氏　　名　貝淵 響

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学 位 論 文 題 目　Quantitative Risk Management Using Extreme Value Theory

論 文 審 査 委 員　主　査　二宮 嘉行
統計科学専攻 教授
加藤 昇吾
統計科学専攻 准教授
川﨑 能典
統計科学専攻 教授
塚原 英敦
成城大学 経済学部 教授
吉田 拓真
鹿児島大学 学術研究院 准教授

（様式３）

博士論文の要旨

氏　　名　貝淵　響

論文題目　Quantitative Risk Management Using Extreme Value Theory

This thesis considers about quantitative risk management using Extreme Value Theory (EVT). In this thesis the focus is on the use of EVT to study extreme financial market risk, which is the risk of losses arising from movements in market prices, from a quantitative point of view. This is because quantitative risk management has now become a standard requirement for all financial institutions due to increase in number of extreme market risk events, especially post 1980s. Such events include the Black Monday of 1987, the Dot-Com Bubble of 2000, the Global Financial Crisis of 2007-2008, and the recent COVID-19 recession of 2020. Extreme market events are rare but have high severity. The risk stemming from these extreme events is called tail risk, which contributes to the propagation of deep and unpredictable financial crises. Tail risk is clearly related to extreme events. The estimation of risk measure heavily relies on accurate estimation of a tail of the underlying distribution and hence the use of EVT is natural and effective.

The standards of quantitative risk management are laid down by Basel Committee on Banking Supervision (BCBS). Financial institutions are asked to estimate specific risk measures so that they can protect themselves against future extreme market catastrophes. Risk measures can be understood as providing a risk assessment in the form of capital amount that are set aside to absorb unexpected future losses. Recently, the BCBS announced a change in the risk measure used for capital requirements in internal market risk models, moving from the Value-at-Risk (VaR) to the Expected Shortfall (ES). VaR is defined as a measure of the potential losses on a portfolio of financial instruments resulting from market movements over a given time horizon and for a probability level. Similarly, ES is a measure of the mean of the losses exceeding VaR at a given probability level. The amendment is driven by the fact that VaR could not predict or cover the extreme losses during the turbulence of 2007-2008 crisis and mathematically does not satisfy the important coherence property.

It is no surprise that the switching from VaR to ES has generated many reactions from both the practical sector and the academic sector as evidenced by the numerous literatures. The backtesting approach established by the BCBS, which tests the accuracy of ES estimates, is causing the problem. More specifically, financial institutions now face the paradox of using ES for computing their market risk capital requirements and using VaR for backtesting ES. For this reason, both estimation and backtesting of VaR are still important nowadays because sensible ES estimates are

based on correctly specified VaR estimates by the definition of ES. This was the motivation for the proposal of a two-step bias-reduced conditional EVT approach called GARCH-UGH for the estimation of one-step ahead dynamic extreme VaR. At the same time, there has not been sufficient investigation to establish the superiority of a certain estimator of ES relative to the others in the literature and no particular type of ES model is prescribed in the framework of the BCBS. We thus considered the estimation of dynamic extreme ES based on our proposed GARCH-UGH approach and the use of the first-order asymptotic equivalence between VaR and ES. Moreover, we also tackled an urgent problem of which ES backtesting methods can be used in practice as we can expect that upcoming regulations will require financial institutions to backtest ES without using VaR backtesting methods.

Outline of this thesis is as follows. Chapter 1 is the introduction of this thesis. In Chapter 2, we briefly describe the statistical aspects of EVT focusing on the tail estimation methods for heavy-tail, i.e., Pareto-type, distributions that are the cornerstone of the use of EVT in finance. We review the important concepts of EVT such as extreme value index, extreme value condition and second-order condition. For tail estimation methods, we rely on the heavy-tail property and estimate extreme value index, extreme quantile (VaR) and second-order parameter, which is required for bias-reduction procedures. In particular, we focus on the famous Hill estimator, Weissman quantile estimator, Peaks-Over-Threshold method using the generalized Pareto distribution and Gomes's estimator of second-order parameter for the purpose of introducing our EVT-type method for the estimation of VaR and ES. We also review both unconditional and conditional estimation methods based on EVT, and the limitations of EVT in finance.

