Gauge Theory as Noncritical Strings*

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Abstract

We consider bosonic noncritical strings as QCD strings and we present a basic strategy to construct them in the context of Liouville theory. We show that Dirichlet boundary conditions play important roles in generalized Liouville theory. Specifically, they are used to stabilize the classical configuration of strings and also utilized to treat tachyon condensation in our model. We point out that Dirichlet boundary conditions for the Liouville mode maintains Weyl invariance, if an appropriate condition is satisfied, in the background with a (non-linear) dilaton.

Contents

Chapter 1

Introduction

Gauge theories and string theories are very important fields of modern theoretical physics. Construction of a non-perturbative formulation of string theories is one of the main subjects which has not yet been completely understood. Although non-perturbative formulations of gauge theories were constructed as lattice gauge theories, we are still interested in analytical studies on non-perturbative effects of gauge theories. The relation between gauge theories and string theories has been studied for a long time, and both theories have developed while affecting each other deeply. Thus, studies on the connection between them seems to be important and is expected to be fruitful. With this in mind, we consider QCD strings in this thesis, from the viewpoint of Liouville theory.

Investigation of non-perturbative effects of QCD (for example, confinement of quarks) has been one of the motivations for string theories. In the context of QCD strings, confinement of quarks is explained in terms of a linear potential produced by the "string" stretched between quark and antiquark. There are many reasons why we consider such a string theory [2], although this naive explanation has not been completed nor proved to this time.[1]

In the strong coupling expansion of lattice gauge theories, the quark-antiquark potential is given by the expectation value of the Wilson loop, each term of which corresponds to different configuration of the lattice surfaces. This reminds us of the sum of the open-string "world-sheet surfaces" whose boundary is the Wilson loop. We can find another description of gauge theories in terms of strings in the large-N limit. In large-N gauge theories, Feynman diagrams are classified by the "Euler number," and the physical observables are expressed as a series with respect to the "topology" of the graph. The dominant terms correspond to planar diagrams. This also reminds us of a "world-sheet" description. Furthermore, in the description of the dual Meissner effect, the fluxes of color electric charge are collimated, and form a "string" between quarks. We also find similar correspondence between gauge theories and string theories in the contexts of D-branes and AdS/CFT.[2]

In this chapter we review the connections between gauge theories and string theories from various viewpoints. After the overview of them, we make our basic presuppositions and assumptions to promote further consideration. Our main assumption is that the four-dimensional non-SUSY large-N pure Yang-Mills (YM) theory corresponds to some suitable four-dimensional bosonic string which has not yet been found.

The organization of this thesis is as follows. In chapter 2, we review Liouville theory which we utilize to describe noncritical strings. Noncritical strings, in the spacetime where the dimension is lower than one, can be consistently quantized in the framework of Liouville theory. The problems

[1] For a review of these topics, see also Ref. [3] and the references therein.

[2] We have found other connections between gauge theories and strings [2, 3].

in the construction of noncritical strings in higher-dimensional spacetime (dimension of which is higher than one) will be presented in chapter 2: one is the instability of the Liouville mode, and the other is the existence of tachyonic mode.

In chapter 3 and chapter 4, we attempt to generalize Liouville theory in order to describe noncritical strings in higher-dimensional spacetime. We point out that one of the problems, the instability of the Liouville mode, can be overcome if we introduce Dirichlet boundary conditions for the Liouville mode at the boundary of the world-sheet. In general, Dirichlet boundary conditions for Liouville mode are forbidden, since they break Weyl invariance. However we point out that we have a special case which we can introduce Dirichlet boundary conditions for it with maintaining Weyl invariance.

In chapter 4, we treat tachyon condensation. Although exact treatment of this problem is very difficult, we present a basic strategy for it in the framework of the generalized Liouville theory. Our proposal is to utilize Dirichlet boundary conditions again. Tachyon condensation is one of the hot topics of recent string theories even for critical strings. We briefly review the recent works related to open-string tachyon condensation on unstable D-branes, and also comment shortly about the author's work on tachyon condensation for critical bosonic strings.

1.1 Connections between gauge theories and string theories

1.1.1 Large-N gauge theory

One of the indication that gauge theories are related to strings is found in the context of large-N gauge theories considered by 't Hooft [4] for the first time.

Let's consider SU(N) pure Yang-Mills theory. The Lagrangian is given as

$$\mathcal{L} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}, \tag{1.1.1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_{\rm YM}[A_\mu, A_\nu], \tag{1.1.2}$$

$$A_\mu = A_\mu^a t^a,\ \mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}, \tag{1.1.3}$$

where t^a are the generators of the SU(N) gauge group.

We obtain the β-function in this theory by perturbative calculation as

$$\beta(g_{\rm YM}) = -\frac{g_{\rm YM}^3}{16\pi^2}\frac{11}{3}N + O(g_{\rm YM}^5). \tag{1.1.4}$$

Therefore, the energy scale which characterizes the theory is given as

$$\Lambda_{\rm QCD}^2 = \mu^2 \exp\left(-\frac{24\pi^2}{11 g_{\rm YM}^2 N}\right), \tag{1.1.5}$$

from (1.1.4) with the approximation up to $O(g_{\rm YM}^5)$.

Now let's take the limit $N \to \infty$ with $\Lambda_{\rm QCD}$ fixed, namely

$$N \to \infty\ (\lambda = g_{\rm YM}^2 N : \text{fixed}). \tag{1.1.6}$$

In this limit, each Feynman diagram carries a topological factor N^χ, namely

$$(\text{contribution of the graph}) \propto \lambda^{n_{\rm p}-n_{\rm v}} N^\chi, \tag{1.1.7}$$

where $n_{\rm p}$ and $n_{\rm v}$ denote the number of the propagators and the vertexes included in the graph. χ represents the Euler number of the graph. Therefore, if N is sufficiently large, $1/N$ expansion gives good approximation, and each term is categorized by its topology. The sum over graphs of a given topology can perhaps be thought as a sum over "world-sheets" of a hypothetical "string". We expect from (1.1.7) that the "closed string coupling constant" is of oder $1/N$. Furthermore, the boundaries of the hypothetical world-sheet can be seen as the world-lines of quarks whose color charge can be interpreted as Chan-Paton factor.

Thus the hypothetical open string stretched between quarks can be thought as a meson, and the hypothetical closed string can be interpreted as a glue-ball. We call these strings QCD strings in this thesis.

1.1.2 Lattice Strong Coupling Expansion

The correspondence between gauge theories and strings can also be seen in lattice gauge theories.

Suppose that we calculate an expectation value of a Wilson loop. In the strong coupling region, we can obtain it analytically in lattice gauge theory with strong coupling expansion [5]. The leading term of the expansion corresponds to the planar section in the lattice rounded by the Wilson loop. The contribution of each term in the expansion seems to depend on the "area" of the section, and we can optimistically expect that

$$\langle W(c)\rangle \sim \sum_{\text{sections}} \exp\left\{-(\text{the area of the section rounded by } c)\right\}, \quad (1.1.8)$$

where c denotes the contour of the Wilson loop. In the case of the rectangular Wilson loop infinitely long in the time direction, its expectation value is related to the quark-antiquark static potential via

$$\lim_{T\to\infty}\langle W(c)\rangle = \exp\left(-V(r)T\right), \quad (1.1.9)$$

where T is the length of the Wilson loop in the time direction. $V(r)$ denotes quark-antiquark static potential where r is the distance between quark and antiquark. In the leading order of the strong coupling expansion, the quark-antiquark potential is linear with r, which describes the property of confinement of quarks. This property also reminds us of the hypothetical string stretched between quarks, the tension of which is the proportional factor in front of r in the linear potential. This picture also suggests that the area of the section of the lattice rounded by the Wilson loop corresponds to the action of the string, and the expectation value of the Wilson loop seems to be obtained as the partition function of the string.

Similar relation between a string partition function and a expectation value of a Wilson loop is also found in the context of AdS/CFT, described in section 1.1.4.

1.1.3 Regge Phenomenology

The relation between gauge theories and string theories was also pointed out in the context of Regge phenomenology.[3]

Regge Trajectory

Regge trajectory represents the relations between mass and angular momentum of hadrons as

$$J \leq \alpha' m^2 + \alpha_0, \tag{1.1.10}$$

where J denotes the angular momentum of a hadron, the mass of which is m. α' and α_0 are constants, especially α' is called "Regge slope".

Correctness of Regge trajectory (1.1.10) has been checked experimentally both for mesons of $J \leq 6$ and baryons of $J \leq \frac{11}{2}$ [6, 7] as far as the author know, and it has good accuracy in this region. It is also known that the approximate value of α' for hadrons is $1\mathrm{GeV}^2$. A similar relation between the angular momentum of glueballs and their mass is also expected. Regge

[3]For a review of subjects in this subsection, see Ref. [6]

trajectory for glueballs has been researched with both experimental method and numerical method in lattice gauge theory. In this case, the suggested value of α' for glueballs is almost the half of that for mesons [7].

In the picture of QCD strings presented in the previous sections, Regge trajectories can easily derived as the relation between the angular momentum and the mass of the strings. In this picture, the tension τ of the string is $(2\pi\alpha')^{-1}$ and $\tau \sim \Lambda_{\mathrm{QCD}}$. The approximate ratio of Regge slopes for mesons and that for glueballs is around 2, and it agrees with the ratio of the tension of open strings and that of closed strings.

Veneziano Amplitude

We can evaluate scattering amplitude of hadrons with QCD string model. For example, the four-point scattering amplitude A can be obtained as [8]

$$A(s,t) \;=\; \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s)-J(t))}, \tag{1.1.11}$$

where s and t denote square of the momentum flowing in s-channel and t-channel, respectively. This amplitude has s-t duality of hadrons. In the limit of large-s with fixed t, the amplitude behaves like as

$$A(s,t) \sim \Gamma(-J(t)) \cdot (-J(s))^{J(t)}, \tag{1.1.12}$$

and it describes the behavior of hadrons well. Therefore, the string description of QCD seems to be successful at least within the above consideration.

