{Xi}. \tag{B.35}$$

Appendix C

LFV effective coupling constants in the MSSM

I LFV effective coupling constants

The formulas of the effective coupling constants for the $\mu \to e\gamma$ and $\mu \to 3e$ processes written in the MSSM variables are given in Ref.[51]. We present these formulas for completeness while taking care of the CP violating phases.

Each coupling constant is divided into a neutralino-charged-slepton-loop contribution and chargino-sneutrino-loop contribution. The four-fermi coupling constants are given as follows:

$$\hat{g}_i = \hat{g}_i^n + \hat{g}_i^c \; (i = 1-6). \tag{C.1}$$

The coupling constant $\hat{g}_1$ comes only from box diagrams.

$$\begin{aligned}
\hat{g}_1^n &= -\frac{\sqrt{2}}{64\pi^2 G_F} \sum_{A,B=1}^{4} \sum_{X,Y=1}^{6} (N_{2AX}^L N_{1AY}^{R*} N_{1BY}^L N_{1BX}^{R*} - 2N_{2AX}^L N_{1AY}^L N_{1BY}^{R*} N_{1BX}^{R*}) \\
&\quad m_{\tilde{\chi}_A^0} m_{\tilde{\chi}_B^0} d_0({m_{\tilde{\chi}_A^0}}^2, {m_{\tilde{\chi}_B^0}}^2, {m_{\tilde{e}_X}}^2, {m_{\tilde{e}_Y}}^2),
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
\hat{g}_1^c &= -\frac{\sqrt{2}}{64\pi^2 G_F} \sum_{A,B=1}^{2} \sum_{X,Y=1}^{3} C_{2AX}^L C_{1AY}^{R*} C_{1BY}^L C_{1BX}^{R*} \\
&\quad m_{\tilde{\chi}_A^-} m_{\tilde{\chi}_B^-} d_0({m_{\tilde{\chi}_A^-}}^2, {m_{\tilde{\chi}_B^-}}^2, {m_{\tilde{\nu}_X}}^2, {m_{\tilde{\nu}_Y}}^2).
\end{aligned} \tag{C.3}$$

The coupling constant $\hat{g}_3$ is divided into three parts. $\hat{g}_{31}$ is a contribution of box diagrams and $\hat{g}_{32}$ is that of Z-penguin diagrams. $\hat{g}_{33}$ is a contribution of off-shell photon-penguin diagrams.

$$\hat{g}_3 = \hat{g}_{31} + \hat{g}_{32} + \hat{g}_{33}, \tag{C.4}$$

$$\begin{aligned}
\hat{g}_{31}^n &= -\frac{\sqrt{2}}{64\pi^2 G_F} \sum_{A,B=1}^{4} \sum_{X,Y=1}^{6} \{N_{2AX}^L N_{1AY}^{L*} N_{1BY}^L N_{1BX}^{L*} d_2({m_{\tilde{\chi}_A^0}}^2, {m_{\tilde{\chi}_B^0}}^2, {m_{\tilde{e}_X}}^2, {m_{\tilde{e}_Y}}^2) \\
&+\frac{1}{2} N_{2AX}^L N_{1AY}^L N_{1BY}^{L*} N_{1BX}^{L*} m_{\tilde{\chi}_A^0} m_{\tilde{\chi}_B^0} d_0({m_{\tilde{\chi}_A^0}}^2, {m_{\tilde{\chi}_B^0}}^2, {m_{\tilde{e}_X}}^2, {m_{\tilde{e}_Y}}^2)\},
\end{aligned} \tag{C.5}$$

$$\begin{aligned}
\hat{g}_{32}^n &= -\frac{1}{16\pi^2} Z_R^e [\sum_{A,B=1}^{4} \sum_{X=1}^{6} N_{2AX}^L N_{1BX}^{L*} \{4(Y_{\tilde{\chi}_L^0})_{AB} c_2({m_{\tilde{e}_X}}^2, {m_{\tilde{\chi}_A^0}}^2, {m_{\tilde{\chi}_B^0}}^2) \\
&-2 m_{\tilde{\chi}_A^0} m_{\tilde{\chi}_B^0} (Y_{\tilde{\chi}_R^0})_{AB} c_0({m_{\tilde{e}_X}}^2, {m_{\tilde{\chi}_A^0}}^2, {m_{\tilde{\chi}_B^0}}^2)\} \\
&+2 \sum_{A=1}^{4} \sum_{X,Y=1}^{6} N_{2AX}^L N_{1AY}^{L*} (X_{\tilde{e}_L})_{XY} c_2({m_{\tilde{\chi}_A^0}}^2, {m_{\tilde{e}_X}}^2, {m_{\tilde{e}_Y}}^2)],
\end{aligned} \tag{C.6}$$

$$\hat{g}_{33}^n = -\frac{\sqrt{2} e^2}{1152\pi^2 G_F} \sum_{A=1}^{4} \sum_{X=1}^{6} N_{2AX}^L N_{1AX}^{L*} \frac{1}{m_{\tilde{e}_X}^2} b_0^n (\frac{m_{\tilde{\chi}_A^0}^2}{m_{\tilde{e}_X}^2}), \tag{C.7}$$

