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博士論文の要旨

氏　　名　小嶋　涼太

論文題目　Triangulation of the Amplituhedron from Sign Flips

The traditional formulation of quantum field theory (QFT) is constructed from the two principles: Locality and Unitarity. The standard calculations of the scattering amplitudes from lagrangians or path integrals make these two principles manifest. However, because of this, a large amount of unphysical redundancies (field redefinitions, gauge redundancies) are introduced.

In 1985, Park and Taylor discovered a surprisingly simple result for the tree scattering amplitude of six gluons by using the spinor-helicity variables. In the original Feynman diagram calculation, there are 220 Feynman diagrams and 100 pages of the calculation are needed. They also generalized this for n gluons tree amplitude and this result showed the same simplicity. This is totally hidden in the original Feynman diagram calculation. This means that the original formalism of QFT is completely hiding some properties of the physical observables we have never known. To reveal these hidden properties, new alternative methods for calculations both tree-level and loop amplitudes are proposed. In 2005 Britto, Cachazo, Feng, and Witten found new on-shell recursion relations (BCFW) for tree-level amplitudes of gluons. These recursion relations are derived from the contour integrations on the complex momentum and the factorization property which comes from the unitarity.

Recently many developments have been driven in the planar limit of the N = 4 Super Yang-Mills (SYM). It has been known that this theory has super conformal symmetry. In 2008, the hidden symmetry "Dual super conformal symmetry" is discovered, which is not manifest in the ordinal formulation and had not been found for long years. Combining these two symmetries, the infinite symmetry "Yangian symmetry" is constructed. The discovery of these hidden symmetries motivated to find new alternative representations that make all symmetries manifest. In 2009, the new variable "Momentum twistor variable" is proposed by Hodges. When scattering amplitudes are written in this variable, all symmetries become manifest. In 2010, the BCFW recursion relations are generalized into the integrand of loop amplitudes in planar N = 4 SYM by using this momentum twistor space. By using this, all loop integrands are systematically and efficiently constructed from lower-loop integrands recursively. In this BCFW representation, there are non-local spurious poles that have never appeared in the ordinal Feynman diagrams expansion. Instead of these spurious poles, this representation makes all symmetries of the amplitude manifest term-by-term, including the Yangian symmetry which is hidden in the original formulation.

The important progress is the discovery of the geometric structure of the scattering amplitudes. The first example of this geometric picture is found by Hodges. He found that the tree NMHV amplitude in this momentum twistor variable can be interpreted as a volume of a polytope and the BCFW recursion relation for the tree NMHV amplitude is interpreted as a triangulation of this polytope. In 2012, the connection between on-shell scattering amplitudes in planar N = 4 SYM and the fundamental object in algebraic geometry "Positive Grassmannian $G_+(k,n)$" is found. In 2013, Arkani-Hamed and Trnka generalized these geometric pictures and found a completely new geometric object: Amplituhedron. This is defined as a generalization of the positive Grassmannian, this means that in the definition of the amplituhedron is purely geometric. However, it is conjectured that the scattering amplitude (loop integrand) of planar N = 4 SYM at any loop order is given by a "canonical form" on the amplituhedron which has logarithmic singularities on all of its boundaries. The BCFW recursion relation of all loop integrands is interpreted as one of the triangulation of the amplituhedron. The remarkable point is that Unitarity and Locality of the scattering amplitudes are derived from the "positivity", which is a property of the amplituhedron. This is the first example of the connection between the physical observable and new geometric structure "positive geometry". Recently many example of this relation are found: the wavefunction of the universe and the cosmological polytope, tree amplitudes in the bi- adjoint φ^3 scalar theory and its generalization and the conformal bootstrap and the cyclic polytope.

The amplituhedron has been explored from a variety of perspectives in the past few years. The amplituhedron gives a geometric picture for tree amplitudes and loop integrands as a canonical form of the amplituhedron. Calculating amplitudes or loop integrands starting from the amplituhedron requires the construction of the canonical form associated to the geometry. To do this, we need to triangulate the amplituhedron into a more simple one that it is easy to obtain the form. However, already at tree level and much more at loop level, the geometry of the amplituhedron is highly non-trivial and because of this, it is difficult to triangulate general amplituhedron and it remains an open problem.