However, a simple QCD string model fails to represent the behavior of hadrons in high energy region with fixed-angle scattering. In this case, we obtain

$$A(s,t) \sim F(\theta)^{-J(s)}, \tag{1.1.13}$$

where θ is the center-of-mass scattering angle. The relations (1.1.13) and (1.1.10) tell us that the amplitude decreases exponentially as s grows. However, experimental data of the fixed angle scattering, in the region $s \to \infty$ and $t \to -\infty$ with fixed s/t, suggest that it decreases in power law of s [9]. As a matter of fact, the behavior of hadrons in this region was explained by parton model instead of QCD strings. Some attempts to describe the properties of hadron amplitude in this region with a QCD string model are found in [53, 54, 55], and it is suggested that the role of Dirichlet boundary conditions on the world-sheet is important. It was also shown that the free energy of large-N YM theory [57] does not agree with that obtained from QCD string at high temperature. However, it was pointed out that Dirichlet boundaries on world-sheets of QCD strings make the free energy more similar to that of YM theory [56]. Important roles of Dirichlet boundaries on world-sheets mentioned above are discussed in chapter 5.

Another disagreement between predictions of QCD strings and the actual behavior of hadrons arises at the quantum level. The value of α_0 have to be unity if we make quantum critical strings unitary in flat spacetime. Therefore, QCD string model has tachyonic ground state, in general. This problem has not yet been solved, and the treatment of tachyon condensation is one of the main themes in this thesis.

1.1.4 AdS/CFT

The relations between gauge theories and strings mentioned above are based on somewhat conceptual discussions. Furthermore, we do not know how to describe the string theory which represents large-N QCD in elementary terms, namely no one knows the exact vacuum of QCD string, even if it exists or not.

However, the vacuum of the string which represents $N = 4$ super Yang-Mills (SYM) theory was presented by Maldacena [10]. According to the Maldacena's conjecture, $N = 4$ SYM theory is described by type IIB superstring theory living in ten-dimensional spacetime which is compactified on the special manifold: $AdS_5 \otimes S^5$. (Here AdS_5 denotes five-dimensional anti-de Sitter spacetime.)

We briefly review this correspondence, in this subsection. In AdS/CFT picture, D-branes play an important role which connect the physics of supergravity and that of SYM theory.

A system of N coincident Dp-branes is a classical solution of the low energy effective action of superstring. The solution is called as brack p-brane solution, and given as follows [11],

$$\begin{aligned} ds^2 &= H(r)^{-\frac{1}{2}}(-dt^2 + dx_{\|}^2) + H(r)^{\frac{1}{2}}(dr^2 + r^2 d\Omega^2) \\ e^{\Phi(r)} &= g_\infty H(r)^{\frac{3-p}{4}} \\ H(r) &= 1 + \left(\frac{R}{r}\right)^{7-p} \\ R^{7-p} &= c_p g_\infty N l_s^{7-p} \\ c_p &= 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right), \end{aligned} \tag{1.1.14}$$

where $l_s^2 = \alpha'$, r is the isotropic radial coordinate on the transverse space, $x_{\|}$ denotes the spatial coordinates along the brane, and g_∞ is open string coupling constant at $r = \infty$.

On the other hand, a system of N coincident Dp-branes is considered to be described by the non-abelian version of the Born-Infeld action, which is suggested as [12]

$$S_{\mathrm{BI}} = -\tau_p^0 \int d^{p+1}x e^{-\Phi} \mathrm{STr}\sqrt{-\det(G_{ij} + 2\pi\alpha' F_{ij})},$$

$$\tau_p \equiv \frac{\tau_p^0}{g_s} = \frac{2\pi\sqrt{\alpha'}}{2\pi\alpha' g_s}, \tag{1.1.15}$$

where G_{ij} is the pull back of the metric $G_{\mu\nu}$, F_{ij} is the field strength of the gauge field on the brane, and $g_s = e^{\Phi}$. STr denotes the symmetrized trace over the gauge group matrices. We obtain the action for the gauge field in the standard form by expand $S_{\rm BI}$ in powers of F_{ij}:

$$\begin{aligned} S_{\rm BI} &= -\frac{1}{4g_{\rm YM}^2}\int d^{p+1}x F_{ij}^a F^{aij}, \\ g_{\rm YM}^2 &= 2g_s(2\pi)^{p-2}(\alpha')^{\frac{p-3}{2}}, \end{aligned} \tag{1.1.16}$$

where we normalized the $U(N)$ matrices as $\mathrm{Tr}(t_i t_j) = \frac{1}{2}\delta_{ij}$.

Let us try to connect these two descriptions of the system of D-branes. First, we look deeper into the case we take near-horizon limit:

$$\begin{aligned} &r \to 0, \alpha' \to 0, \\ &U \equiv \frac{r}{\alpha'} : \text{fixed}. \end{aligned} \tag{1.1.17}$$

In this limit, the metric in (1.1.14) can be rewritten as

$$\frac{ds^2}{\alpha'} = \frac{U^2}{\sqrt{4\pi N g_s}} dx_{\parallel}^2 + \frac{\sqrt{4\pi N g_s}}{U^2} dU^2 + \sqrt{4\pi N g_s} d\Omega^2. \tag{1.1.18}$$

This is the metric of the manifold $AdS_5 \otimes S^5$. One can find that the radius of AdS_5 and the radius of S^5 has the same value b^2 as

$$\begin{aligned} b^2 &= \alpha'\sqrt{4\pi N g_s} = \alpha'\sqrt{g_{\rm YM}^2 N} \\ &= \alpha'\sqrt{\lambda}. \end{aligned} \tag{1.1.19}$$

The black p-brane solution is valid only in the classical region where $\alpha'\mathcal{R} \ll 1$, namely

$$1 \gg \frac{\alpha'}{b^2} \Longrightarrow \lambda \gg 1. \tag{1.1.20}$$

Here $\mathcal{R}$ is the scalar curvature of the target space. In this region, the coupling constant of SYM and that of IIB string are

$$g^2_{\rm YM} = 4\pi g_s = \frac{\lambda}{N}, \tag{1.1.21}$$

and we find that the perturbative method is valid for both SYM and string theory, in 't Hooft limit $N \to \infty$. Therefore, in this region, we can expect that both SYM theory and supergravity are good descriptions for the system of D-branes.

Furthermore, the isometry group of AdS_5 is $O(4,2)$ which corresponds exactly to the conformal symmetry in $N = 4$ SYM, and the isometries of S^5 correspond exactly to the R-symmetry group $SU(4)$ of $N = 4$ SYM. Thus, Maldacena conjectured that $N = 4$ super Yang-Mills theory is equivalent to type IIB string theory compactified on the special background $AdS_5 \otimes S^5$.

In the framework of AdS/CFT, the Green functions in the SYM can be given in the string theory as follows [13, 14],

$$\langle e^{\int d^4x \Phi_0(x) Q(x)} \rangle = \int_{\Phi_0} \mathcal{D}\Phi e^{-S(\Phi)}. \tag{1.1.22}$$

Here $Q(x)$ denotes boundary field (a field living on the boundary of AdS spacetime), and Φ is bulk field with boundary value Φ_0. In the left-hand-side, Φ_0 acts as a source of boundary field. The right-hand-side is the partition function of type IIB string theory obtained by the functional integral over Φ with the restriction that its boundary value is Φ_0.

Expectation value of a Wilson loop in $N = 4$ SYM can be obtained as follows [15],

$$\langle W(c) \rangle \sim e^{-S}, \tag{1.1.23}$$

where c denotes the contour of the Wilson loop and S is the classical value of the action of IIB open string with the world-sheet boundary c.

In fact, this equivalence has been checked by various calculations and it seems to be correct (for example, see Refs. [16]).

There are also some attempts to generalize this string/YM correspondence to non-SUSY case [17].[4] However, the generalization to non-SUSY case has not fully succeeded.

1.1.5 Other connections between gauge theories and strings

We can find other several connections between gauge theories and strings. We briefly introduce them here.

Dual Meissner effects

Nielsen and Olsen [18, 19] presented non-trivial solutions of abelian Higgs model. The solutions, called vortex solutions, have structure similar to that of strings; they are tubes of magnetic flux with constant energy per unit length.

In the dual superconductor picture, dual Meissner effect confines electric color flux to narrow tubes connecting quark-antiquark pairs [20, 21, 22]. These narrow tubes are dual objects of Nielsen-Olsen vortices. String picture also arise from dual Meissner effect like this.

Two-dimensional gauge theories

Two-dimensional gauge theories are exactly solvable, both in a lattice [23, 24] and a continuum formulation [25, 26, 27]. They have two important properties; they are invariant under area-preserving diffeomorphism, and they have

[4] See also sub-subsection "Type 0 strings" in 1.1.5 and the references therein.

no propagating degrees of freedom. These properties reveal their almost topological nature, and they are interpreted in terms of strings [28, 29].

The coefficients of the terms of the order $(\frac{1}{N})^{\chi}$, in the $1/N$ expansion of the partition function of two-dimensional gauge theory, count the number of maps from a world-sheet of Euler number χ to the two-dimensional target space. The number of maps correspond to the number of string configurations, and each map is weighted by the area of its world-sheet. Therefore, two-dimensional gauge theories can be represented as two-dimensional string theories. The connection between them seems to provide an interesting arena for understanding YM/string correspondence in arbitrary dimensions.

Type 0 strings

Type 0 strings are superstrigs which has no supersymmetry in target space [35]. There are some attempts to take type 0 strings as dual models to certain four-dimensional non-SUSY large-N gauge theories [30, 31, 32, 33]. Although this is one of the natural extension of AdS/CFT to non-SUSY version, type 0 strings has tachyons. Some proposal to stabilize these tachyons in appropriate background are found in [30, 34]. This is one of the attractive directions to construct QCD strings, though exact dual models which describe four-dimensional non-SUSY large-N gauge theories has not yet been obtained in this method.

1.2 The basic assumptions and a presuppositions for QCD strings

1.2.1 The basic assumptions and a presuppositions in this thesis

With the above overviews in mind, we make the following presuppositions and we study QCD strings with the following assumptions.