$$\begin{aligned}
\hat{g}_{31}^c &= -\frac{\sqrt{2}}{64\pi^2 G_F} \sum_{A,B=1}^{2} \sum_{X,Y=1}^{3} C_{2AX}^L C_{1AY}^{L*} C_{1BY}^L C_{1BX}^{L*} \\
&\quad d_2({m_{\tilde{\chi}_A^-}}^2, {m_{\tilde{\chi}_B^-}}^2, {m_{\tilde{\nu}_X}}^2, {m_{\tilde{\nu}_Y}}^2),
\end{aligned} \tag{C.8}$$

$$\begin{aligned}
\hat{g}_{32}^c &= -\frac{1}{16\pi^2} Z_R^c \sum_{A,B=1}^{2} \sum_{X=1}^{3} C_{2AX}^L C_{1BX}^{L*} \{4(Y_{\tilde{\chi}_L^-})_{AB} c_2({m_{\tilde{\nu}_X}}^2, {m_{\tilde{\chi}_A^-}}^2, {m_{\tilde{\chi}_B^-}}^2) \\
&-2 m_{\tilde{\chi}_A^-} m_{\tilde{\chi}_B^-} (Y_{\tilde{\chi}_R^-})_{AB} c_0({m_{\tilde{\nu}_X}}^2, {m_{\tilde{\chi}_A^-}}^2, {m_{\tilde{\chi}_B^-}}^2)\},
\end{aligned} \tag{C.9}$$

$$\hat{g}_{33}^c = -\frac{\sqrt{2} e^2}{1152\pi^2 G_F} \sum_{A=1}^{2} \sum_{X=1}^{3} C_{2AX}^L C_{1AX}^{L*} \frac{1}{m_{\tilde{\nu}_X}^2} b_0^c (\frac{m_{\tilde{\chi}_A^-}^2}{m_{\tilde{\nu}_X}^2}). \tag{C.10}$$

The coupling constant $\hat{g}_5$ is divided into three parts. $\hat{g}_{51}$ is a contribution of box diagrams and $\hat{g}_{52}$ is that of Z-penguin diagrams. $\hat{g}_{53}$ is a contribution of off-shell photon-penguin diagrams.

$$\hat{g}_5 = \hat{g}_{51} + \hat{g}_{52} + \hat{g}_{53}, \tag{C.11}$$

$$\begin{aligned}
\hat{g}^n_{51} &= -\frac{\sqrt{2}}{64\pi^2 G_F} \sum_{A,B=1}^{4} \sum_{X,Y=1}^{6} \{(N^L_{2AX} N^{L*}_{1AY} N^R_{1BY} N^{R*}_{1BX} - N^L_{2AX} N^R_{1AY} N^{R*}_{1BY} N^{L*}_{1BX} \\
&\quad + N^L_{2AX} N^R_{1AY} N^{L*}_{1BY} N^{R*}_{1BX}) d_2(m_{\tilde{\chi}^0_A}{}^2, m_{\tilde{\chi}^0_B}{}^2, m_{\tilde{e}_X}{}^2, m_{\tilde{e}_Y}{}^2) \\
&\quad - \frac{1}{2} m_{\tilde{\chi}^0_A} m_{\tilde{\chi}^0_B} N^L_{2AX} N^{R*}_{1AY} N^R_{1BY} N^{L*}_{1BX} d_0(m_{\tilde{\chi}^0_A}{}^2, m_{\tilde{\chi}^0_B}{}^2, m_{\tilde{e}_X}{}^2, m_{\tilde{e}_Y}{}^2)\}, && \text{(C.12)} \\
\hat{g}^n_{52} &= -\frac{1}{16\pi^2} Z^e_L [\sum_{A,B=1}^{4} \sum_{X=1}^{6} N^L_{2AX} N^{L*}_{1BX} \{4(Y_{\tilde{\chi}^0_L})_{AB} c_2(m_{\tilde{e}_X}{}^2, m_{\tilde{\chi}^0_A}{}^2, m_{\tilde{\chi}^0_B}{}^2) \\
&\quad - 2 m_{\tilde{\chi}^0_A} m_{\tilde{\chi}^0_B} (Y_{\tilde{\chi}^0_R})_{AB} c_0(m_{\tilde{e}_X}{}^2, m_{\tilde{\chi}^0_A}{}^2, m_{\tilde{\chi}^0_B}{}^2)\} \\
&\quad + 2 \sum_{A=1}^{4} \sum_{X,Y=1}^{6} N^L_{2AX} N^{L*}_{1AY} (X_{\tilde{e}_L})_{XY} c_2(m_{\tilde{\chi}^0_A}{}^2, m_{\tilde{e}_X}{}^2, m_{\tilde{e}_Y}{}^2)], && \text{(C.13)} \\
\hat{g}^n_{53} &= \hat{g}^n_{33}, && \text{(C.14)} \\
\hat{g}^c_{51} &= -\frac{\sqrt{2}}{64\pi^2 G_F} \sum_{A,B=1}^{2} \sum_{X,Y=1}^{3} \{C^L_{2AX} C^{L*}_{1AY} C^R_{1BY} C^{R*}_{1BX} d_2(m_{\tilde{\chi}^-_A}{}^2, m_{\tilde{\chi}^-_B}{}^2, m_{\tilde{\nu}_X}{}^2, m_{\tilde{\nu}_Y}{}^2) \\
&\quad - \frac{1}{2} C^L_{2AX} C^{R*}_{1AY} C^R_{1BY} C^{L*}_{1BX} m_{\tilde{\chi}^-_A} m_{\tilde{\chi}^-_B} d_0(m_{\tilde{\chi}^-_A}{}^2, m_{\tilde{\chi}^-_B}{}^2, m_{\tilde{\nu}_X}{}^2, m_{\tilde{\nu}_Y}{}^2)\}, && \text{(C.15)} \\
\hat{g}^c_{52} &= -\frac{1}{16\pi^2} Z^e_L \sum_{A,B=1}^{2} \sum_{X=1}^{3} C^L_{2AX} C^{L*}_{1BX} \{4(Y_{\tilde{\chi}^-_L})_{AB} c_2(m_{\tilde{\nu}_X}{}^2, m_{\tilde{\chi}^-_A}{}^2, m_{\tilde{\chi}^-_B}{}^2) \\
&\quad - 2 m_{\tilde{\chi}^-_A} m_{\tilde{\chi}^-_B} (Y_{\tilde{\chi}^-_R})_{AB} c_0(m_{\tilde{\nu}_X}{}^2, m_{\tilde{\chi}^-_A}{}^2, m_{\tilde{\chi}^-_B}{}^2)\}, && \text{(C.16)} \\
\hat{g}^c_{53} &= \hat{g}^c_{33}. && \text{(C.17)}
\end{aligned}$$