In Chapter 3, we tackle the question of estimating the VaR of loss return distribution at extreme levels, which is an important question in financial applications, both from operational and regulatory perspectives. In particular, the dynamic estimation of extreme VaR given the recent past has received substantial attention because the occurrence of extreme financial events has increased since 1980s. Moreover, accurate estimation of VaR is still essential in practice even if the BCBS changed the risk measure for the calculation of capital requirements from VaR to ES. This is because sensible estimation of ES is based on correctly specified VaR estimates. We propose here a new two-step bias-reduced estimation methodology for the estimation of one-step ahead dynamic extreme VaR, called GARCH-UGH (Unbiased Gomes-de Haan), whereby financial returns are first filtered using an AR-GARCH model, and then a bias-reduced estimator of extreme quantiles is applied to the standardized residuals. We analyze the performance of our approach on four financial time series, which are the Dow Jones, Nasdaq and Nikkei stock indices, and the Japanese Yen/British Pound exchange rate. Our results indicate that the GARCH-UGH estimates of the dynamic extreme VaR are more accurate than those obtained either by historical simulation,

conventional AR-GARCH filtering with Gaussian or Student-t innovations, or AR-GARCH filtering with standard extreme value estimates, both from the perspective of in-sample and out-of-sample traditional VaR backtestings, which are the unconditional and conditional coverage tests. The numerical results of comparative VaR backtesting, which is based on the Diebold-Mariano test, also support the use of the GARCH-UGH approach by yielding definitive answers to the cases when GARCH-UGH and GARCH-EVT approaches are either all accepted, or all rejected in the traditional VaR backtestings. In addition, our bias-reduction procedure will be designed to be robust to departure from the independence assumption, and as such will be able to handle residual dependence present after filtering in the first step. Our finite-sample results also illustrate that the GARCH-UGH method leads to one-step ahead extreme conditional VaR estimates that are less sensitive to the choice of sample fraction, and hence mitigates the difficulty in selecting the optimal number of observations for the estimations. Finally, the computational cost of GARCH-UGH is lower than that of conventional GARCH-EVT: the extreme value step in the GARCH-UGH method is semiparametric with an automatic and fast recipe for the estimations of the one-step ahead extreme conditional VaR, while the competing GARCH-EVT method is based on a parametric fit of the Generalized Pareto Distribution (GPD) to the residuals using Maximum Likelihood Estimation.

In Chapter 4, we extend the GARCH-UGH approach used in dynamic extreme VaR estimation to the dynamic extreme ES estimation by means of the asymptotic equivalence between quantile (VaR) and ES. This is motivated by the fact that there has not been sufficient investigation to establish the superiority of a certain estimator of ES relative to the others in the literature and no particular type of ES model is prescribed in the framework of the BCBS. Our results show that the GARCH-UGH approach produces more accurate ES estimates than those obtained by basic estimation methods, both from the perspective of traditional and comparative ES backtestings. We use the exceedance residual test, the conditional calibration test and the expected shortfall regression test for traditional backtestings, and Diebold-Mariano test again based on the joint elicitability of VaR and ES for comparative backtesting. When compared to other EVT-type methods, comparative backtestings with chosen two scoring functions result in a good agreement with the GARCH-UGH approach being the best estimator of ES, while traditional backtestings are not always in line with the superiority of our proposed approach.

Chapter 5 is the conclusion of this thesis. In contrast to the estimation of dynamic extreme ES where most of the existing methods including the ones we referred and proposed for VaR estimation can easily be adapted to the ES, such adaptions are not straight-forward for backtesting ES estimates. Based on the strict definition of backtesting, we understand that a backtesting for specific risk measure should only require its estimates and realized returns as input variables. In contrast to the VaR,

fulfilling this definition for ES is very difficult task because ES is strongly related to the VaR through its definition and joint elicitability. As in every statistical method, each of different ES backtesting methods has its strengths and weaknesses. We thus strongly suggest adopting a two-stage backtesting framework, i.e., the use of both traditional and comparative backtestings for risk measures that will enhance the regulatory framework for financial institutions by providing the correct incentives for accuracy of risk measure estimates. More precisely, the comparative backtesting methods can be used by financial institutions internally to select better performing methods among competing alternatives when traditional backtestings methods do not yield definitive answers as competitive methods are all accepted, or all rejected. Supplementing with comparative backtestings is essential, and hence can adequately quantify the risks even though they still have some drawbacks to consider for the practical use, e.g., there exists no optimal scoring function with any theoretical guarantee. We think that the major challenge of the regulations of BCBS in the implementation of the ES as a risk measure for market risk is the unavailability of simple tools for its evaluation. We also believe that the findings of the estimation and backtesting of risk measures for tail risks in financial extreme market given in Chapter 3 and 4 would be useful for developing regulatory framework of the BCBS and monetary policies aimed at mitigating tail risks.