Presupposition 1

We assume that the color charge is attached to the ends of open strings, and we regard the world-sheet boundaries as Wilson loops.

This presupposition seems to be very natural in the viewpoints of previous subsections.

Presupposition 2

We treat non-dynamical Wilson loops, namely we attempt to describe *pure* YM theories in which quarks are *not* dynamical.

In other words, all the open-string boundaries correspond non-dynamical Wilson loops which we set as input; we presuppose that no intermediate open-string state is created even in higher order corrections of string perturbation.

We have several reasons for this presupposition. One of them is that we can simplify the discussions if we neglect the dynamical freedom of quarks. Another reason is as follows. If quarks are dynamical, many mesons can be created when we make the distance between quark and antiquark sufficiently large. In this situation, the "string" between quark and antiquark is cut into many pieces and QCD-string picture get worse obviously.

Furthermore, we do not have string models which have degrees of freedom of spin on the edges of open strings. This is one of the reasons why we do not treat dynamical quarks which have spin, here.

Assumption 1

We assume that the strings are bosonic, and they exist in four-dimensional spacetime.

This is because our purpose is to describe four-dimensional (large-N) non-SUSY gauge theories. Although there are some attempts to generalize AdS/CFT for non-SUSY cases, they have not yet succeeded. We consider that one of the most natural candidate for non-SUSY QCD strings is bosonic string.

Furthermore, we set the target-space dimensions to be four on purpose. Naively, this is natural choice at the classical level. At the quantum level, our choice makes strings noncritical. However, as seen in chapter 2, Weyl anomaly of noncritical strings reproduces another freedom of an extra spacetime dimension. Therefore the holographic picture similar to that in AdS/CFT, where strings live in higher-dimensional target space while gauge theory is in the lower-dimensional subspace, arise naturally in noncritical strings.

Assumption 2

We assume that the connection between gauge theories and QCD strings is described by the following relation,

$$\langle W(c)\rangle \sim Z(c). \tag{1.2.24}$$

Here c denotes the contour of a Wilson loop, and $\langle W(c) \rangle$ is the expectation value of the Wilson loop. $Z(c)$ is the partition function of the dual open string with the boundary c. The left-hand-side and the right-hand-side coincides up to overall factor.

This assumption is natural in the viewpoint of large-N gauge theories, lattice gauge theories, and AdS/CFT, mentioned above.

1.2.2 Some comments for other directions

Polyakov has proposed dual string models which represent *dynamical* Wilson loops as closed strings [36, 37]. The correspondence between gauge theories and string theories in this model is pursued along loop equations [38] for Wilson loops. Although this is another attractive direction, we do not discuss QCD strings along this model. Because, we made the basic presupposition that we treat Wilson loops as *non*-dynamical objects here.

Chapter 2

Review of Liouville theory

As mentioned in the previous chapter, it is most natural to assume that QCD strings are bosonic and four-dimensional objects. Therefore, we must to find some nontrivial way to construct a consistent noncritical string theory.

Quantization of noncritical strings has been considered in Liouville theories [39, 40]. In particular, strings for $d \leq 1$ have been consistently quantized using DDK theory presented by Distler and Kawai [41] and David [42]. (Here d denotes the spacetime dimension in which strings exist.) For this reason, we first review the work on the DDK theory.[1]

2.1 DDK theory without boundaries

Let us start with a d-dimensional ($d \neq 26$) bosonic string without boundaries for simplicity. We use the Euclidean signature both for the world-sheet metric and for the spacetime metric. The Polyakov action is

$$S_M = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{g} (g^{ab} \partial_a X^\mu \partial_b X_\mu), \tag{2.1.1}$$

[1] For a review of DDK, see Ref. [43].

where μ runs from 1 to d. The partition function with respect to this action is diffeomorphism invariant, and we can fix the world-sheet metric as

$$g_{ab} = \hat{g}_{ab} e^{\phi(\xi)}. \tag{2.1.2}$$

However, the Weyl invariance of the partition function is broken for $d \neq 26$ at the quantum level, and the freedom of the Weyl transformation is no longer a gauge freedom. Therefore, we have to perform the path integral with respect to ϕ rigorously. However, the measure $[d\phi]_g$ of the path integral with respect to ϕ is given by the norm

$$||\delta\phi||_g = \int d^2\xi \sqrt{g} (\delta\phi)^2 = \int d^2\xi \sqrt{\hat{g}} e^{\phi(\xi)} (\delta\phi)^2, \tag{2.1.3}$$

which depends on ϕ itself in a complicated manner. Thus, to perform the path integral with this measure is difficult and seems to be almost impossible.

In Louville theory based on the DDK argument, the measure of the path integral is redefined with respect to some fixed world-sheet metric $\hat{g}_{ab}$ to avoid the above described difficulty. Namely, we use the measure $[d\phi]_{\hat{g}}$ given by the norm

$$||\delta\phi||_{\hat{g}} = \int d^2\xi \sqrt{\hat{g}} (\delta\phi)^2, \tag{2.1.4}$$

which does not depend on ϕ. With this redefinition, we have to use a Jacobian J to maintain consistency:

$$[dX]_g [db]_g [dc]_g [d\phi]_g = [dX]_{\hat{g}} [db]_{\hat{g}} [dc]_{\hat{g}} [d\phi]_{\hat{g}} J, \tag{2.1.5}$$

$$J = e^{-S_L}, \tag{2.1.6}$$

where b and c are the ghost fields. We have the relations [39]

$$[dX]_g [db]_g [dc]_g = [dX]_{\hat{g}} [db]_{\hat{g}} [dc]_{\hat{g}} e^{-S_{\mathrm{Jac}}} \tag{2.1.7}$$

$$S_{\rm Jac} = \frac{1}{2}\frac{26-d}{48\pi}\int d^2\xi\sqrt{\hat{g}}\{\hat{g}^{ab}\partial_a\phi\partial_b\phi + 2\hat{R}\phi\}, \tag{2.1.8}$$

where $\hat{R}$ is the world-sheet scalar curvature with respect to the metric $\hat{g}_{ab}$. (Note that (2.1.7) does not contain $[d\phi]$, while (2.1.5) *does* contain $[d\phi]$.) Therefore, the Jacobian J can be naturally assumed to be

$$S_L = u\int d^2\xi\sqrt{\hat{g}}\{\hat{g}^{ab}\partial_a\phi\partial_b\phi + q\hat{R}\phi\}, \tag{2.1.9}$$

where u and q are some constants.

Then, the total action, including the Jacobian term, is

$$\begin{aligned} S &= S_M + S_L + S_{\rm ghost} \\ &= \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{\hat{g}}\{\hat{g}^{ab}\partial_a\varphi\partial_b\varphi + \hat{g}^{ab}\partial_a X^\mu\partial_b X_\mu + \alpha' Q\hat{R}\varphi\} \\ &\quad + S_{\rm ghost}, \end{aligned} \tag{2.1.10}$$

where we have redefined the factor q and the field ϕ as

$$Q = \sqrt{\frac{4\pi u}{\alpha'}}\, q\,, \tag{2.1.11}$$

$$\varphi(\xi) = \sqrt{4\pi\alpha' u}\,\phi(\xi) = \frac{q}{Q\alpha'}\,\phi(\xi). \tag{2.1.12}$$

In the action (2.1.10), the field φ has a kinetic term and can be regarded as a new "coordinate" of the target space[2]; we obtain one more spacetime dimension and a linear dilaton term through the process of quantization. The world-sheet metric g_{ab} has been replaced with $\hat{g}_{ab}$ in the process.

Let us consider the partition function Z with respect to the action (2.1.10),

$$Z = \int[d\varphi][dX]e^{-S}, \tag{2.1.13}$$

[2]Note that φ has the dimension of length, though ϕ is a dimensionless field.

where $[d\varphi]$ and $[dX]$ stand for $[d\varphi]_{\hat{g}}$ and $[dX]_{\hat{g}}$. (We omit $\hat{g}$ from expressions of the measure in the following.) Here $[dX]$ represents the measure with respect to the matter and the ghost. It also represents the measure for the modular integration if $N_g > 0$, where N_g is the number of the genus of the world-sheet.

We note that the partition function does not change under the simultaneous transformations that keep $g_{ab} = \hat{g}_{ab} e^{\frac{Q\alpha'}{q}\varphi}$ invariant,

$$\hat{g}_{ab} \mapsto \hat{g}_{ab} e^{\delta(\xi)}, \tag{2.1.14}$$

$$\varphi(\xi) \mapsto \varphi(\xi)' = \varphi(\xi) - \frac{Q\alpha'}{q}\,\delta(\xi), \tag{2.1.15}$$

if the boundary condition of the integration in (2.1.13) allows the shift of φ given by (2.1.15). This is because the action in the initial formulation (2.1.1) depends only on g_{ab} and X^μ, which do not change under the above transformations. Thus,

$$Z = \int [d\varphi][dX] e^{-S[\hat{g},X,\varphi]} = \int [d\varphi'][dX] e^{-S[\hat{g}e^{\delta(\xi)},X,\varphi']}. \tag{2.1.16}$$

After rewriting the dummy variable φ' as φ, we note the very important fact that the partition function is now represented in a Weyl *invariant* way *with respect to the new metric* $\hat{g}_{ab}$.

Now we are in a position to determine the value of Q. We found that the partition function is Weyl invariant. Therefore, the total central charge of the theory should be zero:

$$C_\varphi + C_M + C_{\text{ghost}} = 0\,, \tag{2.1.17}$$

where C_φ is the central charge for S_L, C_M is the central charge for S_M, which is equal to d, and C_{ghost} is the central charge for S_{ghost}, which is calculated to be -26. We can evaluate C_φ with a standard technique of conformal field theory as

$$C_\varphi = 1 + 6\alpha' Q^2. \tag{2.1.18}$$

Thus (2.1.17) and (2.1.18) give us

$$Q = \pm\sqrt{\frac{25-d}{6\alpha'}}\,. \tag{2.1.19}$$

We choose the positive branch, in this thesis.