Various mixing matrices and Z coupling constants which appear in the above formulas are given as follows:

$$(Y_{\tilde{\chi}^0_L})_{AB} = -\frac{1}{2} \{(O_{NL})_{A3} (O_{NL})^*_{B3} - (O_{NL})_{A4} (O_{NL})^*_{B4}\}, \tag{C.18}$$

$$(Y_{\tilde{\chi}^0_R})_{AB} = \frac{1}{2}\{(O_{NR})_{A3}(O_{NR})^*_{B3} - (O_{NR})_{A4}(O_{NR})^*_{B4}\}, \tag{C.19}$$

$$(Y_{\tilde{\chi}^-_L})_{AB} = -\frac{1}{2}(O_{CL})_{A2}(O_{CL})^*_{B2}, \tag{C.20}$$

$$(Y_{\tilde{\chi}^-_R})_{AB} = -\frac{1}{2}(O_{CR})_{A2}(O_{CR})^*_{B2}, \tag{C.21}$$

$$(X_{\tilde{e}_L})_{XY} = -\sum_{k=1}^{3}(U_e)_{Xk}(U_e)^*_{Yk}, \tag{C.22}$$

$$(X_{\tilde{e}_R})_{XY} = \sum_{k=1}^{3}(U_e)_{Xk+3}(U_e)^*_{Yk+3}. \tag{C.23}$$

$$Z^e_L = (-\frac{1}{2} + \sin^2\theta_W), \tag{C.24}$$

$$Z^e_R = \sin^2\theta_W. \tag{C.25}$$

The photon-penguin coupling constant is written as follows:

$$A_R = A^n_R + A^c_R, \tag{C.26}$$

$$A^n_R = \frac{\sqrt{2}e}{256\pi^2 G_F}\sum_{A=1}^{4}\sum_{X=1}^{6}\frac{1}{m^2_{\tilde{e}_X}}\{\frac{1}{6}N^R_{2AX}N^{R*}_{1AX}b^n_1(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{e}_X}}) + N^L_{2AX}N^{R*}_{1AX}\frac{m_{\tilde{\chi}^0_A}}{m_\mu}b^n_2(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{e}_X}})\}, \tag{C.27}$$

$$A^c_R = -\frac{\sqrt{2}e}{128\pi^2 G_F}\sum_{A=1}^{2}\sum_{X=1}^{3}\frac{1}{m^2_{\tilde{\nu}_X}}\{\frac{1}{6}C^R_{2AX}C^{R*}_{1AX}b^c_1(\frac{m^2_{\tilde{\chi}^-_A}}{m^2_{\tilde{\nu}_X}}) + C^L_{2AX}C^{R*}_{1AX}\frac{m_{\tilde{\chi}^-_A}}{m_\mu}b^c_2(\frac{m^2_{\tilde{\chi}^-_A}}{m^2_{\tilde{\nu}_X}})\}. \tag{C.28}$$

The other coupling constants can be obtained by simply exchanging the suffix of the above formulas:

$$\hat{g}_2 = \hat{g}_1(L \leftrightarrow R), \tag{C.29}$$

$$\hat{g}_4 = \hat{g}_3(L \leftrightarrow R), \tag{C.30}$$

$$\hat{g}_6 = \hat{g}_5(L \leftrightarrow R), \tag{C.31}$$

$$A_L = A_R(L \leftrightarrow R). \tag{C.32}$$

II Mass Functions

The mass functions used in the effective coupling constants of the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes are defined as follows:

$$b_0^n(x) = \frac{1}{2(1-x)^4}(2 - 9x + 18x^2 - 11x^3 + 6x^3 \ln(x)), \tag{C.33}$$

$$b_1^n(x) = \frac{1}{(1-x)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln(x)), \tag{C.34}$$

$$b_2^n(x) = \frac{1}{(1-x)^3}(1 - x^2 + 2x \ln(x)), \tag{C.35}$$

$$b_0^c(x) = \frac{1}{2(1-x)^4}(-16 + 45x - 36x^2 + 7x^3 + 6(3x-2) \ln(x)), \tag{C.36}$$