We focus on this problem and investigate the triangulation of the general amplituhedron by using the topological definition of the amplituhedron. Recently the topological definition of the amplituhedron is proposed. In this definition, the amplituhedron is defined from the boundary inequalities and the sign flip characterization. This new definition gives us a completely new and clear understanding of the geometry of the loop amplituhedron. For example, in the MHV case, the higher loop amplituhedron is decomposed into the one loop MHV amplituhedron and conditions of the positivity among the different loop momentum variables. This extremely simple picture of the loop MHV amplituhedron makes it easy to consider the triangulation of the amplituhedron. The topological definition of the N^kMHV loop amplituhedron is more interesting. For example, N^kMHV one loop amplituhedron is constructed as an intersection of the two lower-dimensional amplituhedra. This is not obvious from the original definition. The remarkable point is that we can triangulate the lower-dimensional amplituhedron by using this topological definition. These will lead us to a triangulation of the 1-loop NMHV amplituhedron.

In this thesis, we investigate the triangulation of the loop amplituhedron. First, we consider the 2-loop MHV amplituhedron. We see that the 2-loop MHV amplituhedron can be triangulated by using the topological definition. From this, we obtain the canonical form of the n-point 2-loop MHV amplituhedron. The representation of the 2-loop MHV integrand from this canonical form looks completely different from the BCFW representation which obtained from the planar N = 4 SYM. This is a new feature that starts from the 2-loop level, the 1-loop MHV case the canonical form obtained from the geometry is corresponding to the BCFW representation.

Next, we consider the 1-loop NMHV amplituhedron. We obtain an explicit representation of the n-point 1-loop NMHV amplituhedron as a product of two lower-dimensional amplituhedra by using the topological definition. This is a completely new representation that we have never known from the planar N = 4 SYM or BCFW triangulation. From this, we triangulate this 1-loop NMHV amplituhedron explicitly and obtain the canonical form. We see that this canonical form is expressed as a product of two canonical forms of the lower-dimensional amplituhedra. We will also give another new representation of the 1-loop NMHV amplituhedron, "super-local representation". The super-local means both of external poles and internal poles are local. In this representation, the positivity of this form is manifest term-by-term. The positivity of the canonical form is related to the existence of a "dual amplituhedron". This positivity suggests the existence of a dual amplituhedron for the 1-loop NMHV amplituhedron.

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Results of the doctoral thesis screening

博士論文審査結果

Name in Full
氏　名　小嶋　涼太

Title
論文題目　Triangulation of the Amplituhedron from Sign Flips

小嶋涼太氏の博士論文は、N=4 超対称ヤン・ミルズ理論のプラナー散乱振幅と関連する幾何学的物象「アンプリチューヒドロン」(amplituhedron、「振幅多面体」)に関する研究成果をまとめたものである。

場の理論の散乱振幅は、通常摂動論を用いてファインマン図形を計算して得られるが、それに必要な図形の数は摂動の次数と共に急速に増大するため、高次の散乱振幅を求めるのは非常に困難である。一方、特に N=4 超対称ヤン・ミルズ理論のプラナー散乱振幅に関しては、「アンプリチューヒドロン」と呼ばれる仮想的な複素射影空間内の凸多面体に付随した量に関してある種の積分を実行することにより、摂動計算とは全く異なる手法により枝線散乱振幅ならびに低次ループ振幅の非積分関数を得ることが知られていた。この方法は 2013 年に Arkani-Hamed と Trnka によって発見されたものであり、80 年代から知られていた 6-グルーオン枝線散乱振幅のスピナーヘリシティ変数による簡明化や、今世紀になってからの双対共形対称性・ヤンギアン対称性の発見、運動量ツイスター変数の開発などに基づいていた。

凸多面体から散乱振幅を得るためには、その境界で log 的発散する「正準（canonical）形式」を求める必要がある。小嶋氏は n 点 2-ループの「極大にヘリシティが破れた」(multi-helicity violating, MHV）振幅に対応する凸多面体を、「符号反転」（sign flip）という方法を用いて単体に「三角形分割」し、得られた量の総和を取ることで正準形式を求めた。さらに、この論文では n 点 1-ループ超振幅展開で極大の次の項である（next-to-MHV）振幅に対する新しい表式も求め、その正値性などの性質も議論した。どちらも学術的に十分な価値が認められるものであり、査読付き単名英文論文として JHEP 1904 (2019) 085 に既に掲載されている。

小嶋氏は海外滞在経験もあって英語での学術発表もあり、外国人研究者とも共同研究を行なうなど、英語による研究遂行能力を十分有することが認められる。以上の理由により、審査委員会は全会一致で本論文が学位の授与に値すると判断した。