We point out that our action is exactly the same as that of the "linear dilaton string" in $d+1$ dimensional spacetime. In the linear dilaton string, φ is regarded as one of the spacetime coordinates, and Q is given as a factor which appears in the dilaton term. However, from the viewpoint of Liouville theory, we do not regard φ as a real physical coordinate of spacetime. It is the parameter of the Weyl transformation and becomes the new spacetime coordinate through the quantization process. Thus, we call φ (or ϕ) the "Liouville mode" in this thesis.

Now we have some problems. First, the Liouville mode φ does not have a stable vacuum with the action (2.1.10). In Refs. [41] and [42], a cosmological constant is added to the action to obtain a stable vacuum. The Polyakov action with the (renormalized) cosmological constant μ is

$$S_M = \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{g}(g^{ab}\partial_a X^\mu\partial_b X_\mu + \alpha'\mu). \tag{2.1.20}$$

It is natural to assume that the cosmological constant term is deformed by the quantum effect as

$$\begin{aligned}\frac{\mu}{4\pi}\int d^2\xi\sqrt{\hat{g}}e^{\phi} \quad\longmapsto\quad & \frac{\mu}{4\pi}\int d^2\xi\sqrt{\hat{g}}e^{\gamma\phi}\\ &= \frac{\mu}{4\pi}\int d^2\xi\sqrt{\hat{g}}e^{\alpha\varphi},\end{aligned} \tag{2.1.21}$$

where γ is a constant indicating an anomalous dimension, and we defined α as $\alpha \equiv \frac{Q\alpha'}{q}\gamma$. To preserve the Weyl invariance of the theory, we choose α so that the cosmological constant term is Weyl invariant. This is realized if the conformal dimension of the operator $e^{\alpha\varphi}$ is 2. Namely, we demand $e^{\alpha\varphi}$ to be a (1,1)-primary operator. In complex coordinates, the holomorphic part of the energy-momentum tensor given by the action (2.1.10) is

$$T_{ZZ} = -\frac{1}{\alpha'}(\partial\varphi\partial\varphi - Q\partial^2\varphi), \tag{2.1.22}$$

and the conformal weight Δ of $e^{\alpha\varphi}$ is given by

$$\Delta = \frac{\alpha'}{2}\alpha\left(Q - \frac{\alpha}{2}\right), \tag{2.1.23}$$

which should be 1. Thus we obtain two solutions:

$$\alpha_\pm = Q \pm \sqrt{Q^2 - \frac{4}{\alpha'}} = \sqrt{\frac{25-d}{6\alpha'}} \pm \sqrt{\frac{1-d}{6\alpha'}}\,. \tag{2.1.24}$$

In the classical limit ($d \to -\infty$), α should be zero, and we take the branch of α_-. Then we have a stable vacuum for a negative world-sheet curvature $\hat{R}$. This is because the potential for φ is given by

$$V(\varphi) = \frac{1}{4\pi}\int d^2\xi(Q\hat{R}\varphi + \mu e^{\alpha\varphi}), \tag{2.1.25}$$

and has a minimum value if the factor of φ in the first term is negative. (We have assumed a constant-φ configuration as a classical solution here.)

We have a second problem with the strings for $d > 1$. In this region, α is a complex number. Thus we have to regard $\mu e^{\alpha\varphi}$ as a tachyon vertex operator with momentum in the φ direction, rather than a cosmological constant term. Furthermore, the composite operator $e^{\alpha\varphi}$ becomes non-normalizable [44]. We also have to consider the condensation of the target-space tachyons, since we treat a noncritical bosonic string in which the tachyonic mode is not projected out. We point out that we do not have target-space tachyons if $d \leq 1$. This is because the Liouville theory is described as a $d+1$-dimensional string theory, and the world-sheet oscillation can be fixed completely by the gauge symmetry if $d+1 \leq 2$. Thus the tachyonic mode cannot appear.

For this reason, we can construct a Weyl invariant string theory with a cosmological constant for the case $d \leq 1$. A consistent model for quantized noncritical strings for $d > 1$ has not yet been constructed.

2.2 Liouville theory with boundaries

For noncritical strings with a boundary, we assume S_L to be

$$S_L = \frac{1}{4\pi\alpha'} \int_{\mathcal{M}} d^2\xi \sqrt{\hat{g}} \{\hat{g}^{ab}\partial_a\varphi\partial_b\varphi + \alpha' Q \hat{R}\varphi\} + \frac{1}{2\pi\alpha'} \int_{\partial\mathcal{M}} ds \sqrt{\hat{g}_{ss}} \alpha' Q \hat{k}\, \varphi \,, \tag{2.2.26}$$

where s is a parameter of the boundary $\partial\mathcal{M}$ and $ds\sqrt{\hat{g}_{ss}}$ denotes an invariant infinitesimal length on it [40]. The quantity $\hat{k}$ is the extrinsic curvature with respect to the metric $\hat{g}_{ab}$. If the world-sheet has many boundaries, $\partial\mathcal{M}$ denotes all of them. The only difference between this and the boundaryless case is the existence of the boundary terms. We can also add a boundary cosmological constant term,

$$\mu_b \int_{\partial\mathcal{M}} ds \sqrt{\hat{g}_{ss}} e^{\frac{\alpha}{2}\varphi}. \tag{2.2.27}$$

For strings with boundaries, we also need to consider the boundary conditions. We can consider several types of boundary conditions for φ.[3] Now we set $\hat{k} = 0$ for simplicity. The reason why this choice is natural is discussed in section 3.1. In this case, we have two choices for the boundary conditions.

[3] Boundary conditions and boundary states for linear dilaton theory have been considered by several authors. (For example, see Refs. [40],[45],[46] and the references therein.)

One of them is Neumann boundary conditions, and the other is Dirichlet boundary conditions.

Neumann boundary conditions allow the ends of open strings to move freely. In other words, Neumann boundary conditions can be regarded as an equation of motion for the endpoint. Therefore, it does not restrict the discussion in section 2.1, and the argument given for the boundaryless case still holds. On the other hand, we have to be careful if we consider Dirichlet boundary conditions. This is because the shift of φ (2.1.15) is not consistent with Dirichlet boundary conditions, and thus Weyl invariance is broken, [46] in general.

However, we show in chapter 4 that we can use Dirichlet boundary conditions without breaking Weyl invariance in some special case.

Chapter 3

Liouville theory as a QCD string: a trial to go beyond the $d = 1$ barrier

3.1 Liouville theory as a QCD string

We presented our basic assumptions for noncritical QCD strings in chapter 1. Here we make these more precise before proceeding with a further argument.

The boundaries of the world-sheet correspond to non-dynamical rigid Wilson loops. In this sense, the boundary conditions for X^μ should be Dirichlet boundary conditions. A nontrivial problem is how to choose the boundary conditions for φ.

We choose the topology of the classical world-sheet to be a cylinder, and we set $\hat{k} = 0$ for simplicity. This is reasonable for the calculation of the static quark-antiquark potential. We usually consider a rectangular Wilson loop of infinite length along the time direction to calculate it. However, $\hat{k}$ diverges at the corners of the rectangle, and this makes calculations difficult. To avoid this, we connect the shorter sides (the space-like sides) of the rectangle and thereby make it periodic, like a ring. This configuration consists of two parallel circular Wilson loops, and is like a cylinder. Then, the corners disappear, and the boundaries are straight from the two-dimensional viewpoint on the world-sheet. Then we can set $\hat{k} = 0$ naturally on the world-sheet. Of course, the color charges on the loops are set to make a color singlet. In the last step of the calculation, the periodicity of the loops is set to infinity, and thus the calculated value of the potential should attain the same value as that for an infinitely long rectangular Wilson loop.

Next, we make our problems clear. To go beyond the $d = 1$ barrier, we have mainly two problems to solve. First, we must stabilize the vacuum of the field φ, or fix the zero mode of φ to stabilize the classical configuration of the string. Second, we have to treat the condensation of the target space tachyons if they exist.

We propose some basic ideas to solve these problems in the following sections. We find that Dirichlet boundary conditions play important roles in the generalized Liouville theory.

3.1.1 Generalization of Liouville theory

We saw that the cosmological constant term becomes tachyonic for $d > 1$. Thus we cannot use it to stabilize φ.

One idea to stabilize φ without a cosmological constant is to add another Weyl invariant term to the action, which generates the minimum of $V(\varphi)$. We point out that one of the simplest candidates for such a term is

$$\frac{\mu'}{4\pi\alpha'}\int d^2\xi\, e^{2Q\varphi}\hat{g}^{ab}\partial_a X^\mu \partial_b X_\mu, \tag{3.1.1}$$

where μ' in (3.1.1) is a constant. This is because one can verify that the operator

$$e^{2Q\varphi}\hat{g}^{ab}\partial_a X^\mu \partial_b X_\mu \tag{3.1.2}$$

has conformal dimension 2, namely $\Delta(e^{2Q\varphi}) = 0$ from (2.1.23). Then we obtain a new action with (3.1.1):

$$S = \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{\hat{g}}\{(1+\mu' e^{2Q\varphi})\hat{g}^{ab}\partial_a X^\mu \partial_b X_\mu + \hat{g}^{ab}\partial_a\varphi\partial_b\varphi + \alpha' Q\hat{R}\varphi\} + S_{\rm ghost}. \tag{3.1.3}$$

However, one finds that the above action is not exactly Weyl invariant.[1]

Let us consider the most general action for Liouville theory. We naturally impose $SO(d)$ symmetry on d-dimensional spacetime, and we then obtain

$$S = \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{\hat{g}}\{a^2(\varphi)(\partial X^\mu)^2 + (\partial\varphi)^2 + \alpha' R^{(2)}\Phi(\varphi)\}, \tag{3.1.4}$$

where we have fixed the $\varphi\varphi$-component of the target-space metric to be 1, with appropriate redefinition (or target space general coordinate transformation) of φ. Now the cosmological constant term is assumed to be zero. (We have suppressed the ghost term here. Also, note that we have dropped the boundary terms, since we set $\hat{k} = 0$.)