$$b_1^c(x) = \frac{1}{2(1-x)^4}(2 + 3x - 6x^2 + x^3 + 6x \ln(x)), \tag{C.37}$$

$$b_2^c(x) = \frac{1}{2(1-x)^3}(-3 + 4x - x^2 - 2 \ln(x)), \tag{C.38}$$

$$\begin{aligned} c_0(x,y,z) &= -\frac{x\ln(x)}{(y-x)(z-x)} - \frac{y\ln(y)}{(x-y)(z-y)} - \frac{z\ln(z)}{(x-z)(y-z)}, \\ c_2(x,y,z) &= \frac{1}{4}[\frac{3}{2} - \frac{x^2\ln(x)}{(y-x)(z-x)} - \frac{y^2\ln(y)}{(x-y)(z-y)} - \frac{z^2\ln(z)}{(x-z)(y-z)}], \end{aligned} \tag{C.39}$$

(C.40)

$$d_0(x,y,z,w) = \frac{x\ln(x)}{(y-x)(z-x)(w-x)} + \frac{y\ln(y)}{(x-y)(z-y)(w-y)} + \frac{z\ln(z)}{(x-z)(y-z)(w-z)} + \frac{w\ln(w)}{(x-w)(y-w)(z-w)}, \quad (C.41)$$

$$d_2(x,y,z,w) = \frac{1}{4}\{\frac{x^2\ln(x)}{(y-x)(z-x)(w-x)} + \frac{y^2\ln(y)}{(x-y)(z-y)(w-y)} + \frac{z^2\ln(z)}{(x-z)(y-z)(w-z)} + \frac{w^2\ln(w)}{(x-w)(y-w)(z-w)}\}. \quad (C.42)$$

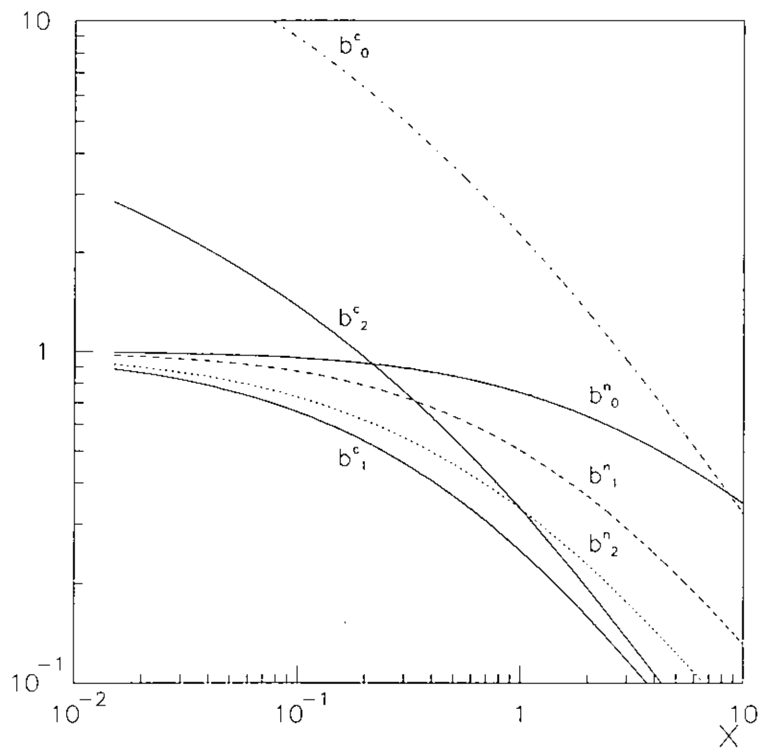

FIG. C.1: Mass functions which appear in the calculation of the photon-penguin amplitudes.

Appendix D

Approximate expressions of the photon-penguin amplitudes for the SO(10) model

In this Appendix we discuss the $\mu^+ \to e^+\gamma$ amplitude for SO(10) GUT using approximate formulas. Although we used full formulas for a numerical analysis, more transparent expressions are obtained in a special parameter region.

The expressions for A_R and A_L are simplified if we use the following approximations:

1. Keep only dominant contributions. These are parts of terms proportional to m_τ in the neutralino and charged-slepton loop diagrams for both A_R and A_L. For A_R, a part of the chargino-sneutrino loop contribution can also give a large contribution.

2. Use the fact that, except for the left-right slepton mixings, the slepton mass matrix is almost diagonalized in the basis where y_u is diagonal and the diagonal elements for the first-two-generations are almost the same. The third generation sleptons become lighter because of the effect of the GUT interaction. We treat diagonal elements for the first two generation in the slepton mass matrix exactly degenerate in the approximate formulas and the difference between the third and the first components are denoted as $\Delta m^2_{\tilde{e}_R}$, $\Delta m^2_{\tilde{e}_L}$ and $\Delta m^2_{\tilde{\nu}_L}$, respectively. Neglecting the renormalization effects between the GUT and electroweak scales by small lepton Yukawa coupling constants, these

difference are given by Eq.(4.6). We take into account the left-right mixing of the slepton mass matrix as a perturbation.

3. Take the limit $m_{\tilde{e}_L} \simeq m_{\tilde{e}_R} \simeq m_{\tilde{\nu}_L} = \overline{m} >> M_{\tilde{\chi}_A^-}, M_{\tilde{\chi}_A^0}$; namely, the average slepton mass is much larger than the chargino and neutralino masses.