Form8 : Separate Sheet
（様式８・別紙１）

Results of the doctoral thesis defense

博士論文審査結果

Name in Full
氏 名　　貝淵　響

Title
論文題目　Quantitative Risk Management Using Extreme Value Theory

【論文の概要】

論文は 5 章 213 ページからなり，英語で書かれている．本論文の目的は，金融資産の収益率時系列に対し，条件付分散不均一モデルと極値理論を組み合わせる枠組みで，より高い精度が期待できる新たな手法を提案することである．第 1 章では，金融リスク管理の基本概念に加え，幾つかのリスク尺度とバックテストと呼ばれる検証法を整理し，論文の動機が述べられている．第 2 章では裾指数をはじめとする統計的極値理論の基本事項が述べられ，とりわけ本論文で重要な役割を果たす二次条件に関して説明し，パレート型分布に対する裾指数の推定法をレビューした上で，ファイナンスにおける統計的極値理論の利用法を概観している．第 3 章では，GARCH-UGH 法と名付けた，金融リスク管理の新たな方法が提示され，その有効性が実証分析で示されている．提案手法にとって最も重要な先行研究は，McNeil and Frey (2000, *J Empiric Financ*)の GARCH-EVT 法である．この方法は，収益率（実際にはその符号を変えた損失率時系列）に対して AR(1)-GARCH(1,1)モデルをあてはめて分散不均一性を除去したあと，標準化残差の右裾に一般化パレート分布をあてはめて高分位点を外挿し，AR(1)-GARCH(1,1)モデルと組み合わせて極端な損失の予測を行う．これに対し本論文では，標準化残差の順序統計量に基づく Hill 推定量に対してバイアス補正を行う二次推定量を利用することで，GARCH-EVT 法の極値理論パートの精度を上げる新しい方法を提案した．実証分析では，複数の日次金融時系列に対して 1 期先外挿予測を繰り返し，所与の信頼水準のもとでの期待超過数と経験超過数の比較，超過率とクラスター性に関する仮説検定を行い，GARCH-EVT 法，GARCH-UGH 法，GARCH フィルタなしの UGH 法の 3 つで比較して，60 ケース中 47 ケースで提案手法が分位点予測の観点から最も優れていると結論している．第 3 章がリスク尺度として Value-at-Risk (VaR, 分位点)に焦点をあてたのに対し，第 4 章は期待ショートフォール(ES)，すなわちある分位点より先の裾部分の期待値をリスク尺度に，GARCH-UGH 法の性能を実証分析で比較している．比較の枠組みは，この分野で既存の Residual Exceedance 法，Conditional Calibration 法，ES 回帰法が取り上げられており，結果は概ね VaR での結論を追認するものである．第 5 章は結論と今後の展望となっている．

【論文の評価】

GARCH-EVT 法の登場から 20 年間，GARCH パートを様々な変種で置き換える論文は多かったが，EVT パートを改善する試みはなかった．提案手法は，Hill 推定量のバイアス補正を行う二次推定量を援用し，高分位点における高精度のリスク管理が期待できる新提案として独自性のある貢献と評価できる．

【その他】

第 3 章の内容をまとめた研究論文が，査読付き英文学術誌 Quantitative Finance に採択され，第 22 巻 7 号に掲載されている．また，この研究を様々な角度から担保するための検証が，雑誌には掲載されない形で論文には含まれている．データを生成する真の構造が想定したモデルに含まれていないときの,提案手法の頑健性を数値的に検証したことや,ターゲットとしている金融時系列を用いた実証分析で，裾指数を区間推定することにより提案手法の理論を採用することの妥当性を検証したことは，その一部である．

以上をもち，審査委員会は，本論文が博士（統計科学）の学位を授与するに十分な水準を達成するものであると判定した．