The β-functions which should vanish for Weyl invariance are

$$\beta^G_{MN} = \alpha'(\mathcal{R}_{MN} + 2\nabla_M\nabla_N\Phi) + O(\alpha'^2), \tag{3.1.5}$$

$$\beta^\Phi = \alpha'\left(-Q^2 - \frac{1}{2}\nabla^2\Phi + (\nabla\Phi)^2\right) + O(\alpha'^2), \tag{3.1.6}$$

[1] Although the composite operator (3.1.2) is a (1,1)primary operator, multiple insertion of them creates another divergence, which causes breakdown of the scale invariance. Thus the insertion of the operator (3.1.2) into the action breaks Weyl invariance.

In the case of a cosmological constant term, the multiple insertions do not give any divergence.

where M and N run from 1 to $d+1$, and $X^{d+1} = \varphi$. $\mathcal{R}_{MN}$ is the Ricci tensor of the $d+1$-dimensional target space.

Collecting the above results, the equation of motion up to order α'^2 become

$$0 = \mathcal{R}_{MN} + 2\nabla_M \nabla_N \Phi, \tag{3.1.7}$$

$$0 = -Q^2 - \frac{1}{2}\nabla^2\Phi + (\nabla\Phi)^2, \tag{3.1.8}$$

where Q is given by (2.1.19). We stress that we can use these equations only in the region $\alpha'\mathcal{R} \ll 1$, where the perturbative approximation for the β-functions is valid.

The solution of these equations is given in Ref. [47] as

$$a(\varphi) = a_0\sqrt{\frac{1+\lambda e^{2Q\varphi}}{1-\lambda e^{2Q\varphi}}}\,, \tag{3.1.9}$$

where λ and a_0 are constants, and

$$\Phi = Q\varphi - \frac{3}{2}\log(1-\lambda e^{2Q\varphi}) + \frac{1}{2}\log(1+\lambda e^{2Q\varphi}) + \text{constant}\,. \tag{3.1.10}$$

The allowed region for φ is given by $|\lambda e^{2Q\varphi}| < 1$, or

$$\varphi < -\frac{1}{2Q}\log|\lambda|. \tag{3.1.11}$$

We have to check the validity of this solution before further calculations. The target space scalar curvature for the solution (3.1.9) is given by

$$\begin{aligned}
\mathcal{R} &= -3\left(\frac{a'}{a}\right)^2 - 5\frac{a''}{a} \\
&= \frac{-4Q^2 b}{(1-b^2)^2}(5b^2+8b+5) \longrightarrow \begin{cases} -\infty & (\lambda > 0) \\ 0 & (\lambda = 0) \\ +\infty & (\lambda < 0) \end{cases} \text{as } |b| \to 1,
\end{aligned} \tag{3.1.12}$$

where $b = \lambda e^{2Q\varphi}$ and a' denotes $\frac{\partial a}{\partial \varphi}$. Therefore, the solution is valid only in the region satisfying $|b| \ll 1$, or equivalently

$$\varphi \ll -\frac{1}{2Q} \log |\lambda| , \tag{3.1.13}$$

if $\lambda \neq 0$. In the case $\lambda = 0$, the solution is exactly the same as the "linear dilaton string" considered for $d \leq 1$. We investigate the $\lambda \neq 0$ solution here. In the region satisfying (3.1.13), the solution is expanded as

$$a^2(\varphi) = a_0^2\{1 + 2\lambda e^{2Q\varphi}\} + O(b^2), \tag{3.1.14}$$

$$\Phi(\varphi) = Q\varphi + 2\lambda e^{2Q\varphi} + O(b^2). \tag{3.1.15}$$

We find that the above solution is consistent with the proposed action given in (3.1.3), if we set $\mu' = 2\lambda$ and rescale X in (3.1.4) to $\frac{X}{a_0}$.

Although the best method is to find a solution for the exact equation of motion, this seems to be very difficult. Thus we consider the physics only in the region defined by (3.1.13), where the perturbation with respect to α' is valid, and we use the action

$$\begin{aligned} S &= \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{\hat{g}} \{(1 + 2\lambda e^{2Q\varphi})\hat{g}^{ab}\partial_a X^\mu \partial_b X_\mu + \hat{g}^{ab}\partial_a\varphi\partial_b\varphi + \alpha'\hat{R}(Q\varphi + 2\lambda e^{2Q\varphi})\} \\ &\quad + S_{\text{ghost}} \end{aligned} \tag{3.1.16}$$

in this region.

Chapter 4

Stabilization of the vacuum of Liouville mode φ

In this chapter, we attempt to stabilize the vacuum for φ in the action (3.1.16). We have two strategies. One is the standard stabilization using the minimum of the potential. However, we find that the potential in which we are interested has no minimum in some cases. The other one is to impose Dirichlet boundary conditions for φ and fix its zero mode. To use this method, we have to check the consistency of the Dirichlet boundary conditions and Weyl invariance.

4.1 Stabilization using the potential minimum

In the case that Dirichlet boundary conditions are not imposed, the zero mode of φ must be stabilized at the minimum point of the potential.

The vacua of X^μ are stable, and only the stabilization of φ is needed in our model. Thus, let us consider the effective action which we obtain after the path integration with respect to X^μ,

$$S(\varphi) = \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{\hat{g}}\{(1+2\lambda e^{2Q\varphi})\langle\hat{g}^{ab}\partial_a X^\mu\partial_b X_\mu\rangle+\hat{g}^{ab}\partial_a\varphi\partial_b\varphi+\alpha'\hat{R}(Q\varphi+2\lambda e^{2Q\varphi})\}, \tag{4.1.1}$$

where $\langle\mathcal{O}\rangle$ denotes the expectation value of the operator $\mathcal{O}$ obtained by the path integration with respect to X^μ. The equation of motion with respect to the zero mode of φ is given by

$$\delta S = \delta\varphi_c\int d^2\xi\sqrt{\hat{g}}v'(\varphi_c) = 0, \tag{4.1.2}$$

where φ_c is the zero mode of φ, and $v(\varphi_c)$ is expressed by

$$v(\varphi_c) = \alpha'\hat{R}Q\varphi + 2\lambda e^{2Q\varphi_c}(\alpha'\hat{R} + \langle\hat{g}^{ab}\partial_a X^\mu\partial_b X_\mu\rangle). \tag{4.1.3}$$

The Gauss-Bonnet theorem states that

$$\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\hat{R} = 4\pi\chi\,, \tag{4.1.4}$$

where χ is the Euler number of the world-sheet $\mathcal{M}$. It is given by $\chi = 2 - 2N_g - N_b$, where N_g is the number of the genus of $\mathcal{M}$, and N_b is the number of boundaries of $\mathcal{M}$. Then the equation of motion for φ_c is

$$4\pi\alpha' Q\chi + 4\lambda Q e^{2Q\varphi_c}(4\pi\alpha'\chi + A) = 0, \tag{4.1.5}$$

where

$$A = \int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\langle\hat{g}^{ab}\partial_a X^\mu\partial_b X_\mu\rangle \quad > 0. \tag{4.1.6}$$

Since $\lambda \neq 0$ and $Q \neq 0$, we obtain

$$e^{2Q\varphi_c} = -\frac{4\pi\alpha'\chi}{4\lambda(4\pi\alpha'\chi + A)} \quad > 0, \tag{4.1.7}$$

and we immediately note that we have no solution for $\chi = 0$. We also obtain

$$v''(\varphi_c) = -8\pi\alpha' Q^2 \chi \qquad (4.1.8)$$

from (4.1.3) and (4.1.5), and we have no stable vacuum for $\chi > 0$.

Note that the leading string diagram in our model is a cylinder, for which we have $\chi = 0$. In general, the leading term can be a disk ($\chi = 1$) or a cylinder in the calculation of the correlation functions of Wilson loops. Thus, unfortunately, we do not have a stable vacuum suitable for these $\chi \geq 0$ cases.[1]

4.2 The Weyl invariant Dirichlet boundary conditions and stabilization of the vacuum of φ

We have seen in our model that we cannot make a consistent potential for φ with disk and cylinder diagrams. However, if we impose Dirichlet boundary conditions and fix the zero mode φ_c, we are free of this difficulty.

The problem in this case is the compatibility of the Dirichlet boundary conditions and the Weyl invariance of the world-sheet. In a linear dilaton string, Dirichlet boundary conditions break Weyl invariance [46]. Therefore, we have to find a method to introduce Dirichlet boundary conditions into our theory without breaking Weyl invariance.

A good place to start is to recall the origin of the Weyl-invariance breaking in a general dilatonic string with Dirichlet boundary conditions. In dilatonic string theory, we usually need a freedom of field redefinition to preserve Weyl invariance. For example, we have to make a constant shift (a field

[1] We have the same problem for $\chi \geq 0$ in DDK. However, in the calculation of string susceptibility, insertion of a δ-function into the path integral, which keeps the world-sheet area constant, allows us to obtain the correct value for $d \leq 1$.

redefinition) of φ, are in (2.1.15), to cancel the variation caused by the Weyl transformation (2.1.14) for $d \leq 1$. Dirichlet boundary conditions for φ forbids such a shift at the boundary, and breaks Weyl invariance. On the other hand, Dirichlet boundary conditions for X^μ do not break Weyl invariance, because no shift of X^μ is needed.

However, **we point out that we can employ Dirichlet boundary conditions for φ in a general dilatonic string if the criterion stated below is satisfied.** In the theory with a dilaton, the required shift of the field is not a constant in general. For example, the shifts we need for the fields in (3.1.4) at the one-loop level are given as

$$\delta\varphi = -\frac{\alpha'}{2}\partial_\varphi\Phi(\varphi)\delta\sigma, \tag{4.2.9}$$

$$\delta X^\mu = 0, \tag{4.2.10}$$

where ∂_φ stands for $\frac{\partial}{\partial\varphi}$.[2] We can check that (4.2.9) gives the correct constant shift required in the linear dilaton case (2.1.15) if we set $q = 2$.[3] We look deeper into the origin of the field redefinition in the Appendix.