Within these approximations A_R and A_L are given by

$$A_R \simeq -\frac{e\tan^2\theta_W}{32\pi^2}[e^{-i(\phi_2-\phi_3)}a^n + a^c], \tag{D.1}$$

$$A_L \simeq -\frac{e\tan^2\theta_W}{32\pi^2}[e^{-i(\phi_3-\phi_1)}a^{n*}], \tag{D.2}$$

where

$$\begin{aligned} a^n &= (V_{CKM}^0)_{32}(V_{CKM}^0)^{*2}_{33}(V_{CKM}^0)_{31}(\frac{m_\tau}{m_\mu}) \\ &\quad \frac{(\frac{A_{e3}}{y_\tau} + \mu^*\tan\beta)}{\overline{m}}(\frac{m_W}{\overline{m}})^2(\frac{M_1}{\overline{m}})(\frac{\Delta m^2_{\tilde{e}_L}}{\overline{m}^2})(\frac{\Delta m^2_{\tilde{e}_R}}{\overline{m}^2}). \end{aligned} \tag{D.3}$$

$$\begin{aligned} a^c &= (V_{CKM}^0)^*_{32}(V_{CKM}^0)_{31}\frac{\sqrt{2}\cot^2\theta_W}{\cos\beta} \\ &\quad \sum_{A=1}^{2}(O_{CL})^*_{A2}(O_{CL})_{A1}\frac{m_W}{\overline{m}}\ln(\frac{\overline{m}^2}{M^2_{\tilde{\chi}_A^-}})(\frac{M_{\tilde{\chi}_A^-}}{\overline{m}})(\frac{\Delta m^2_{\tilde{\nu}_L}}{\overline{m}^2}). \end{aligned} \tag{D.4}$$

For the neutralino contributions, the difference between the above expression and the exact calculation is within 10% above $m_{\tilde{e}_R} > 500$ GeV for the parameter set of Fig.(6.5). For the chargino contributions the approximation is slightly worse. At $m_{\tilde{e}_R} > 500$ GeV the difference is within a factor of two and becomes about at the 10% level for $m_{\tilde{e}_R} = 1000$ GeV. From the above expression we can see that despite a lack of the factor $\frac{m_\tau}{m_\mu}$ the chargino contribution can become comparable to, or even dominant over, the neutralino contribution when $\overline{m} >> m_W$, because of the enhancement factors $\frac{\sqrt{2}\cot^2\theta_W}{\cos\beta}$ and $(\frac{\overline{m}}{m_W})\ln(\frac{\overline{m}^2}{M^2_{\tilde{\chi}_A^-}})$.

Appendix E

Branching ratios and asymmetries

In this appendix, we give kinematic functions which appear in the calculation of branching ratio and asymmetries of $\mu \to 3e$ process with polarized muons:

$$F_i(x) \equiv 2\int_{1-x}^{x} dx_2 \alpha_i(x, x_2), \tag{E.1}$$

$$G_i(x) \equiv 2\int_{1-x}^{x} dx_2 \beta_i(x, x_2), \tag{E.2}$$

$$H_i(x) \equiv -2\int_{1-x}^{x} dx_2 \gamma_i(x, x_2), \tag{E.3}$$

$$F_1(x) = -\frac{8}{3}(4x-5)(2x-1)^2, \tag{E.4}$$

$$F_2(x) = -\frac{2}{3}(2x-1)(8x^2-8x-1), \tag{E.5}$$

$$F_3(x) = 16\ln(\frac{x}{1-x})(2x^2-2x+1) + \frac{32}{3}\frac{(2x-1)(x^2-x+1)}{1-x}, \tag{E.6}$$

$$F_4(x) = 32(2x-1)^2, \tag{E.7}$$

$$F_5(x) = -8(2x-1)(2x-3), \tag{E.8}$$

$$G_1(x) = -16\,(1-x)^2 \ln 2(1-x) - \frac{2}{3}\,(2x-1)(8x^2-32x+23), \tag{E.9}$$

$$G_2(x) = -16(2x^2-2x-7)\ln 2(1-x) + 16(2x^2-2x+1)\ln 2x$$

$$
\begin{aligned}
& +\frac{32}{3}\frac{(2x-1)(x^2-13x+13)}{1-x}, && \text{(E.10)} \\
H_1(x) &= 2(6-5x)(1-x)\sqrt{2x-1} \\
&\quad -(7x^2-24x+16)\sqrt{1-x}\arccos(\frac{2-3x}{x}) \\
&\quad +16(1-x)^2\arccos(\frac{1-x}{x})\}, && \text{(E.11)} \\
H_2(x) &= -16(6-x)\sqrt{2x-1} \\
&\quad -8\frac{5x^2+8x-16}{\sqrt{1-x}}\arccos(\frac{2-3x}{x}) - 128\arccos(\frac{1-x}{x}), && \text{(E.12)} \\
H_3(x) &= -\frac{4}{3}\sqrt{2x-1}(17x^2-24x+4) + 2\frac{(7x-6)x^2}{\sqrt{1-x}}\arccos(\frac{2-3x}{x}), && \text{(E.13)} \\
H_4(x) &= +\frac{2}{3}\sqrt{2x-1}(17x^2-30x+16) \\
&\quad -\frac{(7x^2-16x+8)x}{\sqrt{1-x}}\arccos(\frac{2-3x}{x}). && \text{(E.14)}
\end{aligned}
$$