We note that we do *not* need field redefinition for φ if $\partial_\varphi\Phi = 0$. Therefore, we can use the Dirichlet boundary conditions

$$\varphi|_{\partial\mathcal{M}} = \varphi_0\,, \tag{4.2.11}$$

where

$$\partial_\varphi\Phi(\varphi)|_{\varphi=\varphi_0} = 0, \tag{4.2.12}$$

[2]The required field redefinition to preserve Weyl invariance appears in articles which discuss the β-functions for non-linear sigma models. For the models with boundaries, see Refs. [48] and [49]. The condition (4.2.9) for the shift is also found in Ref. [46].

[3]If $q = 2$, we get $u = \frac{1}{2}\frac{d-25}{48\pi}$. The denominator naturally coincides with the denominator of the factor in (2.1.8). In this linear dilaton case, the Weyl anomaly is obtained exactly at the one-loop level.

without breaking Weyl invariance at the one-loop level. The above stated criterion for consistent Dirichlet boundary conditions is one of the most important assertions of this thesis.

As a next step, let us examine whether our model (3.1.16) has such a φ_0 or not. Our dilaton term is

$$\Phi(\varphi) = Q\varphi + 2\lambda e^{2Q\varphi} + O(b^2). \tag{4.2.13}$$

Thus the condition (4.2.12) is

$$\partial_\varphi \Phi = Q + 4\lambda Q e^{2Q\varphi} + O(b^2) = 0, \tag{4.2.14}$$

and we obtain

$$\varphi_0 = -\frac{1}{2Q}\log(-4\lambda) + O(b^2) \tag{4.2.15}$$

for $\lambda < 0$. The condition (4.2.14) is valid only in the region satisfying $|b| = |\lambda e^{2Q\varphi}| \ll 1$, and now $\lambda e^{2Q\varphi_0} = -1/4$. Thus the above result suggests that we have an appropriate point φ_0 for the Dirichlet boundary conditions (4.2.11) if $\lambda < 0$.

For this reason, we choose $\lambda < 0$ and impose Dirichlet boundary conditions on the ends of the string to stabilize its configuration for arbitrary χ. Note that the above argument naturally selects the branch of λ uniquely.

4.3 Consistency with black p-brane solutions

In this section, we check the consistency of the statement in the previous section from the viewpoint of black p-brane solutions in supergravity.

Black p-branes in supergravity theories are interpreted as Dp-branes in superstring theories. Generally, black p-brane solutions contain a dilaton field which depends on the position in target space. Black p-brane solutions are given as follows [11],

$$\begin{aligned}
ds^2 &= H(r)^{-\frac{1}{2}}(-dt^2 + dx_{\|}^2) + H(r)^{\frac{1}{2}}(dr^2 + r^2 d\Omega^2) \\
e^{\Phi(r)} &= g_\infty H(r)^{\frac{3-p}{4}} \\
H(r) &= 1 + \left(\frac{R}{r}\right)^{7-p} \\
R^{7-p} &= c_p g_\infty N l_s^{7-p} \\
c_p &= 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right),
\end{aligned} \tag{4.3.16}$$

where $l_s^2 = \alpha'$, r is the isotropic radial coordinate on the transverse space, $x_{\|}$ denotes the spatial coordinates along the brane, and g_∞ is open string coupling constant at $r = \infty$.

In the case of $p \neq 3$, the dilaton has nontrivial dependence on r. Therefore, in the string theory side, the position where branes can exist should be restricted by the consistency condition for Weyl invariance on the worldsheet, as discussed in the previous section. Therefore, we will check how the consistency condition works in the black p-brane solutions (4.3.16).

The consistency condition for Weyl invariance here is $\nabla^r\Phi(r) = 0$, and it can be written as

$$\begin{aligned}
\nabla^r\Phi(r) &= G^{rm}\nabla_m\Phi(r) \\
&= G^{rr}\partial_r\Phi(r) \\
&= H^{-\frac{1}{2}}\partial_r\Phi(r) \\
&= \frac{3-p}{4}H(r)^{-\frac{3}{2}}(p-7)\frac{R^{7-p}}{r^{8-p}} = 0.
\end{aligned} \tag{4.3.17}$$

Let us take the limit $r \to 0$ for (4.3.17). After short calculations, we obtain

$$\nabla^r\Phi(r) \longrightarrow \begin{cases} \infty & (p=6) \\ 0 & (p \neq 0) \end{cases} \quad \text{as } |r| \to 0\,. \tag{4.3.18}$$

Therefore, the left-hand-side of (4.3.17) diverges near the brane position $r = 0$ and the condition does not seem to be satisfied in the case of $p = 6$, while the condition (4.3.17) is satisfied at $r = 0$ in the case of $p \neq 6$.

However, the above divergence does not mean the contradiction between the consistency condition and the black p-brane solutions. When we derive the consistency condition, we performed perturbative calculations in the non-linear sigma model on the world-sheet. The perturbative method is only valid when $\alpha' \mathcal{R} \ll 1$, where $\mathcal{R}$ is the scalar curvature of the target space. The behavior of $\mathcal{R}$ near $r = 0$ in the solutions (4.3.16) is as follows

$$\begin{aligned} \mathcal{R} &= -\frac{1}{4}(p^2 - 4p - 17)(\partial_r H)^2 H^{-\frac{5}{2}} \\ &\sim r^{\frac{3-p}{2}} \longrightarrow \begin{cases} \infty & (p > 3) \\ \text{const.} & (p = 3) \\ 0 & (p < 3) \end{cases} \quad \text{as } |r| \to 0 \,. \end{aligned} \qquad (4.3.19)$$

Therefore, our perturbative analysis for the condition (4.3.17) is not applicable to the case of $p > 3$, and the divergence in (4.3.18) in the $p = 6$ case does not mean the contradiction to our result. On the other hand, the condition (4.3.17) is satisfied in the region $p \leq 3$ where perturbative analysis is valid.

Thus, our criterion for Dirichlet boundary conditions maintaining Weyl invariance is consistent, and has no contradiction to black p-brane solutions.

Chapter 5

Tachyon condensation: another role of Dirichlet boundary conditions in Liouville theory

5.1 Tachyon condensation with Dirichlet boundary conditions

We found that we can use Dirichlet boundary conditions to stabilize the string configuration, while preserving Weyl invariance for arbitrary χ, at least up to $O(\alpha'^2)$.

However, we have another big problem: If the target-space dimension ($d+1$ for our model) is greater than 2, we cannot fix all of the freedom of the world-sheet oscillation with gauge symmetry, and we have physical oscillation. It is well known that we have a tachyonic oscillation in the bosonic string in flat spacetime. In our model (3.1.16), the target space becomes asymptotically flat in the region $\varphi \ll 0$. Now we are considering the case $d > 1$. Thus a tachyonic ground state appears at least in the region in which the spacetime is almost flat. If we have a tachyonic mode, we have to handle tachyon condensation.

Tachyon condensation has been discussed by many authors (for example, see Ref. [51][1]), but this subject is difficult and has not yet been solved completely. We present some ideas to treat tachyon condensation here [52]. We stress that Dirichlet boundary conditions play an important role in this subsection too.

First, let us recall field condensation in usual quantum field theories. In field theory with field condensation (that is, in the theory with nonzero expectation value of the field), we can calculate correct quantities if we know the correct expectation value of the field, even with the perturbation around an incorrect vacuum. For example, we can calculate the exact propagator with the perturbation around an incorrect vacuum by attaching tadpole diagrams to the tree propagator. Even though the mass squared is negative in a description around such a vacuum, tadpoles with appropriate weight create an additional shift of the mass squared, and make the total mass squared positive. In such a case, although the tachyonic mode exists in a perturbative theory around an incorrect vacuum, the theory is never wrong, and only the "vacuum" is wrong. In field theories, the true vacuum or exact expectation value of the field can be given by the Schwinger-Dyson equation. Thus we can get the correct weight of the tadpoles, we can calculate the correct propagator, and so on.

We now make an analogy between field theories and tachyonic string theories. Although we have a tachyonic mode, we believe that the string theory is not fatally flawed, and the problem is that we do not know the true vacuum of it. The analogy with field theories tells us that we may be able to obtain correct quantities if we attach correct "tadpoles" to the world-sheet.

[1]See also section 5.2 in this thesis.

Then, the question is what is the "tadpole" in string theories.

We guess here that the tadpole in string theories is a macroscopic hole with Dirichlet boundary conditions in the world-sheet. We assume that the Dirichlet boundary condition for them is

$$X^{\mu}(\xi_i) = a_i^{\mu}, \tag{5.1.1}$$

$$\varphi(\xi_i) = \varphi_0 , \tag{5.1.2}$$

where i distinguishes each tadpole, a_i^{μ} is a constant, and φ_0 is a constant which satisfies the condition (4.2.12). The translational invariance in d-dimensional spacetime is recovered after the integration over the moduli space which is the region where "holes" can exist in the spacetime. Of course, we have to consider the proper weights of the string wave functions on it. The above assumption results from the following considerations.

The tachyonic tadpole is an off-shell state, because it does not carry momentum. We also know that off-shell states in string theory do not correspond to local emission vertexes. Thus we naturally assume that the tachyonic tadpole is a non-local macroscopic hole on the world-sheet.[2] We also have to preserve Weyl invariance, and it is natural to impose the above Dirichlet boundary conditions (5.1.1) and (5.1.2) on the edge of the hole. Note that the above Dirichlet boundary conditions map the macroscopic hole on the world-sheet into a single point in the target space, and the "holes" are invisible in the target space. We set boundary conditions like this because visible "holes" seem to be unphysical. Neumann boundary conditions cannot be taken for a tadpole for the following reason. If we impose Neumann boundary conditions at a hole, the value of X^{μ} changes along the edge of the

[2] Some argument for the macroscopic hole as a tachyonic state is given in Ref. [44].

hole. This means that we observe a macroscopic hole even in d-dimensional spacetime. Although the Neumann boundary conditions for φ do not make a macroscopic hole in the visible d-dimensional spacetime, the hole can have momentum in the φ direction. This allows us to make an on-shell state, and the hole can break into on-shell open strings moving along the φ direction. These situations do not seem to be natural for our model. Contrastingly, the tadpole with the Dirichlet boundary conditions (5.1.1) and (5.1.2) does not leak any momentum from the world-sheet, and this gives a natural property for the tadpole.

The macroscopic holes with Dirichlet boundary conditions on the world-sheet (or D-instantons in the target space) and the non-perturbative effects induced by them have already discussed in Refs. [53, 54, 55] and Ref. [46] for critical strings. However, we insist that the macroscopic holes discussed here play the role of "tadpoles" naturally even in Liouville theory.

Unfortunately, we do not have the Schwinger-Dyson equation of string theory, and we do not know how to obtain the correct weight which should be attached to the tadpole. Thus, we cannot give a rigorous discussion to treat tachyon condensation, but we present a rough argument regarding tachyon condensation.

To treat a macroscopic hole on the world-sheet is rather difficult, and we therefore approximate it as a point on the world-sheet which couples to the Dirichlet boundary conditions. In this case, the insertion of the tadpole is regarded as the insertion of

$$h \int_{\mathcal{M}} d^2\xi_1 \sqrt{g}\, \delta(X^M(\xi_1) - a^M(\xi_1)) \tag{5.1.3}$$

into the world-sheet, where h is the weight of the tadpole, and ξ_1 denotes the insertion *point* on the world-sheet. We have $X^{d+1} = \varphi$ and $a^{d+1} = \varphi_0$. In usual strings without a dilaton, the δ-function in (5.1.3) becomes 1 after the integration over the moduli a^M, and the string propagator with the tadpoles can be estimated as

$$\begin{aligned}\text{propagator} &= \frac{1}{L_0+\overline{L_0}} + \frac{1}{L_0+\overline{L_0}}h\frac{1}{L_0+\overline{L_0}} + \frac{1}{L_0+\overline{L_0}}h\frac{1}{L_0+\overline{L_0}}h\frac{1}{L_0+\overline{L_0}} + \cdots \\ &= \frac{1}{L_0+\overline{L_0}-h}, \end{aligned} \tag{5.1.4}$$

where L_0 $(\overline{L}_0)$ is the (anti)holomorphic part of the Hamiltonian of the corresponding conformal field theory. Thus, the insertion of the tadpoles (5.1.3) seems to make an additional shift to the energy of the tachyonic state.

However, we cannot apply the above estimation directly to Liouville theory. Although we should integrate over a^μ to get Poincaré invariance in the d-dimensional spacetime, we never integrate over φ_0 in our model (because it is fixed). Therefore, the expected non-perturbative effects induced by the tadpoles in Liouville theory seem to be different from those of strings with a constant dilaton. strings. In any case, we must develop a technique to estimate the effects of the insertion of Dirichlet boundaries.

5.2 Some comments on tachyon condensation in critical strings

In the previous section, we have presented a basic strategy to handle tachyon condensation in the generalized Liouville theory. However, we need further consideration to reach exact treatment.

In this section, we make a survey of tachyon condensation of critical strings. Consideration of tachyon condensation in critical string theory is as important as that in Liouville theory. Although we do not yet understand the relation between tachyon condensation of critical strings and that of noncritical strings, consideration of critical case seems to be significant, even for our purpose here. It might give us some important suggestions for construction of Liouville theory.

In this section, we briefly review the recent developments on tachyon condensation for critical bosonic open strings. We also comment on some new results obtained by the author.

5.2.1 Brief review of tachyon condensation for critical bosonic open strings

Sen presented several important conjectures on tachyon condensation of open bosonic strings [60, 61, 62, 63]. It was conjectured that arbitrary-dimensional bosonic D-brane can decay into the open string vacuum or lower-dimensional D-brane. Moreover, the vacuum energy of the bosonic D-brane is considered to correspond to the tension of the D-brane.

Old-days calculation [64] in the open string field theory [65] has been renewed to discuss the tachyon condensation [66, 67, 68, 69, 70, 71, 72, 73]. Since in these cases all scalar quantities may acquire the vacuum expectation value, we can only analyze the tachyon condensation by truncating the infinite levels of string excitations. Some attempts for the exact manipulation are found in [74, 75, 76, 77].

However, this difficulty is overcome recently [78, 79, 80]. Using another formulation called boundary string field theory (BSFT) [81, 82, 83, 84, 85], we have only to consider the tachyon field in the discussion of the tachyon condensation. This is because the general property of the renormalization group flow ensures that the quadratic modes of the tachyon field decouple from the other modes. Exact analysis was performed in this formulation. Both the D25-brane's decay into the open string vacuum and the D25-brane's decay into lower-dimensional brane were analyzed, and Sen's conjectures were confirmed exactly. As a result, the derivative truncated tachyon potential was found to agree with the toy model proposed in [86, 87]. Several related works are also found in [88, 89, 90, 91, 92, 93].

5.2.2 Some new results for open-string tachyon condensation

Moriyama and the author have obtained some new results on the tachyon condensation for critical bosonic open strings, recently [94][3].

We have considered the decaying processes of D25-brane into lower-dimensional D-brane, and have derived the descent relation of effective tachyon potential of bosonic open strings, in the framework of BSFT. We have also made field-theoretical analysis, and have calculated the effective tachyon potential. We have shown that the effective tachyon potential on lower-dimensional D-branes has the same profile as that on D25-brane. Namely, there exist self-similarity in the effective tachyon potential on arbitrary-dimensional D-branes in bosonic strings.

The details for this work is presented in Ref. [94].

5.2.3 Toward closed-string tachyon condensation

Although our understanding on tachyon condensation in critical open bosonic string has been deepened through the recent works related to decay of D-branes, closed-string tachyon condensation is still unknown topics even in the critical dimension. When open-string tachyons condense, open-string sector disappears within the treatment mentioned in the previous subsections 5.2.1 and 5.2.2, which does not include consideration of closed-string

[3] It is submitted to GUAS with this thesis.

sector. However, closed-string tachyons still remain open-string vacuum, and we have to treat closed-string tachyon condensation to reach fully stable vacuum. Therefore, to consider closed-string tachyon condensation is extremely important.

The basic strategy to treat tachyon condensation presented in section 5.1 has been applied to closed-string tachyons in Liouville theory, although exact analysis has not yet been performed. To treat tachyon condensation with "tadpoles" (macroscopic holes on world-sheets with Dirichlet boundary conditions) can be one of the directions which solve problems related on closed-string tachyons, both for critical and noncritical cases.

As a matter of fact, Green has already tried to insert such macroscopic holes into world-sheets of critical strings to obtain a modified QCD string theory which has properties similar to those in YM theories. For example, point-like states of the Dirichlet boundaries were shown to produce power-like behavior of fixed angle scattering at high energy [53, 54, 55]. It was also pointed out in Ref. [56] that at high temperature the Dirichlet boundaries make a string free energy similar to that of large-N YM theory found by Polchinski [57]. Furthermore, we find an argument that logarithmic dependence of physical quantities on energy scale is produced by the Dirichlet boundaries in calculations of critical strings [58], which might lead us to a suitable QCD string model that reproduces correct asymptotic-freedom behavior of YM theories.

The results obtained there seem to be indications that we are in the correct way toward a description of closed strings at stable vacuum.

Chapter 6

Conclusion and discussions

6.1 Conclusion

We attempted to quantize a noncritical (four dimensional) bosonic string as a natural candidate of a (large-N) pure QCD string. We considered the generalized Liouville action (3.1.4) as such a string.

One of the main problems here is the stabilization of the Liouville mode φ while preserving Weyl invariance, and we found that we can stabilize it with the Dirichlet boundary conditions (4.2.11). The criterion for consistent Dirichlet boundary conditions at the one-loop level is given by (4.2.12), and the stabilized value of φ is independent on the topology of the world-sheet in this method. We also analyzed the perturbative solutions of the equation of motion for the backgrounds. It was shown that we have a suitable solution, at the one-loop level, which satisfies the criterion. Furthermore, this argument leads us to the unique selection of the branch of the solutions; although we have several branches of the solutions, we can select the unique branch among them by examining whether it allows consistent Dirichlet boundary conditions or not.

We also discussed tachyon condensation. Although the complete treatment of it is very difficult, we presented a simple strategy for it. The idea we presented is to attach "tadpoles" to the world-sheet. We surmised that the "tadpole" in Liouville theory might be represented as a macroscopic hole with Dirichlet boundary conditions, where the condition for Liouville mode is restricted by the criterion (4.2.12). Although a similar proposal to alter string vacuum with D-instantons has already given for critical strings, we insist that we can also use the above method for Liouville theory within the condition (4.2.12), and the introduction of D-instanton-like tadpoles does not break our assumptions and presuppositions presented in chapter 1. Furthermore, we guess that the non-perturbative effects in Liouville theory are different from those of critical strings. This is because the moduli space, namely the regions for the target-space coordinates of the tadpoles which should be integrated over, is different from those for strings with a constant dilaton.

As further directions, the effects of the insertion of Dirichlet boundaries into world-sheets should be investigated more, even for critical cases. They are closely related to closed-string tachyon condensation. Studies on critical-string vacua should give us some important suggestions for Liouville theory as a QCD string. The relation between open-string tachyon condensation and closed-string tachyon condensation should be studied further, too. Consideration of open-string "tadpoles" would be also fruitful if we compare the results with those from recent analyses of unstable D-branes.

Before we close this thesis, we stress again that Dirichlet boundary conditions have important roles in the generalized Liouville theory, and they can be imposed on the Liouville mode while preserving Weyl invariance if the appropriate condition mentioned above is satisfied. The investigation of

Dirichlet strings in dilatonic backgrounds is very important, and it should yield necessary information about the construction of noncritical strings.