$$I_i[\delta] = \int_{\frac{1}{2}}^{1-\delta} dx F_i(x), \tag{E.15}$$

$$J_i[\delta] = \int_{\frac{1}{2}}^{1-\delta} dx G_i(x) dx, \tag{E.16}$$

$$K_i[\delta] = \int_{\frac{1}{2}}^{1-\delta} dx H_i(x) dx, \tag{E.17}$$

$$I_1[\delta] = \frac{2}{3}(1+2\delta)(1-2\delta)^3, \tag{E.18}$$

$$I_2[\delta] = \frac{1}{3}(1+2\delta-2\delta^2)(1-2\delta)^2, \tag{E.19}$$

$$I_3[\delta] = \frac{16}{3}(1-\delta)(2-\delta+2\delta^2)\ln(\frac{1-\delta}{\delta}) - \frac{8}{9}(1-2\delta)(13-4\delta+4\delta^2), \tag{E.20}$$

$$I_4[\delta] = \frac{16}{3}(1-2\delta)^3, \tag{E.21}$$

$$I_5[\delta] = \frac{8}{3}(1+\delta)(1-2\delta)^2, \tag{E.22}$$

$$J_1[\delta] = -\frac{1}{9} - \frac{2}{3}\delta + 6\delta^2 + \frac{16}{3}(\ln 2\delta - \frac{4}{3})\delta^3 - \frac{8}{3}\delta^4, \tag{E.23}$$

$$\begin{aligned} J_2[\delta] &= -\frac{16}{3}(2+21\delta+3\delta^2-2\delta^3)\ln 2\delta + \frac{16}{3}(1-\delta)(2-\delta+2\delta^2)\ln 2(1-\delta) \\ &\quad -\frac{8}{9}(1-2\delta)(49+68\delta+4\delta^2), \end{aligned} \tag{E.24}$$

$$\begin{aligned} K_1[\delta] &= \frac{4}{315}(8+8\delta-93\delta^2-225\delta^3)\sqrt{1-2\delta} \\ &\quad -\frac{2}{3}\delta^{\frac{3}{2}}(1-6\delta-3\delta^2)\arccos(\frac{3\delta-1}{1-\delta}) \\ &\quad -\frac{16}{3}\delta^3\arccos(\frac{\delta}{1-\delta}), \end{aligned} \tag{E.25}$$

$$\begin{aligned} K_2[\delta] &= \frac{32}{5}(4+9\delta+\delta^2)\sqrt{1-2\delta} - 16\sqrt{\delta}(3+6\delta-\delta^2)\arccos(\frac{3\delta-1}{1-\delta}) \\ &\quad +128\delta\arccos(\frac{\delta}{1-\delta}), \end{aligned} \tag{E.26}$$

$$\begin{aligned} K_3[\delta] &= \frac{8}{105}\sqrt{1-2\delta}(48-57\delta-68\delta^2+85\delta^3) \\ &\quad -4(1-\delta)^3\sqrt{\delta}\arccos(\frac{3\delta-1}{1-\delta}), \end{aligned} \tag{E.27}$$

$$\begin{aligned} K_4[\delta] &= \frac{4}{105}\sqrt{1-2\delta}(64-41\delta+26\delta^2-85\delta^3) \\ &\quad -2(1-\delta-\delta^2+\delta^3)\sqrt{\delta}\arccos(\frac{3\delta-1}{1-\delta}). \end{aligned} \tag{E.28}$$

Appendix F

Electric dipole moments

I Electron EDM

The electron EDM is described by the effective Lagrangian which is given by

$$\mathcal{L}_{\text{eff}} = -\frac{i}{2} d_e \bar{e} \sigma^{\mu\nu} \gamma_5 e F_{\mu\nu}. \tag{F.1}$$

In the MSSM, if we include the SUSY CP violating phases, this effective Lagrangian is induced from 1-loop diagrams which include SUSY particles at the EW scale. There are two contributions from neutralino-selectron and chargino-sneutrino loop diagrams, as follows:

$$d_e = d_e^n + d_e^c, \tag{F.2}$$

$$d_e^n = \frac{e}{32\pi^2} \sum_{A=1}^{4} \sum_{X=1}^{6} Im(N_{1AX}^R N_{1AX}^{L*}) \frac{M_{\tilde{\chi}_A^0}}{m_{\tilde{e}_X}^2} b_2^n\left(\frac{M_{\tilde{\chi}_A^0}^2}{m_{\tilde{e}_X}^2}\right), \tag{F.3}$$

$$d_e^c = -\frac{e}{16\pi^2} \sum_{A=1}^{2} \sum_{X=1}^{3} Im(C_{1AX}^R C_{1AX}^{L*}) \frac{M_{\tilde{\chi}_A^-}}{m_{\tilde{\nu}_X}^2} b_2^c\left(\frac{M_{\tilde{\chi}_A^-}^2}{m_{\tilde{\nu}_X}^2}\right). \tag{F.4}$$

A similar discussion can be applied to the quark sector and various CP-violating operators are induced at the EW scale. They result in the neutron EDM at a low-energy scale, whereas QCD corrections for these operators cannot be neglected.

II Neutron EDM

In this section, we discuss the QCD correction in the calculation of the neutron EDM [46, 52]. The neutron EDM is calculated by the following effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \sum_q C_q^E(M)\mathcal{O}_q^E(M) + \sum_q C_q^C(M)\mathcal{O}_q^C(M) + C^G(M)\mathcal{O}^G(M), \tag{F.5}$$

where $\mathcal{O}_q^E$, $\mathcal{O}_q^C$, $\mathcal{O}^G$ correspond to the quark electric dipole, chromomagnetic dipole, and gluonic Weinberg's operators, respectively, which are given by

$$\mathcal{O}_q^E = -\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}, \tag{F.6}$$

$$\mathcal{O}_q^C = -\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 T^a q G^{a\mu\nu}, \tag{F.7}$$

$$\mathcal{O}^G = -\frac{1}{6}f^{abc}\epsilon^{\mu\nu\lambda\rho}G^a_{\lambda\rho}G^b_{\mu\alpha}G^{c\alpha}_{\nu}. \tag{F.8}$$

Here, $\epsilon^{0123} = 1$, and f^{abc} is the structure constant of the SU(3) group.

In SUSY models, we can obtain the Wilson coefficients at the electroweak scale by evaluating 1-loop diagrams. C_q^E is induced by the photon-penguin diagram. C_q^C is induced by the gluon-penguin diagram. There are three types of SUSY contributions: chargino-squark loop, neutralino-squark loop and gluino-squark loop diagrams which are given by

$$C_d^E(M_W) = C_d^{E(\tilde{\chi}^-)} + C_d^{E(\tilde{\chi}^0)} + C_d^{E(\tilde{G})}, \tag{F.9}$$

$$C_d^{E(\tilde{\chi}^-)} = -\frac{e}{32\pi^2}\sum_{A=1}^{2}\sum_{X=1}^{6} Im(C_{iAX}^{dR}C_{iAX}^{dL*})\frac{M_{\tilde{\chi}_A^-}}{m^2_{\tilde{u}_X}} \tag{F.10}$$

$$\{Q_u b_2^n(\frac{M^2_{\tilde{\chi}_A^-}}{m^2_{\tilde{u}_X}}) - 2(Q_d - Q_u)b_2^c(\frac{M^2_{\tilde{\chi}_A^-}}{m^2_{\tilde{u}_X}})\}, \tag{F.11}$$

$$C_d^{E(\tilde{\chi}^0)} = -\frac{eQ_d}{32\pi^2}\sum_{A=1}^{4}\sum_{X=1}^{6} Im(N_{iAX}^{dR}N_{iAX}^{dL*})\frac{M_{\tilde{\chi}_A^0}}{m^2_{\tilde{d}_X}}b_2^n(\frac{M^2_{\tilde{\chi}_A^0}}{m^2_{\tilde{d}_X}}), \tag{F.12}$$

$$C_d^{E(\tilde{G})} = -\frac{eQ_d}{12\pi^2}\sum_{X=1}^{6} Im(\Gamma_{iX}^{dR}\Gamma_{iX}^{dL*})\frac{M_3}{m^2_{\tilde{d}_X}}b_2^c(\frac{M_3^2}{m^2_{\tilde{d}_X}}), \tag{F.13}$$

$$C_u^E(M_W) = C_d^E(M_W) \quad (d \leftrightarrow u), \tag{F.14}$$

$$C_d^C(M_W) = C_d^{C(\tilde{\chi}^-)} + C_d^{C(\tilde{\chi}^0)} + C_d^{C(\tilde{G})}, \tag{F.15}$$

$$C_d^{C(\tilde{\chi}^-)} = -\frac{g_3}{32\pi^2}\sum_{A=1}^{2}\sum_{X=1}^{6} Im(C_{iAX}^{dR}C_{iAX}^{dL*})\frac{M_{\tilde{\chi}_A^-}}{m_{\tilde{u}_X}^2}b_2^n(\frac{M_{\tilde{\chi}_A^-}^2}{m_{\tilde{u}_X}^2}), \tag{F.16}$$

$$C_d^{C(\tilde{\chi}^0)} = -\frac{g_3}{32\pi^2}\sum_{A=1}^{4}\sum_{X=1}^{6} Im(N_{iAX}^{dR}N_{iAX}^{dL*})\frac{M_{\tilde{\chi}_A^0}}{m_{\tilde{d}_X}^2}b_2^n(\frac{M_{\tilde{\chi}_A^0}^2}{m_{\tilde{d}_X}^2}), \tag{F.17}$$

$$C_d^{C(\tilde{G})} = -\frac{g_3}{32\pi^2}\sum_{X=1}^{6} Im(\Gamma_{iX}^{dR}\Gamma_{iX}^{dL*})\frac{M_3}{m_{\tilde{d}_X}^2}\{-\frac{1}{6}b_2^n(\frac{M_3^2}{m_{\tilde{d}_X}^2}) - 3b_2^c(\frac{M_3^2}{m_{\tilde{d}_X}^2})\}, \tag{F.18}$$

$$C_u^C(M_W) = C_d^C(M_W) \quad (d \leftrightarrow u). \tag{F.19}$$

The gluonic Weinberg's operator is induced at a 2-loop level and the diagram involving the stop and the gluino gives dominant contribution. On the basis where y_u is diagonalized, the Wilson coefficient is written using the matrix elements in Eq.(B.15):

$$C^G = -\frac{3g_3}{16\pi^2}\alpha_3^2\frac{Im\{(m_{\tilde{u}}^2)_{63}\}}{m_t M_3}\frac{1}{M_3^2}z_t H(z_1, z_2, z_t) \tag{F.20}$$