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Appendix A

The Origin of the Field Redefinition and the Derivation of (4.2.9)

Here we look deeper into the origin of the necessity of the field redefinition, at the one-loop level [namely, up to $O(\alpha'^2)$]. Let us consider a general string action in $d+1$ dimensions,

$$\begin{aligned} S &= \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\{G_{MN}^{bare}(\boldsymbol{X})\hat{g}^{ab}\partial_a X^M\partial_b X^N + \alpha'\hat{R}\Phi^{bare}(\boldsymbol{X})\} \\ &\quad +\frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\alpha'\hat{k}\Phi^{bare}(\boldsymbol{X}(s))\,, \end{aligned} \tag{A.0.1}$$

where we have included the boundary term, as it is needed in following calculation.[1] The couplings with the superscript "$bare$" are the bare quantities, while those without such a superscript denote the renormalized quantities, in this appendix. The renormalized action with dimensional regularization up to two loops is expressed as

$$S = \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^{2+\epsilon}\xi\sqrt{\hat{g}}\left(G_{MN}^{bare}(\boldsymbol{X}) + \alpha'\sum_{n=1}^{\infty}\frac{1}{\epsilon^n}C_{MN}^{(n)}(G_{MN},\Phi,\boldsymbol{X})\right)\hat{g}^{ab}\partial_a X^M\partial_b X^N$$

[1] For the non-linear sigma model with boundaries, see Refs. [48] and [49].

$$
\begin{aligned}
&+\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^{2+\epsilon}\xi\sqrt{\hat{g}}\alpha'\hat{R}\left(\Phi^{bare}(\boldsymbol{X})+\sum_{n=1}^{\infty}\frac{1}{\epsilon^n}C_{\Phi}^{(n)}(G_{MN},\Phi,\boldsymbol{X})\right)\\
&+\frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} d^{1+\epsilon}s\sqrt{\hat{g}_{ss}}\alpha'\hat{k}\left(\Phi^{bare}(\boldsymbol{X}(s))+\sum_{n=1}^{\infty}\frac{1}{\epsilon^n}\tilde{C}_{\Phi}^{(n)}(G_{MN},\Phi,\boldsymbol{X}(s))\right),
\end{aligned}
\tag{A.0.2}
$$

where the symmetric tensor $\sum_{n=1}^{\infty}\frac{1}{\epsilon^n}C_{MN}^{(n)}$ is the counterterm of the kinetic term, and the scalar $\sum_{n=1}^{\infty}\frac{1}{\epsilon^n}C_{\Phi}^{(n)}$ ($\sum_{n=1}^{\infty}\frac{1}{\epsilon^n}\tilde{C}_{\Phi}^{(n)}$) is the counterterm of the dilaton term in $\mathcal{M}$ ($\partial\mathcal{M}$). If we perform the Weyl transformation $\hat{g}_{ab}\mapsto\hat{g}_{ab}e^{\delta\sigma}$ and take the limit of $\epsilon\to 0$, the finite variation of the action at $O(\delta\sigma)$ is

$$
\begin{aligned}
\delta_\sigma S &= \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\alpha'\left\{\frac{1}{2}C_{MN}^{(1)}+\Phi\frac{\partial C_{MN}^{(1)}}{\partial\Phi}+G_{KL}\frac{\partial C_{MN}^{(1)}}{\partial G_{KL}}\right\}\hat{g}^{ab}\partial_a X^M\partial_b X^N\delta\sigma\\
&\quad+\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\alpha'\hat{R}\left\{\frac{1}{2}C_{\Phi}^{(1)}+\Phi\frac{\partial C_{\Phi}^{(1)}}{\partial\Phi}+G_{KL}\frac{\partial C_{\Phi}^{(1)}}{\partial G_{KL}}\right\}\delta\sigma\\
&\quad-\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\alpha'\left(\Phi(\boldsymbol{X})+C_{\Phi}^{(1)}\right)\nabla^2(\delta\sigma)\\
&\quad+\frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\alpha'\hat{k}\left\{\frac{1}{2}\tilde{C}_{\Phi}^{(1)}+\Phi\frac{\partial\tilde{C}_{\Phi}^{(1)}}{\partial\Phi}+G_{KL}\frac{\partial\tilde{C}_{\Phi}^{(1)}}{\partial G_{KL}}\right\}\delta\sigma\\
&\quad+\frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\alpha'\frac{1}{2}(\hat{n}^a\partial_a(\delta\sigma))\left(\Phi(\boldsymbol{X}(s))+\tilde{C}_{\Phi}^{(1)}\right),
\end{aligned}
\tag{A.0.3}
$$

where $\hat{n}^a$ is the unit outward normal vector, and K, L, M and N are the indices of the coordinates of spacetime.[2] The term which contains $\nabla^2(\delta\sigma)$ in (A.0.3) is rewritten as

$$
\begin{aligned}
&-\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\hat{g}^{ab}\alpha'\left\{\nabla_a\nabla_b X^M\nabla_M\left(\Phi(\boldsymbol{X})+C_{\Phi}^{(1)}\right)\right.\\
&\qquad\qquad\qquad\qquad\left.+\ \partial_a X^M\partial_b X^N\nabla_M\nabla_N\Phi(\boldsymbol{X})\right\}\delta\sigma\\
&-\frac{1}{4\pi\alpha'}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\alpha'(\hat{n}^a\partial_a(\delta\sigma))\left(\Phi(\boldsymbol{X}(s))+C_{\Phi}^{(1)}\right)
\end{aligned}
$$

[2]The divergent terms of $O(1/\epsilon)$ are set to zero by pole equation [50].

$$+\quad \frac{1}{4\pi\alpha'}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\alpha'\hat{n}^a\partial_a X^M\nabla_M\left(\Phi(\boldsymbol{X}(s))+C^{(1)}_\Phi\right)\delta\sigma\,, \qquad \text{(A.0.4)}$$

where ∂_M stands for $\frac{\partial}{\partial X^M}$.

Therefore we get

$$\begin{aligned}
\delta_\sigma S &= \frac{1}{4\pi}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\left\{\frac{1}{2}C^{(1)}_{MN}+\Phi\frac{\partial C^{(1)}_{MN}}{\partial\Phi}+G_{KL}\frac{\partial C^{(1)}_{MN}}{\partial G_{KL}}-\nabla_M\nabla_N\Phi(\boldsymbol{X})\right\}\hat{g}^{ab}\partial_a X^M\partial_b X^N\delta\sigma \\
&\quad+\frac{1}{4\pi}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\hat{R}\left\{\frac{1}{2}C^{(1)}_\Phi+\Phi\frac{\partial C^{(1)}_\Phi}{\partial\Phi}+G_{KL}\frac{\partial C^{(1)}_\Phi}{\partial G_{KL}}\right\}\delta\sigma \\
&\quad+\frac{1}{2\pi}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\hat{k}\left\{\frac{1}{2}\tilde{C}^{(1)}_\Phi+\Phi\frac{\partial\tilde{C}^{(1)}_\Phi}{\partial\Phi}+G_{KL}\frac{\partial\tilde{C}^{(1)}_\Phi}{\partial G_{KL}}\right\}\delta\sigma \\
&\quad+\frac{1}{4\pi}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}(\hat{n}^a\partial_a(\delta\sigma))\left(\tilde{C}^{(1)}_\Phi-C^{(1)}_\Phi\right) \\
&\quad-\frac{1}{4\pi}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\hat{g}^{ab}\nabla_a\nabla_b X^M\nabla_M\left(\Phi(\boldsymbol{X})+C^{(1)}_\Phi\right)\delta\sigma \\
&\quad+\frac{1}{4\pi}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}\hat{n}^a\partial_a X^M\nabla_M\left(\Phi(\boldsymbol{X}(s))+C^{(1)}_\Phi\right)\delta\sigma. \qquad \text{(A.0.5)}
\end{aligned}$$

The last three terms cannot be absorbed into any counterterm. The third term of (A.0.5) goes to zero if $\tilde{C}^{(1)}_\Phi = C^{(1)}_\Phi$. This is realized if all the β-functions, except for that of the dilaton, vanish [49]. The problem is to determine how to deal with the last two terms.

Fortunately, we can cancel them using field redefinition. If we perform a field redefinition $X^M \mapsto X^M + \delta X^M$, the variation of the action is

$$\begin{aligned}
\delta_X S &= \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\;2\hat{g}^{ab}\partial_a X^M\partial_b\delta X_M \\
&\quad+(\text{terms proportional to }\delta X_M) \\
&= -\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\xi\sqrt{\hat{g}}\;2\hat{g}^{ab}\nabla_a\nabla_b X^M\delta X_M \\
&\quad+\frac{1}{4\pi\alpha'}\int_{\partial\mathcal{M}} ds\sqrt{\hat{g}_{ss}}2\hat{n}^a\partial_a X^M\delta X_M \\
&\quad+(\text{terms proportional to }\delta X_M). \qquad \text{(A.0.6)}
\end{aligned}$$

Therefore, if we set

$$\delta X^M = -\frac{\alpha'}{2}\nabla^M(\Phi(\boldsymbol{X}) + C_\Phi^{(1)}(\boldsymbol{X}))\delta\sigma, \tag{A.0.7}$$

we can cancel the last two terms of (A.0.5). The remaining terms proportional to δX_M in (A.0.6) can be absorbed into the counterterms. Thus, after a proper field redefinition, $\delta_\sigma S + \delta_X S$ contains only terms proportional to the β-functions, and we can preserve Weyl invariance if we set each of the β-functions to zero. We know that the counterterm $C_\Phi^{(1)}$ at the one-loop level corresponds to the central charge, and is a constant. Thus the required shift of X^M is

$$\delta X^M = -\frac{\alpha'}{2}\nabla^M\Phi(\boldsymbol{X})\delta\sigma. \tag{A.0.8}$$

Now we emphasize the very important fact that we do *not* need field redefinition at the special point $\boldsymbol{X}_0$ where $\nabla^M\Phi(\boldsymbol{X}_0) = 0$.

In our model (3.1.16), G_{MN}, Φ and the counterterms depend only on $X^{d+1} = \varphi$, so that (A.0.8) can be written as

$$\delta\varphi = -\frac{\alpha'}{2}\nabla^\varphi\Phi(\varphi)\delta\sigma \tag{A.0.9}$$

$$\delta X^\mu = 0, \tag{A.0.10}$$

where μ runs from 1 to d and $\nabla^\varphi = \nabla^{d+1}$. Now $G^{\varphi\varphi} = 1$, and $\nabla^\varphi\Phi = \nabla_\varphi\Phi$. Thus we obtain (4.2.9).

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