where z_1, z_2 and z_t are defined as follows

$$z_1 = \frac{1}{2M_3^2}\{(m_{\tilde{u}}^2)_{33} + (m_{\tilde{u}}^2)_{66} - \frac{2}{\sin 2\theta}|(m_{\tilde{u}}^2)_{63}|\}, \tag{F.21}$$

$$z_2 = \frac{1}{2M_3^2}\{(m_{\tilde{u}}^2)_{33} + (m_{\tilde{u}}^2)_{66} + \frac{2}{\sin 2\theta}|(m_{\tilde{u}}^2)_{63}|\}, \tag{F.22}$$

$$z_t = (\frac{m_t}{M_3})^2 \tag{F.23}$$

$$\tan 2\theta = -\frac{2|(m_{\tilde{u}}^2)_{63}|}{(m_{\tilde{u}}^2)_{33} - (m_{\tilde{u}}^2)_{66}}. \tag{F.24}$$

A mass function $H(z_1, z_2, z_t)$ is defined so that

$$H(z_1, z_2, z_t) = \frac{1}{2}\int_0^1 dx\int_0^1 du\int_0^1 dy x(1-x)u\frac{N_1 N_2}{D^4}, \tag{F.25}$$

$$N_1 = u(1-x) + z_t x(1-x)(1-u) - 2ux\{z_1 y + z_2(1-y)\}, \tag{F.26}$$

$$N_2 = (1-x)^2(1-u)^2 + u^2 - \frac{1}{9}x^2(1-u)^2, \tag{F.27}$$

$$D = u(1-x) + z_t x(1-x)(1-u) + ux\{z_1 y + z_2(1-y)\}. \tag{F.28}$$

These three contributions are listed in Ref.[46].

We can take into account a QCD correction from the electroweak scale to a hadronic scale (1 GeV) by using the following renormalization group equations for the Wilson coefficients:

$$M\frac{d\vec{C}(M)}{dM} = \frac{\alpha_s(M)}{4\pi}\gamma^T\vec{C}(M), \tag{F.29}$$

where $\vec{C} = (C_q^E, C_q^C, C^G)^T$ and the anomalous dimension matrix γ_{ij} is written by

$$\gamma = \begin{pmatrix} 8/3 & 0 & 0 \\ 32eQ/(3g_s) & (-29+2N_f)/3 & 0 \\ 0 & 6m_q & 2N_f+3 \end{pmatrix}. \tag{F.30}$$

Here, N_f is the number of the quark flavors and Q denotes the electro-magnetic charge of the quark in unit e ($e > 0$). The RGEs can be solved analytically as follows:

$$\begin{aligned} C_q^E(M) &= \eta^{\frac{8}{33-2N_f}}\left[C_q^E(M_i) + 8eQ\left(1-\eta^{\frac{-4}{33-2N_f}}\right)\frac{C_q^C(M_i)}{g_s(M_i)}\right. \\ &\left. - \frac{72eQm_q(\mu_0)}{7+2N_f}\left(1-\eta^{\frac{-4}{33-2N_f}} + \frac{2}{2N_f+5}(1-\eta^{\frac{10+4N_f}{33-2N_f}})\right)\frac{C_G(M_i)}{g_s(M_i)}\right] \end{aligned} \tag{F.31}$$

$$\begin{aligned} C_q^C(M) &= \eta^{\frac{-29+2N_f}{33-2N_f}}\left[C_q^C(M_i)\right. \\ &\left. -\frac{9}{7+2N_f}\left(1-\eta^{\frac{14+4N_f}{33-2N_f}}\right)m_q(M_i)C^G(M_i)\right], \end{aligned} \tag{F.32}$$

$$C^G(M_i) = \eta^{\frac{9+6N_f}{33-2N_f}}C^G(M_i), \tag{F.33}$$

where $\eta = g_s(M_i)/g_s(M)$.

We solve RGE from m_W to m_b, m_b to m_c and m_c to the 1 GeV scale. When the heavy quarks (c, b) decouple at their mass threshold, C^G is induced through the chromo-electric dipole moment of the heavy quarks. The difference C^G below and above the threshold is given by [53]

$$C^G(m_q)_{below} - C^G(m_q)_{above} = +\frac{\alpha_s(m_q)}{8\pi m_q(m_q)} C_q^C(m_q). \tag{F.34}$$

Taking into account the QCD and threshold corrections, we obtain the effective Lagrangian at the hadronic scale. It is then straightforward to evaluate the effective $\mathcal{L}$ at the 1 GeV scale from the m_W scale.

The neutron EDM (d_n) is given by the Wilson coefficients at a hadronic scale as follows:

$$d_n = d_n^E + d_n^C + d_n^G, \tag{F.35}$$

$$d_n^E = \frac{1}{3}\left(4C_d^E - C_u^E\right), \tag{F.36}$$

$$d_n^C = \frac{1}{3}\frac{e}{4\pi}\left(4C_d^C - C_u^C\right), \tag{F.37}$$

$$d_n^G = \frac{eM}{4\pi} C^G, \tag{F.38}$$

where M is a chiral symmetry-breaking parameter, which is estimated to be 1.19 GeV. In the above we use a non-relativistic quark model for d_n^E and a naive dimensional analysis for d_n^C and d_n^G.

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